Group Work 1, Section 3.1
Doing a Lot with a Little

Section 3.1 introduces the Power Rule: $\frac{d}{dx}x^n = nx^{n-1}$, where n is any real number. The good news is that this rule, combined with the Constant Multiple and Sum Rules, allows us to take the derivative of even the most formidable polynomial with ease! To demonstrate this power, try Problem 1:

1. *A formidable polynomial:*

$$f(x) = x^{10} + \tfrac{7}{9}x^9 + \tfrac{1}{2}x^8 - 5x^7 - 0.33x^6 + \pi x^5 - \sqrt{2}x^4 - 42$$

Its derivative:

$$f'(x) = 10x^9 + 7x^8 + 4x^7 - 35x^6 - 2x^5 + 5\pi x^4 - 4\sqrt{2}\,x^3 + 0$$

The ability to differentiate polynomials is only one of the things we've gained by establishing the Power Rule. Using some basic definitions, and a touch of algebra, there are all kinds of functions that can be differentiated using the Power Rule.

$h(x) = x^{\frac{9}{2}} - 3 + 2x^{-\frac{1}{2}}$

2. *All kinds of functions:*

$f(x) = \sqrt[3]{x} + \sqrt[5]{2}$ $\qquad g(x) = \frac{1}{x^3} - \frac{1}{\sqrt[4]{x^3}}$ $\qquad h(x) = \frac{x^5 - 3\sqrt{x} + 2}{\sqrt{x}}$ $\qquad j(x) = e^e - x^e$

Their derivatives:

$f(x) = x^{\frac{1}{3}} + \sqrt[5]{2}$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$

$g(x) = x^{-3} - x^{-\frac{3}{4}}$

$g'(x) = -3x^{-4} + \frac{3}{4}x^{-\frac{7}{4}}$

$= \frac{-3}{x^4} + \frac{3}{4x^{\frac{7}{4}}}$

$h'(x) = \frac{9}{2}x^{\frac{7}{2}} - 1x^{-\frac{3}{2}} = \frac{9}{2}x^{\frac{7}{2}} - \frac{1}{x^{3/2}}$

$j'(x) = 0 - ex^{e-1} = -ex^{e-1}$

Unfortunately, there are some deceptive functions that look like they should be straightforward applications of the Power and Constant Multiple Rules, but actually require a little thought.

3. *Some deceptive functions:*

$f(x) = (2x)^4 = 16x^4$ $\qquad g(x) = (x^3)^5 = x^{15}$

Their derivatives:

$f'(x) = 4(2x)^3 * 2$

$= 64x^3$

$g'(x) = 15x^{14}$

$5(x^3)^4 \cdot 3x^2 = 15x^{14}$

The process you used to take the derivative of the functions in Problem 3 can be generalized. In the first case, $f(x) = (2x)^4$, we had a function that was of the form $(kx)^n$, where k and n were constants ($k = 2$ and $n = 4$). In the second case, $g(x) = (x^3)^5$, we had a function of the form $(x^k)^n$. Now we are going to find a pattern, similar to the Power Rule, that will allow us to find the derivatives of these functions as well.

4. Show that your answers to Problem 3 can also be written in this form:

$$f'(x) = 4(2x)^3 \cdot 2 \qquad\qquad g'(x) = 5(x^3)^4 \cdot 3x^2$$

And now it is time to generalize the Power Rule. Consider the two general functions, and try to find expressions for the derivatives similar in form to those given in Problem 4. You may assume that n is an integer.

5. *Two general functions*:

$$f(x) = (kx)^n \qquad\qquad g(x) = (x^k)^n$$

Their derivatives:

$f'(x) = n(Kx)^{n-1} \cdot K$

$g'(x) = n(x^K)^{n-1} \cdot Kx^{K-1}$

Group Work 1, Section 3.2
Back and Forth (Form A)

Compute the following derivatives. Write your answers at the bottom of this sheet, where indicated. When finished, fold the top of the page backward along the dotted line and hand to your partner.

DO NOT SIMPLIFY.

1. $f(x) = 5x^4 + \frac{3}{2}x^2 - 4$

2. $g(x) = 2\sqrt{x} - 4\sqrt[4]{x}$ $= 2x^{\frac{1}{2}} - 4x^{\frac{1}{4}} \Rightarrow f'(x) = x^{\frac{1}{2}-1} - 4 \cdot \frac{1}{4} x^{\frac{1}{4}-1} = x^{-\frac{1}{2}} - x^{-\frac{3}{4}}$

$= \frac{1}{x^{\frac{1}{2}}} - \frac{1}{x^{3/4}}$

3. $h(x) = 2xe^x$ $h'(x) = (2x)'e^x + (2x)(e^x)' = 2e^x + 2xe^x$

4. $j(x) = \dfrac{x^4 - 4x + 3}{e^x + 1}$; $j'(x) = \dfrac{f'g - g'f}{g^2} = \dfrac{(4x^3-4)(e^x+1) - e^x(x^4-4x+3)}{(e^x+1)^2}$

5. $k(x) = \dfrac{3}{\sqrt[3]{x}} + 42$ $= 3x^{-\frac{1}{3}} + 42$; $k'(x) = -\frac{1}{3} \cdot 3x^{-\frac{1}{3}-1} = -1x^{-\frac{4}{3}} = \dfrac{-1}{x^{4/3}}$

ANSWERS:

$f'(x) = 20x^3 + 3x$

$g'(x) = \dfrac{1}{x^{1/2}} - \dfrac{1}{x^{3/4}}$

$h'(x) = 2e^x + 2xe^x$

$j'(x) = \dfrac{(4x^3-4)(e^x+1) - e^x(x^4-4x+3)}{(e^x+1)^2}$

$k'(x) = -1x^{-\frac{4}{3}} = \dfrac{-1}{x^{4/3}}$

Group Work 1, Section 3.2
Back and Forth (Form B)

Compute the following derivatives. Write your answers at the bottom of this sheet, where indicated. When finished, fold the top of the page backward along the dotted line and hand to your partner. DO NOT SIMPLIFY.

1. $f(x) = -2x^3 + \frac{\sqrt{8}}{2}x^2 - 8$ $\Rightarrow f'(x) = -6x^2 + \sqrt{8}\,x^1$

2. $g(x) = \dfrac{xe^x + 6x^2}{5}$ $= \frac{1}{5}(xe^x + 6x^2)$; $g'(x) = \frac{1}{5}\left[1e^x + xe^x + 12x^1\right]$

3. $h(x) = \underbrace{(x^3 + x^2 + 2x)}_{f}\,\underbrace{(5x^2 - 2x^4 + 8x)}_{g}$ $= \underbrace{(3x^2+2x+2)}_{f'}\underbrace{(5x^2-2x^4+8x)}_{g} + \underbrace{(10x-8x^3+8)}_{g'}\underbrace{(x^3+x^2+2x)}_{f}$

4. $j(x) = \dfrac{e^x}{\sqrt{x}}$ $\Rightarrow j(x) = e^x \cdot x^{-\frac{1}{2}}$

5. $k(x) = \sqrt{11} - 22e^x$

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ANSWERS:

$f'(x) =$ $-6x^2 + \sqrt{8}\,x$

$g'(x) =$ $\frac{1}{5}\left[1e^x + xe^x + 12x\right]$

$h'(x) =$ $(3x^2+2x+2)(5x^2-2x^4+8x) + (10x-8x^3+8)(x^3+x^2+2x)$

$j'(x) =$ $e^x \cdot x^{-\frac{1}{2}} + -\frac{1}{2}x^{-\frac{3}{2}}e^x = \dfrac{e^x}{\sqrt{x}} - \dfrac{e^x}{2x^{3/2}}$

$k'(x) =$ $0 - 22e^x = -22e^x$

Group Work 2, Section 3.2
Sparse Data

Assume that $f(x)$ and $g(x)$ are differentiable functions about which we know very little. In fact, assume that all we know about these functions is the following table of data:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-2	3	1	-5	8
-1	-9	7	4	1
0	5	9	9	-3
1	3	-3	2	6
2	-5	3	8	?

This isn't a lot of information. For example, we can't compute $f'(3)$ with any degree of accuracy. But we are still able to figure some things out, using the rules of differentiation.

1. Let $h(x) = e^x f(x)$. What is $h'(0)$?

$h'(x) = e^x \cdot f(x) + e^x \cdot f'(x)$

$h'(0) = e^0 \cdot f(0) + e^0 f'(0) = 5 + 9 = \boxed{14}$

2. Let $j(x) = -4f(x)g(x)$. What is $j'(1)$?

$j'(x) = -4f'(x)g(x) + -4f(x)g'(x)$

$j'(1) = -4f'(1)g(1) + -4f(1)g'(1)$

$= -4(-3)(2) + -4(3)(6) = 24 - 72 = \boxed{-48}$

3. Let $k(x) = \dfrac{xf(x)}{g(x)}$. What is $k'(-2)$?

$k'(x) = \dfrac{[1 \cdot f(x) + x f'(x)]g(x) - g'(x) \cdot x f(x)}{g^2(x)}$

$k'(-2) = \dfrac{[f(-2) - 2f'(-2)]g(-2) - g'(-2) \cdot (-2) f(-2)}{[g(-2)]^2} = \dfrac{[3 - 2(1)](-5) + 2(8)(3)}{(-5)^2}$

$= \boxed{\dfrac{43}{25}}$

4. Let $l(x) = x^3 g(x)$. If $l'(2) = -48$, what is $g'(2)$?

$l'(x) = 3x^2 g(x) + x^3 g'(x)$

$l'(2) = 3(2)^2 g(2) + (2)^3 g'(2)$

$-48 = 12(8) + 8(g'(2))$

$\boxed{g'(2) = -18}$

5. Let $m(x) = \dfrac{1}{f(x)^3}$. What is $m'(-1)$?

$$m'(x) = \frac{\cancel{0(f(x^3))} - f'(x^3)\ 3x^2(1)}{[f(x^3)]^2}$$

$$m'(-1) = \frac{-f'(-1)\cdot 3(-1)^2}{[f(-1)]^2} = \frac{-7\cdot 3(-1)^2}{(-9)^2}$$

$$= \frac{-21}{81} = \boxed{\frac{-7}{27}}$$

6. Let $n(x) = x^2 f(x)\, g(x)$. What is $n'(1)$?

$$n'(x) = 2x\,[f(x)g(x)] + x^2\,[f'(x)g(x) + f(x)g'(x)]$$

$$n'(1) = 2(1)\,[f(1)g(1)] + (1)^2\,[f'(1)g(1) + f(1)g'(1)]$$

$$= 2[(3)(2)] + [(-3)(2) + (3)(6)]$$

$$= 2\cdot 6 + [-6 + 18]$$

$$= 12 + 12 \ \boxed{= 24}$$

Group Work 1, Section 3.3
Follow That Particle!

A particle moves according to the position function $f(t) = \frac{1}{3}t^3 - t^2 - 4t$, where t is measured in seconds and f in feet.

Answer the following questions. You can visualize this motion and verify many of your answers using graphing technology. However, do not look at the graph until you have attempted all of the problems by hand.

1. What is the position of the particle at $t = 1$, $t = 3$, $t = 5$, $t = 6$?

$t=1$, $y=-4\frac{2}{3}$ $t=3$, $y=-12$ $t=5$, $y=-3\frac{1}{3}$ $t=6$, $y=12$

2. Find the velocity of the particle at time t. What is the velocity at $t = 1$, $t = 3$, $t = 4$?

$v(t) = t^2 - 2t - 4$

$v(1) = -5$ ft/sec $v(3) = -1$ ft/sec $\boxed{v(4) = 16-8-4 = 4 \text{ ft/sec}}$

3. When is the particle at rest? When is the particle moving forward?

$v(t)=0 \Rightarrow t^2-2t-4=0$ $t = \frac{--2 \pm \sqrt{(-2)^2-4(1)(-4)}}{2} = \frac{2 \pm \sqrt{20}}{2}$

After 3.24 Seconds $= \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}$ 3.24 sec, $\cancel{-1.24 \text{ sec}}$

4. Find the total distance traveled by the particle on the intervals [0, 2] and [2, 4]. Which is larger and why? [0, 2]

$t=0$ $y=0$, $t=2$ $y=-9\frac{1}{3}$ total distance $= 9\frac{1}{3}$ feet

$t=2$ $y=-9\frac{1}{3}$, $t=4$ $y=-10.67$, $t=3.24$ $y=-12.42$

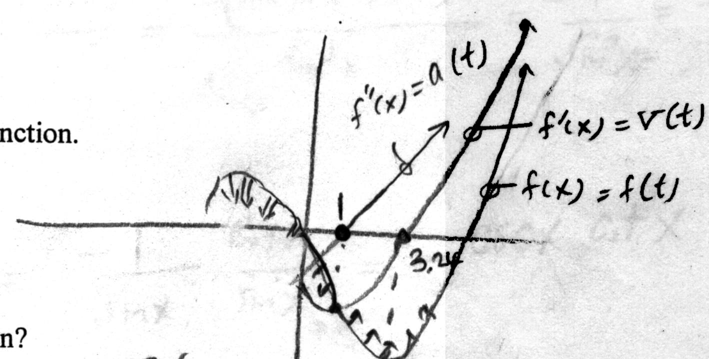
total Distance $= |12.42 - 9\frac{1}{3}| + |12.42 - 10.67| =$ ~~[illegible]~~ $= 4.83$ feet

time can not be negative

5. Find the acceleration of the particle at time t.

$a(t) = 2t - 2$

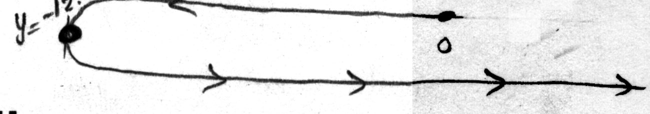
6. Draw a rough graph of the particle's position function.



7. When is the particle speeding up? Slowing down?

* speeding up $(0, 1)$ & $(3.24, \infty)$ $t = 3.24$

Slowing down $(1, 3.24)$



Group Work 4, Section 3.4
The Magnificent Six

The derivative of $f(x) = \sin x$ was derived for you in class. From this one piece of information, it is possible to figure out formulas for the derivatives of the other five trigonometric functions. Using the trigonometric identities you know, and the Product and Quotient Rules, compute the following derivatives. Simplify your answers as much as possible.

1. $(\sin x)' = \lim\limits_{h\to 0} \dfrac{\sin(x+h)-\sin x}{h} = \lim\limits_{h\to 0}\dfrac{\sin x\cos h + \cos x\sin h - \sin x}{h}$

$$= \lim_{h\to 0}\frac{\sin x(\cos h - 1)}{h} + \lim_{h\to 0}\cos x\left(\frac{\sin h}{h}\right) = \boxed{\cos x}$$

2. $(\cos x)' = \lim\limits_{h\to 0}\dfrac{\cos(x+h)-\cos x}{h} = \lim\limits_{h\to 0}\dfrac{\cos x\cos h - \sin x\sin h - \cos x}{h}$

$$\lim_{h\to 0}\frac{\cos x(\cos h - 1)}{h} - \sin x\lim_{h\to 0}\frac{\sin h}{h} = \boxed{-\sin x}$$

3. $(\tan x)' = \left(\dfrac{\sin x}{\cos x}\right)' = \dfrac{\cos x\cdot\cos x - \sin x\sin x}{\cos^2 x} = \dfrac{1}{\cos^2 x} = \sec^2 x$

4. $(\sec x)' = \left(\dfrac{1}{\cos x}\right)' = \dfrac{0.\cos x - \sin x}{\cos^2 x} = \dfrac{1}{\cos x}\cdot\dfrac{\sin x}{\cos x} = \sec x\tan x$

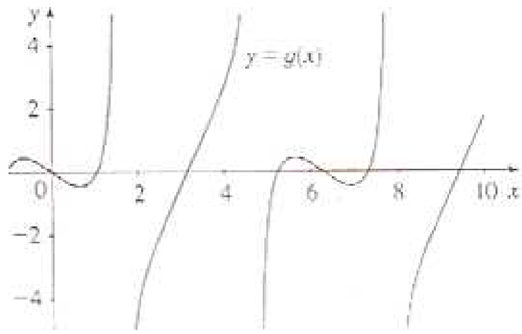
5. $(\cot x)' = \left(\dfrac{\cos x}{\sin x}\right)' = \dfrac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{\sin^2 x} = \dfrac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = \dfrac{-1}{\sin^2 x} = -\csc^2 x$

6. $(\csc x)' = \left(\dfrac{1}{\sin x}\right)' = \dfrac{0.\sin x - \cos x}{\sin^2 x} = -\dfrac{1}{\sin x}\,\dfrac{\cos x}{\sin x} = -\csc x\cot x$

Group Work 5, Section 3.4
Using Our New Knowledge

x	$g(x) = \tan x - 2\sin x$
0	0
π	0
2π	0

The following is a graph of $g(x) = \tan x - 2\sin x$.



1. Compute $g'(0)$, $g'(\pi)$, and $g'(2\pi)$.

$g'(x) = \sec^2 x - 2\cos x$

$g'(0) = \left(\frac{1}{\cos 0}\right)^2 - 2\cos 0 = 1 - 2 = -1$

$g'(\pi) = \left(\frac{1}{\cos \pi}\right)^2 - 2\cos \pi = 1 - 2 = 3$

$g'(2\pi) = \left(\frac{1}{\cos 2\pi}\right)^2 - 2\cos 2\pi = 1 - 2 = -1$

2. Find equations of the lines tangent to this curve at $x = 0$, $x = \pi$, and $x = 2\pi$.

$(0,0)$ $y - 0 = -1(x - 0) \Rightarrow \boxed{y = -x}$

$(\pi, 0)$ $y - 0 = 3(x - \pi) \Rightarrow \boxed{y = 3x - 3\pi}$

$(2\pi, 0)$ $y - 0 = -1(x - 2\pi) \Rightarrow \boxed{y = -1x + 2\pi}$

3. Graph the equations you found in Problem 2, and make sure they look as they should.

4. What happens when you try to find the equation of the line tangent to this curve at $x = \frac{\pi}{2}$? Explain your answer.

$g'\left(\frac{\pi}{2}\right) = $ Undefined