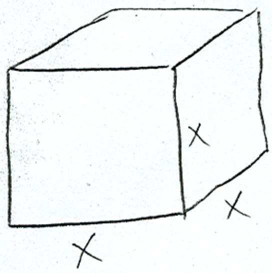
Section 4.1 Work Sheet 3 let x = inch time = sec



$$V = x \cdot x \cdot x = x^3 \quad ; \quad S = 6x^2$$

$$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt} \quad ; \quad \frac{dS}{dt} = 12x \cdot \frac{dx}{dt}$$

a) $\frac{dS}{dt} = (12)(2) \cdot \frac{dx}{dt}$ But $\frac{dx}{dt} = \frac{dV/dt}{3x^2} = \frac{1}{3(2)^2} = \frac{1}{12}$

$$\frac{dS}{dt} = 24 \cdot \frac{1}{12} = 2 \left(\frac{\text{unit}^2}{\text{time}}\right) = 2\left(\frac{\text{inch}^2}{\text{sec}}\right)$$

b) $\frac{dV}{dt} = (3)(2)^2 \cdot \frac{dx}{dt}$ But $\frac{dx}{dt} = \frac{\frac{dS}{dt}}{12x} = \frac{1}{(12)(2)} = \frac{1}{24}$

$$\frac{dV}{dt} = 12 \left(\frac{1}{24}\right) = \frac{1}{2}\left(\frac{\text{unit}^3}{\text{time}}\right) = \boxed{\frac{1}{2} \cdot \frac{\text{inch}^3}{\text{sec}}}$$

#2 Suppose an ice cube of original side length 1 is melting at a rate of 0.01 $\frac{in^3}{sec}$.

$X = 1$ inches

$$\frac{dV}{dt} = -0.01 \frac{in^3}{sec}$$

a) What is the rate of change of side length when the length is 0.5?

$\frac{dx}{dt} = ?$ when $x = 0.5$ inches

$V = x^3$ and $\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$$\Downarrow$$

$$-0.01 = 3(0.5)^2 \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = -0.01\overline{33}$$

$$= \boxed{\frac{-1}{75} \frac{in}{sec}}$$

When the length is 0.1?

$$-0.01 = 3(0.1)^2 \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{-0.01}{3(0.1)^2} = \boxed{\frac{-1}{3} \frac{in}{sec}}$$

When the length is 0.01?

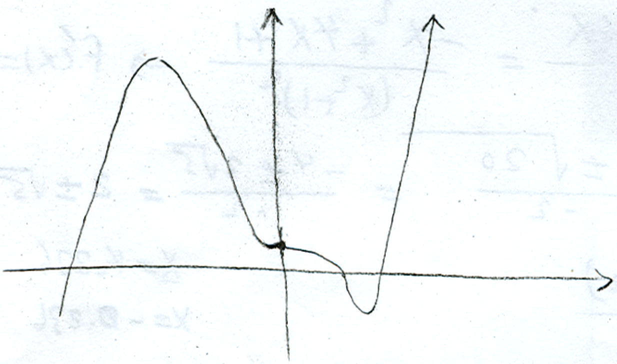
$$-0.01 = 3(0.01)^2 \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{-0.01}{3(0.01)^2} = \boxed{-33\frac{1}{3} \frac{inch}{sec}}$$

Group Work 3, Section 4.2
The Little Dip

Consider $f(x) = \frac{1}{5}x^5 + x^4 - 4x^3 + 3$.

1. Draw a graph of f. Estimate all critical points, local extrema, and absolute extrema.



critical Points $x = 2$ $y = -6.6$

$x = 0$ $y = 3$

$x = -6$ $y = 607.8$

2. Compute the actual values of all critical points, local extrema, and absolute extrema using Fermat's Theorem.

$$f'(x) = 5\left(\frac{1}{5}x^4\right) + 4x^3 - 12x^2 + 0$$

$$\Rightarrow x^4 + 4x^3 - 12x^2 = 0$$

Also $f''(x) = 4x^3 + 12x^2 - 24$

$$x^2(x^2 + 4x - 12) = 0$$

$$x^2(x+6)(x-2) = 0$$

$x = 0$ $x = -6$ $x = 2$

$y = 3$ $y = 607.8$ $y = -6.6$

x	y	y'	y''	Max/
0	3	0	0	Infle Pint
-6	607.8	0	-288	Local
2	-6.6	0	32	Local max

Group Work 1, Section 4.2
Maxima and Minima

1. Find all local and absolute extrema for the function $f(x) = \dfrac{x-2}{x^2+1}$ on the interval $[-1, 2]$.

$$f'(x) = \frac{1(x^2+1) - 2x(x-2)}{(x^2+1)^2} = \frac{x^2+1-2x^2+4x}{(x^2+1)^2} = \frac{-x^2+4x+1}{(x^2+1)^2} \Rightarrow f'(x)=0$$

$$-x^2+4x+1=0 \qquad x = \frac{-4 \pm \sqrt{4^2-4(-1)(1)}}{2(-1)} = \frac{-4\pm\sqrt{20}}{-2} = \frac{-4\pm 2\sqrt{5}}{-2} = 2\pm\sqrt{5}$$

$$f''(x) = \frac{(-2x+4)(x^2+1)^2 - 2(x^2+1)(2x)(-x^2+4x+2)}{(x^2+1)^4}$$

$X = 4.236$

$X = -0.236$

x	y	y'	y''	max / min
4.236	Not in the Interval			
-0.236	-2.118	0	+	Minimum (Local & Absolute Minimum)

2. One model for the food-price index (the price of a representative "basket" of foods) between 1984 and 1994 is the function

$$I(t) = 0.00009045t^5 + 0.001438t^4 - 0.06561t^3 + 0.4598t^2 - 0.6270t + 99.33$$

For this model, t is measured in years since midyear 1984 (so $0 \le t \le 10$) and $I(t)$ is measured in 1987 dollars (hence scaled so that $I(3) = 100$). Estimate the times when food was cheapest during the period 1984–1994. Then estimate the times when food was most expensive.

1984 + 10 1994

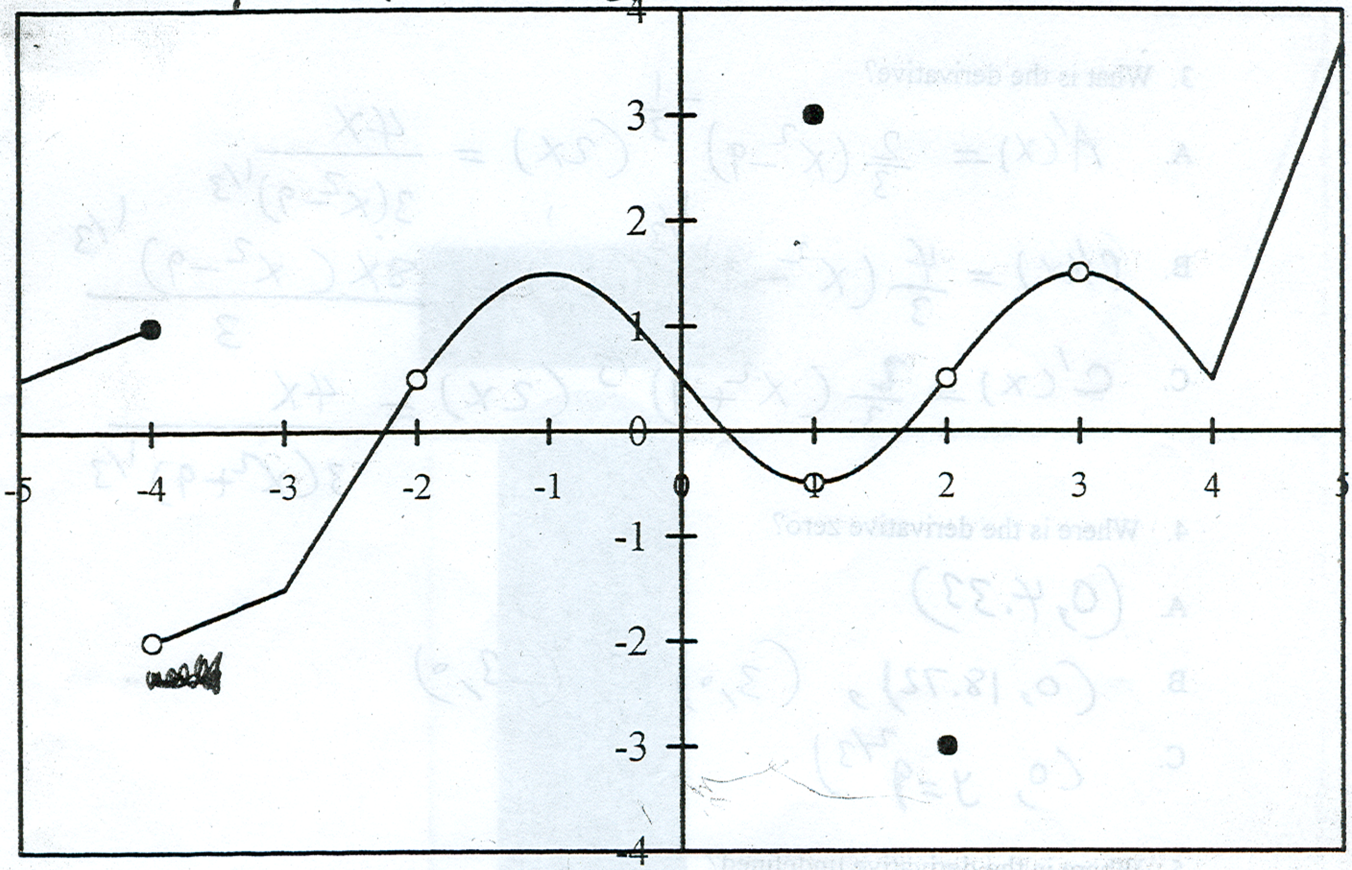
Minimum occured in ~~1991~~; So the food was cheapest in ~~1991~~ and it was \$ ~~99.10~~ per Basket. \$96.86

1984 + 5 = 1989

Maximum occured in ~~1995~~; So the food was most expensive in ~~1995~~ 1989 and it was about \$100.67 per Basket.

See page 272 Def. of Critical Numbers

Def. of Critical Number of f is a Number C in the Domain of f s.t $f'(C)=0$ or $f'(C)$ DNE



1. In the function above, is there a critical point at the x value of

a. -4 b. -3 c. -2 d. -1 e. 0

Since f(x) = ? Not in the Domain

f. 1 g. 2 h. 3 i. 4

f(1) = ~~0~~

See page 162

Consider the functions

$$A(x)=\left(x^2-9\right)^{\frac{2}{3}} \qquad B(x)=\left(x^2-9\right)^{\frac{4}{3}} \qquad C(x)=\left(x^2+9\right)^{\frac{2}{3}}$$

For each function, answer the questions 2 through 6.

2. Where is the function zero?

A. $X=\pm 3,\ y=0$

B. $X=\pm 3,\ y=0$

C. None

3. What is the derivative?

A. $A'(x) = \frac{2}{3}(x^2-9)^{-\frac{1}{3}}(2x) = \frac{4x}{3(x^2-9)^{1/3}}$

B. $B'(x) = \frac{4}{3}(x^2-9)^{1/3}(2x) = \frac{8x(x^2-9)^{1/3}}{3}$

C. $C'(x) = \frac{2}{3}(x^2+9)^{-1/3}(2x) = \frac{4x}{3(x^2+9)^{1/3}}$

4. Where is the derivative zero?

A. (0, 4.33)

B. (0, 18.72), (3, 0) & (−3, 0)

C. $(0,\ y = 9^{2/3})$

5. Where is the derivative undefined?

A. $x = \pm 3$

B. None

C. None

6. Where are the critical points of the functions A, B, and C?

A. (0, 4.33) (x = 3, 0) (x = −3, 0)

B. (0, 18.72) (3, 0) (−3, 0)

C. $(0, 9^{2/3})$

7. Explain the difference between the appearance of the functions A and B at the point x = 3. (Hint: graph it and zoom in.)