Group Work 3, Section 1.6
Irrational, Impossible Relations

1. If $\log_2 x = s$, then what is $\log_{1/2} x$?

$$2^s = x$$

$$\frac{\log x}{\log 2} = s \Rightarrow \log x = s\log 2$$

$$\log_{\frac{1}{2}} x = \frac{\log x}{\log 1/2} = \frac{s \log 2}{\log 1/2}$$

$$\boxed{\log_{\frac{1}{2}} x = -s}$$

2. If $\log_b x = s$, then what is $\log_{1/b} x$ (assuming $b > 1$)?

$$\log_b x = s \Rightarrow \frac{\log x}{\log b} = s \Rightarrow \log x = s \log b$$

$$\log_{\frac{1}{b}} x = \frac{\log x}{\log 1/b} = \frac{s \log b}{\log \frac{1}{b}} = \boxed{-s}$$

3. If $\log_b x = s$, then what is $\log_{b^2} x$?

$$\log_b x = s \Rightarrow \frac{\log x}{\log b} = s \Rightarrow \log x = s \log b$$

$$\log_{b^2} x = \frac{\log x}{\log b^2} = \frac{s \log b}{\log b^2} = \boxed{\frac{1}{2} s}$$

We are going to estimate $\log_2 3$. In pre-calculus, you memorized that $\log_2 3 \approx 1.584962501$. Suppose you didn't have this fact memorized. There is no $\log_2$ button on your calculator! How would you compute it?

$$\log_2 3 = \frac{\log 3}{\log 2} \text{ OR } \frac{\ln 3}{\ln 2}$$

Unfortunately, the calculator gives us only a finite number of digits. If $\log_2 3$ were a rational number, we would be able to express it as a fraction, giving us perfect accuracy. Do you think it is rational or irrational? Try to prove your result.

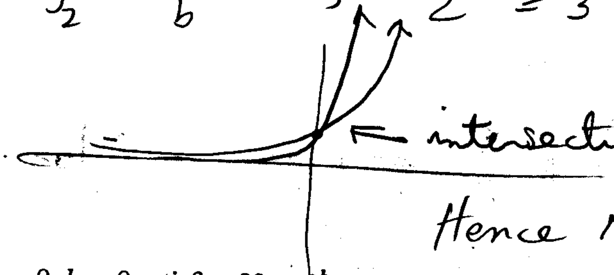
$$\log_2 3 = 1.584962501$$ irrational ~~rational~~ Number

Group Work 3, Section 1.6
Irrational, Impossible Relations (Hint Sheet)

So, you realize that it's not easy to determine whether $\log_2 3$ is rational!
One way to attempt to show that $\log_2 3$ is rational is to assume that it is, and try to find integers a and b such that $\log_2 3 = \frac{a}{b}$. If we can show that there are no such a and b, then $\log_2 3$ *cannot* be rational.

1. Assume that $\log_2 3 = \frac{a}{b}$ for integers $a, b \geq 0$. Show that a and b must then satisfy $2^a = 3^b$.

Assume $\log_2 3 = \frac{a}{b} \implies 2^{\frac{a}{b}} = 3 \implies 2^a = 3^b$



← intersection at $a = b = 0$.

Hence No such a & b exists then $\log_2 3$ is irrational

2. Notice that $a = 0$, $b = 0$ satisfies $2^a = 3^b$. Why doesn't this fact help us?

because $\log_2 3 \neq \frac{0}{0}$

3. Find $a \neq 0$ and $b \neq 0$ that satisfy $2^a = 3^b$, or show that no such a and b exist.

No such a or b exists

4. Is $\log_2 3$ rational or irrational? Why?