

1) Find the most general antiderivative of the following functions.

$$\begin{aligned} \text{a) } f(x) &= 12x^{\frac{3}{4}} + 6x^{\frac{1}{3}} - 5 \Rightarrow \frac{12x^{\frac{3}{4}+1}}{\frac{3}{4}+1} + \frac{6x^{\frac{1}{3}+1}}{\frac{1}{3}+1} - 5x + C \\ &= \frac{12x^{\frac{7}{4}}}{\frac{7}{4}} + \frac{6x^{\frac{4}{3}}}{\frac{4}{3}} - 5x + C = \frac{48}{7}x^{\frac{7}{4}} + \frac{18}{4}x^{\frac{4}{3}} - 5x + C \end{aligned}$$

$$\text{b) } f(x) = \left(2 - \frac{1}{\sqrt{x}}\right)\left(2 + \frac{1}{\sqrt{x}}\right) = 4 - \frac{1}{x}$$

$$F(x) = 4x - \ln|x| + C$$

$$\text{c) } f(t) = \frac{t^5 + 2t^4}{\sqrt{t}} = t^{5-\frac{1}{2}} + 2t^{4-\frac{1}{2}} = t^{\frac{9}{2}} + 2t^{\frac{7}{2}}$$

$$F(t) = \frac{t^{\frac{11}{2}}}{\frac{11}{2}} + \frac{2t^{\frac{9}{2}}}{\frac{9}{2}} + C = \frac{2}{11}t^{\frac{11}{2}} + \frac{4}{9}t^{\frac{9}{2}} + C$$

$$\text{d) } f(x) = \pi^3 + 15x$$

$$F(x) = \pi^3 x + \frac{15x^2}{2} + C$$

$$\text{e) } f(t) = e^t + \sec t \tan t + 2$$

$$F(t) = e^t + \sec t + 2t + C$$

$$f) f(t) = \frac{1}{t^2+1} + \frac{1}{\sqrt{1-t^2}}$$

$$F(t) = \tan^{-1} t + \sin^{-1} t + C$$

The acceleration of an object dropped or thrown on Earth is $-32 \frac{\text{feet}}{\text{sec}^2}$

- 2) A ball is thrown directly upward at a speed of 40 feet per second from a cliff 100 feet above the ground. (2 points each)

- a) Find expressions for the velocity and height of the ball t seconds after it was released.

$$V(t) = -32t + 40 \text{ ft/sec}$$

$$S(t) = -\frac{32t^2}{2} + 40t + 100 \text{ ft}$$

- b) At what time does the ball reach its highest point?

$$\text{when } V(t) = 0 \Rightarrow t = \frac{-40}{-32} = 1.25 \text{ sec}$$

- c) How high above the ground (from the base of the cliff) does the ball reach?

$$S(1.25) = -16(1.25)^2 + 40(1.25) + 100 = 125 \text{ feet}$$

\Rightarrow

- d) When does the ball strike the ground at the base of the cliff?

$$S(t) = 0 \quad t = 4.045 \text{ sec}$$

- e) What is its velocity at that instant (i.e. when the ball hits the ground)?

$$V(4.045) = -32(4.045) + 40 = -89.44 \text{ ft/sec}$$