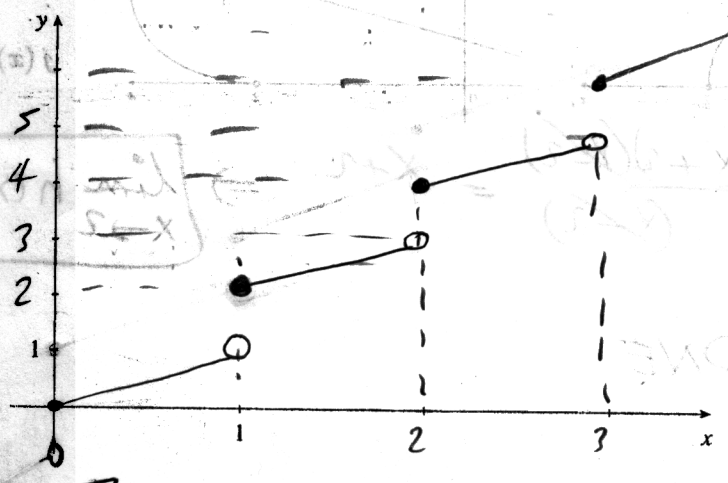
Group Work 1, Section 2.3
Exploring Limits

1. Consider the function $g(x) = x + [\![x]\!]$.

(a) Sketch a graph of $y = g(x)$, $0 \le x \le 3$.



(b) What is $\lim_{x \to 2^-} g(x)$? 3

(c) What is $\lim_{x \to 2^+} g(x)$? 4

(d) Does $\lim_{x \to 2} g(x)$ exist? If so, what is the value of this limit? If not, why not?

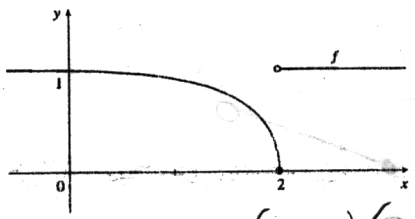
$\lim_{x \to 2} g(x) = DNE$

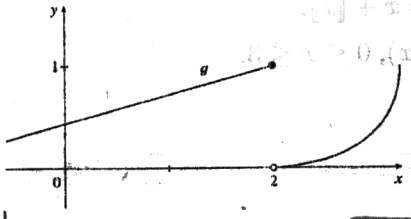
(e) For what values of a with $0 \le a \le 3$ is $\lim_{x \to a} g(x) = g(a)$?

$0 < a < 1 \cup 1 < a < 2 \cup 2 < a < 3$

$(0,1) \cup (1,2) \cup (2,3)$

2. Given the functions f and g (defined graphically below) and h and j (defined algebraically), compute each of the following limits, or state why they don't exist:





$$h(x) = \frac{x^2 - 4}{x - 2}$$

$$j(x) = \begin{cases} 1 & \text{if } x < 2 \\ 0 & \text{if } x \geq 2 \end{cases}$$

(a) $\lim_{x \to 2} h(x) = \dfrac{(x+2)(x-2)}{(x-2)} = x+2 \Rightarrow \lim_{x \to 2} h(x) = 4$

(b) $\lim_{x \to 2} g(x) = DNE$

(c) $\lim_{x \to 2} [g(x) + h(x)] = DNE$

(d) $\lim_{x \to 2} [f(x)\, g(x)] = DNE$

(e) $\lim_{x \to 2} [g(x) - j(x)] = DNE$

3. (a) In general, if $\lim_{x \to a} m(x)$ exists and $\lim_{x \to a} n(x)$ exists, is it true that $\lim_{x \to a} [m(x) + n(x)]$ exists? How about $\lim_{x \to a} [m(x)\, n(x)]$? Justify your answers.

Yes ; Both exist

(b) In general, if $\lim_{x \to a} m(x)$ does not exist and $\lim_{x \to a} n(x)$ does not exist, is it true that $\lim_{x \to a} [m(x) + n(x)]$ does not exist? How about $\lim_{x \to a} [m(x)\, n(x)]$? Compare these with your answers to part (a).

Yes Yes