Professor Katiraie Calculus I Spring 2006; Test I

Name:__________________________

(24 Points, Domain of each problem is worth 3 Points, and Range is worth 3 Points)

1) Find the **Domain** and **Range** of the following functions:

a) $f(x) = \sqrt{(4-3x^2)}$

$4-3x^2 \geq 0$

$-3x^2 \geq -4$

$x^2 \leq \frac{4}{3}$

3pts

Domain $\frac{-2}{\sqrt{3}} \leq x \leq \frac{2}{\sqrt{3}}$

3pts

$\boxed{\text{Range } [0,2]}$

b) $g(x) = \ln(\ln(x+5))$

$\boxed{\text{Domain: } x+5>1 \quad x>-4}$ 3pts

$\boxed{\text{Range : } \mathbb{R}}$ 3pts

c) $g(x) = 1 + \frac{1}{\sin x}$

3pts

Domain All Reals except $x = n\pi ; n = 0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots$

Range : $(-\infty, 0] \cup [2, \infty)$

3pts

d) $g(x) = 1 + \frac{1}{x}$

3pts

$\boxed{\text{Domain All } \mathbb{R} \text{ except Zero}}$

$\boxed{\text{Range All } \mathbb{R} \text{ except } 1}$

3pts

(10 Points)

2) The graph of g is given.

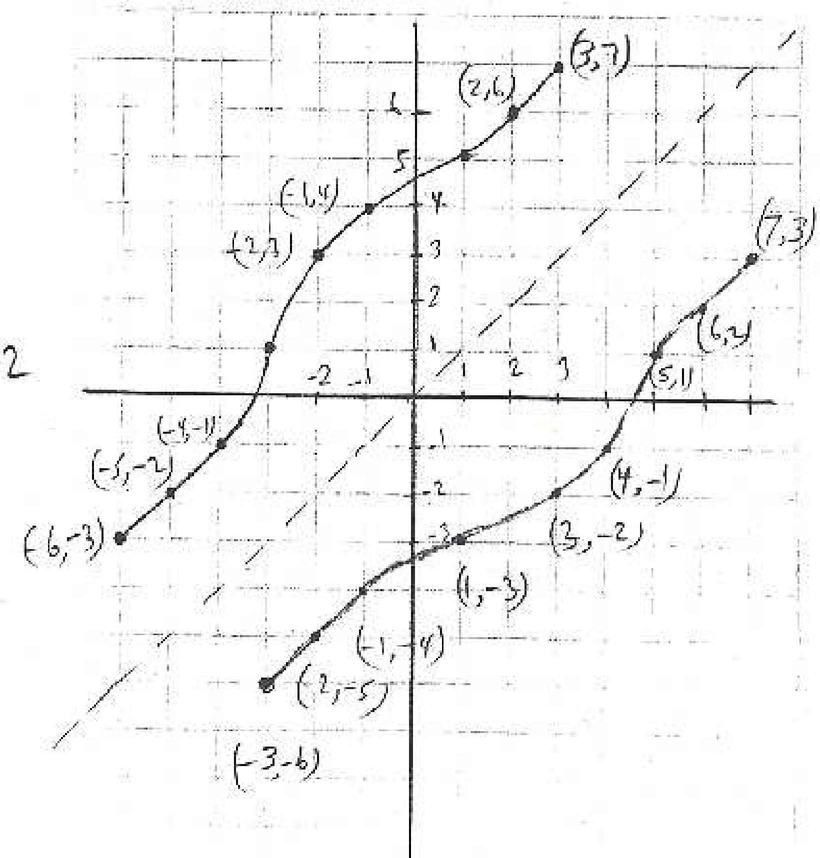
2pts a) State the value of g(2) = 6

2pts b) Is g one-to-one? Yes

2pts c) Estimate the value of $g^{-1}(3)$? −2

2pts d) Estimate the domain of $g^{-1}(x)$ $[-3, 7]$

2pts e) Sketch the graph of $g^{-1}(x)$

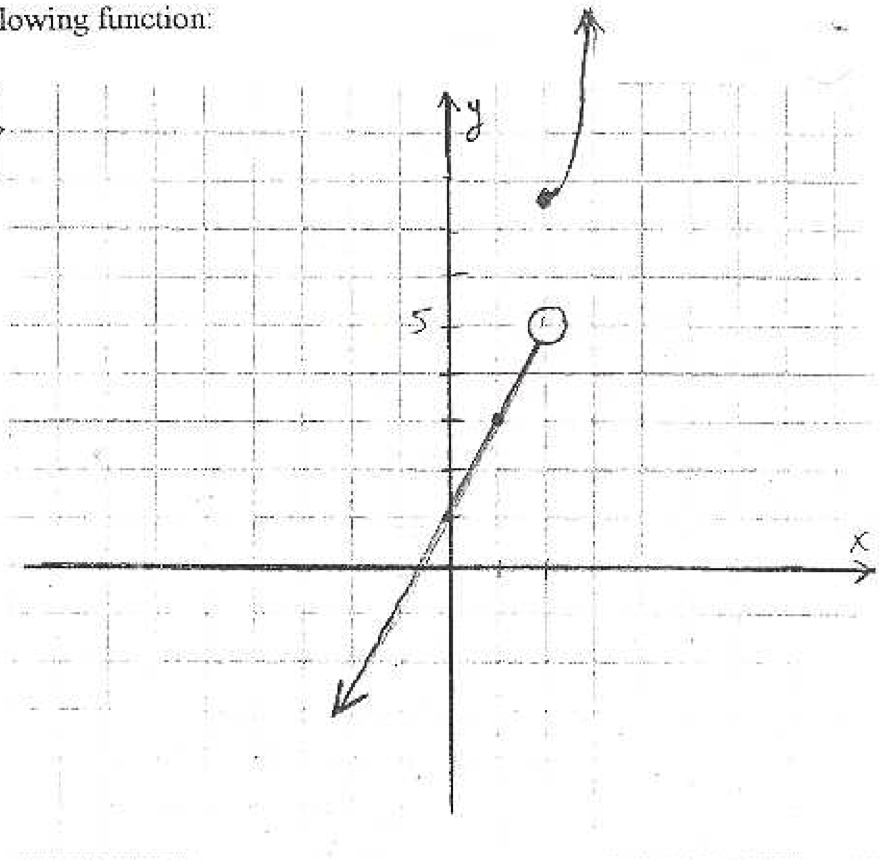


(10 Points) 3) Sketch the graph of the following function:

$$f(x) = \begin{cases} 2x+1 & \text{if } x<2 \\ e^x & \text{if } x \geq 2 \end{cases}$$

X	$y=2X+1$
2	5
1	3
0	1

X	$y=e^X$
2	$e^2 = 7.4$
3	$e^3 = 20.86$



(12 Points) 4) Determine whether f is even, odd, or neither even nor odd;
(Must Use Definition of Even, Odd Functions)

a) $f(x) = 2x^5 - 3x^3 + 2$ $\quad f(-x) = 2(-x)^5 - 3(-x)^3 + 2 = -2x^5 + 3x^3 + 2 \neq f(X)$ or $-f(x)$

3pts Neither

b) $f(x) = e^{-x^2}$

3pts $f(-x) = e^{-(-x)^2} = e^{-x^2} = f(x)$ Even

c) $f(x) = x + \sin(x)$ $\quad f(-x) = -x + \sin(-x) = -x - \sin x$ odd

3pts $= -f(x)$

d) $f(x) = x^4 + 2x^2 + x$ $\quad f(-x) = (-x)^4 + 2(-x)^2 + (-x)$

3pts $= x^4 + 2x^2 - x \neq f(x)$ or $-f(x)$

Neither

(10 Points) 5) A small-appliance manufacturer finds that it costs \$9000 to produce 1000 toaster ovens a week and \$12000 to produce 1500 toaster ovens a week.

a) Express the cost as a function of the number of the toaster ovens produced, assuming that it is linear.

$(1000, \$9000)$ $\Longrightarrow m = 6$

$(1500, \$12000)$

$y = 6x + 3000$

4pts

$C(x) = 6x + 3000$

b) What is the slope of the graph and what does it represent?

3pts

$m = 6$ Every additional toaster costs \$6 to produce.

c) What is the y-intercept of the graph and what does it represent?

$y_{int} = (0, 3000)$ The fixed Cost is \$3000

3pts

(5 Points)

6) If $f(x) = 5x + \log(x+10)$, find $f^{-1}(1)$

5pts let $1 = 5x + \log(x+10)$

Now use Graphing Calculator or try plotting points

$$\boxed{f^{-1}(1) = 0}$$

Because $f(0) = 5(0) + \log(0 + 10) =$

$= 0 + \log 10 = 1$

Since $f(0) = 1$

$f^{-1}(1) = 0$

(10 Points)

7a) Sketch the curve represented by the parametric equation

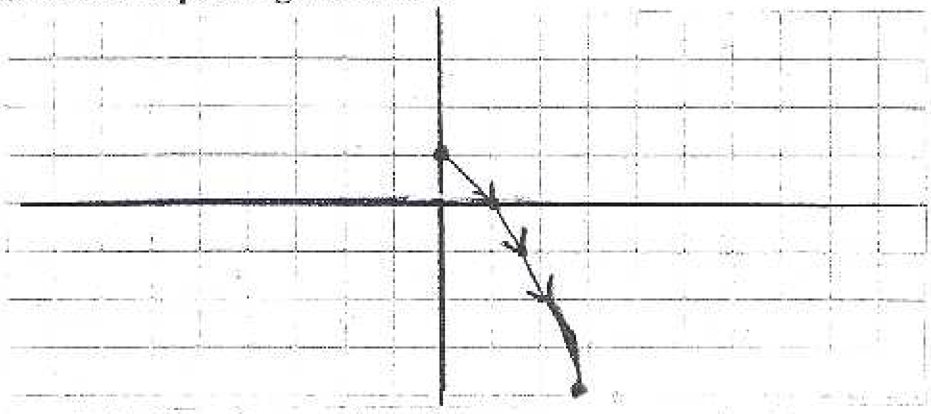
$x = \sqrt{t}, \quad y = 1 - t, \quad 0 \le t \le 5$

Indicate with an arrow the direction in which the curve is traced as t increases.

Make a table of points with the corresponding values of t.

5 pts

t	$x=\sqrt{t}$	$y=1-t$	
0	0	1	(0,1)
1	1	0	(1,0)
2	1.4	-1	
3	1.7	-2	
4	2	-3	
5	2.24	-4	



7b) Eliminate the parameter to find a Cartesian equation of the curve.
(Indicate the Domain and Range of the Cartesian equation)

$0 \le t \le 5$

$t = x^2$

$y = 1 - t = 1 - x^2$

$\boxed{y = 1 - x^2}$

3 pts

$\boxed{\begin{array}{l} 0 \le x \le \sqrt{5} \\ -4 \le y \le 1 \end{array}}$ (1 pt) (1 pt)

(12 Points)

8) Use the following table to evaluate the expressions.

X	1	2	3	4	5	6
f(x)	6	5	4	1	3	5
g(x)	6	3	5	1	2	3

3 pts

a) $f(g(2)) = f(3) = 4$

3 pts

b) $g(g(6)) = g(3) = 5$

3 pts

c) $(g \circ g \circ f)(5) = (g \circ g)(3) = g(5) = 2$

3 pts

d) $(f \circ g \circ f)(6) = (f \circ g)(5) = f(2) = 5$

x	f(x)
	1

(12 Points) 9) Let f be a one-to-one function whose inverse function is given by the formula:

$$f^{-1}(x) = x^5 + 5x^3 + 2x^2 + 1$$

(3pts) a) Compute $f^{-1}(-1) = (-1)^5 + 5(-1)^3 + 2(-1)^2 + 1 = -3$

(3pts) b) Compute $f(1) = 0$ or -0.388 $Y_1 = x^5 + 5x^3 + 2x^2 + 1$ $Y_2 = 1$

(3pts) c) Compute the value of x such that $f(x) = 1$ $\boxed{x = 9}$

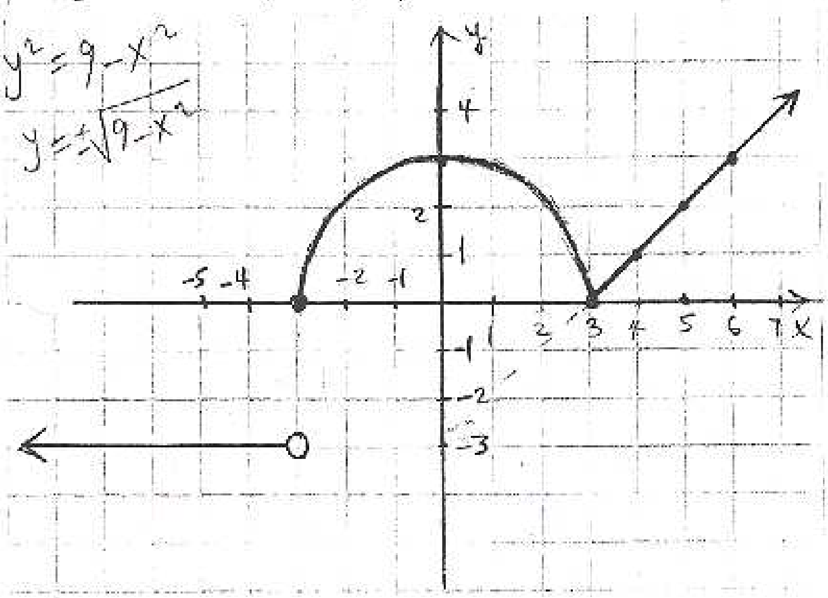
(3pts) d) Compute the value of y such that $f^{-1}(y) = 1$ $y = 0$ or -0.388

$x^2 + y^2 = 3^2$

$y^2 = 9 - x^2$

$y = \pm\sqrt{9 - x^2}$

(12 Points) 10) Find a formula that describes the following function:



$$f(x) = \begin{cases} -3 & x < -3 \\ \sqrt{9 - x^2} & -3 \leq x \leq 3 \\ x - 3 & x > 3 \end{cases}$$

(2pts) (6pts) (4pts)

(12 Points) 11) If $f(x) = 2x^2 - 3x + 1$, find and simplify $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$

$f(x+h) = 2(x+h)^2 - 3(x+h) + 1$

$= 2(x^2 + 2xh + h^2) - 3x - 3h + 1$

$= 2x^2 + 4xh + 2h^2 - 3x - 3h + 1$ (4pts)

$f(x+h) - f(x) = (2x^2 + 4xh + 2h^2 - 3x - 3h + 1) - (2x^2 - 3x + 1) =$

$= \cancel{2x^2} + 4xh + 2h^2 - \cancel{3x} - 3h + \cancel{1} - \cancel{2x^2} + \cancel{3x} - \cancel{1} = h(4x + 2h - 3)$ (4pts)

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{\cancel{h}(4x + 2h - 3)}{\cancel{h}} = 4x + 2h - 3$ (4pts)

(24 Points)

12) Given $f(x) = \ln(x)$ and $g(x) = x^2 - 9$,

Find the following and State their Domains:

6pts a) $fog(x) = \ln(x^2-9) = \ln(x+3)(x-3) = \ln(x+3) + \ln(x-3)$

$\boxed{\text{Domain } x > 3}$ and $x < -3$

6pts b) $gof(x)$ $\boxed{= (\ln(x))^2 - 9 \quad \text{Domain } x > 0}$

6pts c) $fof(x)$ $\boxed{\ln(\ln x)) ; \text{ Domain } x > 1}$

6pts d) $gog(x)$ $(x^2-9)^2 - 9 = x^4 - 18x^2 + 81 - 9 = \boxed{x^4 - 18x^2 + 72 \quad \text{Domain All Reals}}$

Extra Credits:

(6 points)

13) Find an expression for the function whose graph consists of the line segment from the point (-2,2) to the point (-1,0) together with the top half of the circle with the center at the origin and radius 1.

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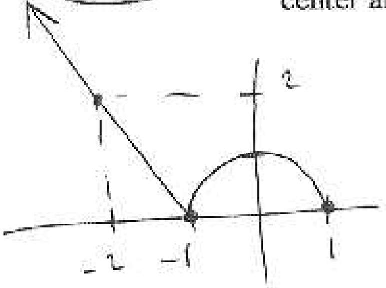

$$f(x) = \begin{cases} -2x-2 & x \leq -1 \\ \sqrt{1-x^2} & -1 < x \leq 1 \end{cases}$$

3pts

3pts

Extra Credits:

(4 points)

14) Express the function $F(x) = \dfrac{1}{\sqrt{x+\sqrt{x}}}$

as a composition of three functions (namely (fogoh)(x)).

(Hint: Find f(x), g(x), and h(x) so that (fogoh)(x) = $\dfrac{1}{\sqrt{x+\sqrt{x}}}$)

4pts

let $\begin{cases} f(x) = \dfrac{1}{x} \\ g(x) = \sqrt{x} \\ h(x) = x + \sqrt{x} \end{cases}$ then $\boxed{(f \circ g \circ h)(x) = \dfrac{1}{\sqrt{x+\sqrt{x}}}}$