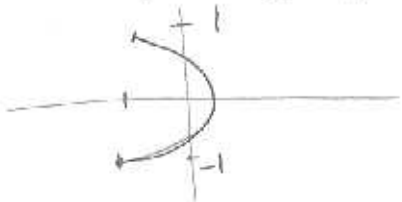
Group Work 1, Section 1.7
Name that Parametrization

1. Consider the graph of the following set of parametric equations:

$$x(t) = \cos 2t \qquad y(t) = \sin t \qquad 0 < t < \infty$$

(a) Graph this curve using technology. Why does it look the way it does?



(b) Write this equation in the form $x = f(y)$. (*Hint:* Use the formula $\cos 2\theta = \cos^2\theta - \sin^2\theta$.)

$x = \cos 2t = \cos^2 t - \sin^2 t$

$= 1 - \sin^2 t - \sin^2 t$

$= 1 - 2\sin^2 t$

$= 1 - 2y^2$

Recall $\sin^2 t + \cos^2 t = 1$

$\cos^2 t = 1 - \sin^2 t$

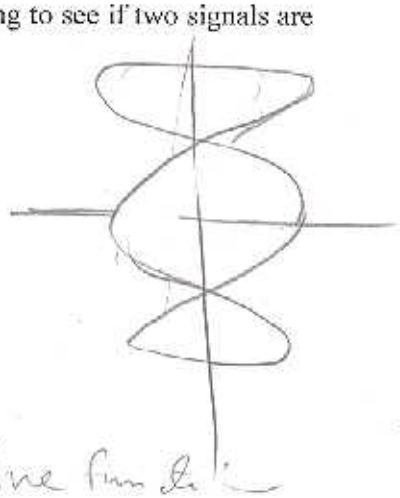
2. Try to guess what the graphs of the following sets of parametric equations look like, and then see if you are right.

(a) $x(t) = \cos 6t \qquad y(t) = \sin 2t \qquad 0 \le t \le 4\pi$

(b) $x(t) = \cos\sqrt{2}t \qquad y(t) = \sin 2t \qquad 0 \le t \le 4\pi$

These curves are called Lissajous figures, and are used in electrical engineering to see if two signals are "in sync".

Tmin = 0
Tmax = 12.56
Tstep = 0.5
Xmin = -1
Xmax = 1 Xscl = 0.1
Ymin = -1
Ymax = 1
Yscl = 0.1



3. Consider these two sets of parametric equations:

Same: Sin function $x(t) = t \qquad y(t) = \sin t \qquad 0 < t < \infty$

Sin function $x(t) = 2t \qquad y(t) = \sin 2t \qquad 0 < t < \infty$

What is the relationship between their associated curves?

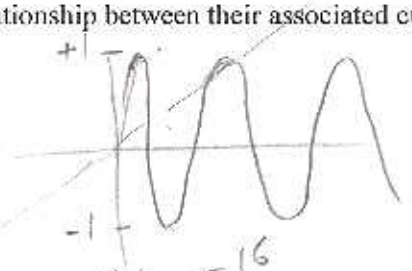
Just a sine function

4. (a) Consider these two sets of parametric equations:

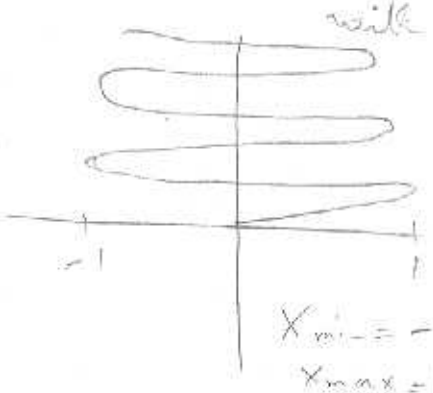
$$x(t) = t \qquad y(t) = \sin t \qquad 0 < t < \infty$$

$$x(t) = \sin t \qquad y(t) = t \qquad 0 < t < \infty$$

What is the relationship between their associated curves?



$X_{min} = -16$
$X_{max} = 16$
$X_{scl} = 0.05$



$X_{min} = -1$
$X_{max} = 1$
$X_{scl} = 0.1$

(b) Given any set of equations of the form

$$x(t) = t \qquad y(t) = f(t) \qquad 0 < t < \infty$$

What does the graph of the set of equations

$$x(t) = f(t) \qquad y(t) = t \qquad 0 < t < \infty$$

look like?

Move it -90° and then flip it w.r.t y axis

5. (a) Use a graphing calculator to check that $f(x) = x^5 - 3x^3 + 5x + 2$ is one-to-one.

(b) Graph its inverse function f^{-1}.

$X = t \qquad y = t^5 - 3t^3 + 5t + 2$

(f)

(f^{-1}) $\quad x = t^5 - 3t^3 + 5t + 2 \qquad y = t$

See Example 6 Page 78