Group Work 1, Section 2.5
Infinite Limits

1. Describe the horizontal asymptotes, if any, of the following functions.

(a) $f(x) = \dfrac{x^4 - x^2 + 2}{3x^4 + x^2 + 5}$ HA $\boxed{y = \frac{1}{3}}$

show $\lim_{x\to\infty} \dfrac{\frac{x^4}{x^4} - \frac{x^2}{x^4} + \frac{2}{x^4}}{\frac{3x^4}{x^4} + \frac{x^2}{x^4} + \frac{5}{x^4}} = \dfrac{1+0+0}{3+0+0} = \dfrac{1}{3}$ $\lim_{x\to\infty} \dfrac{1 - \frac{1}{x^2} + \frac{2}{x^4}}{3 + \frac{1}{x^2} + \frac{5}{x^4}}$

(b) $f(x) = \dfrac{2x^5 - 2x^3 + 18}{x^4 + x^2 - x + 2}$ HA: None $\lim_{x\to\infty} \dfrac{\frac{2x^5}{x^4} - \frac{2x^3}{x^4} + \frac{18}{x^4}}{\frac{x^4}{x^4} + \frac{x^2}{x^4} - \frac{x}{x^4} + \frac{2}{x^4}} = \infty$

(c) $f(x) = \dfrac{2x^5 - 2x^3 + 18}{x^4 + 3x^3 - x + 2} - \dfrac{2x}{1} = \dfrac{2x^5 - 2x^3 + 18 - 2x(x^4 + 3x^3 - x + 2)}{x^4 + 3x^3 - x + 2}$

$= \dfrac{-6x^4 + -2x^3 + 2x^2 - 4x + 18}{x^4 + 3x^3 - x + 2} \Rightarrow \boxed{y = -6 \text{ HA}}$

2. Draw an odd function which has the lines $y = 1$, $x = -4$, and $x = -1$ among its asymptotes.

3. (a) Describe all vertical and horizontal asymptotes of $f(x) = \dfrac{3x^2 + 4x + 5}{\sqrt{16x^4 - 81}}$.

$\sqrt{16x^4 - 81} = 0$

$(4x^2 - 9)(4x^2 + 9) = 0$

$(2x + 3)(2x - 3) = 0 \Rightarrow$ V.A $x = \frac{-3}{2}$ $x = \frac{3}{2}$

$\boxed{\text{H.A. } y = \frac{3}{4}}$

(b) Let $h(x) = \dfrac{g(x)}{2x^2 + 4x - 30}$. Find $g(x)$ such that [redacted] the following conditions hold:

(i) $\lim_{x\to 3} h(x)$ exists. $2x - 6 = g(x)$ $\dfrac{}{(2x-6)(x+5)} \Rightarrow \lim_{x\to 3} h(x) = \dfrac{1}{8}$

(ii) $y = 2$ is a horizontal asymptote for $h(x)$. $g(x) = 4x^2$

(iii) $x = -5$ is a vertical asymptote for $h(x)$. $g(x) = 2x - 6$

(iv) $h(4) = 0$.

$\dfrac{g(x)}{(2x-6)(x+5)} =$

(iv) let $g(x) = x - 4$

4. Find formulas for two functions, f and g, such that $\lim_{x\to\infty} f(x) = \infty = \lim_{x\to\infty} g(x)$ and

(a) $\lim_{x\to\infty} (f(x) - g(x)) = \infty$

let $f(x) = e^x$ and $g(x) = 2^x$

(b) $\lim_{x\to\infty} (f(x) - g(x)) = -\infty$

let $f(x) = 2^x$ and $g(x) = e^x$

(c) $\lim_{x\to\infty} (f(x) - g(x)) = 0$

let $f(x) = e^x + \frac{1}{x}$ and $g(x) = e^x$

(d) $\lim_{x\to\infty} (f(x) - g(x)) = 42$

let $f(x) = e^x + 42$ $g(x) = e^x$

5. Let $P(x) = a_m x^m + \ldots + a_1 x + a_0$, and $Q(x) = b_n x^n + \ldots + b_1 x + b_0$ be polynomials of degree m and n, respectively.

(a) Find $\lim_{x\to\infty} \frac{P(x)}{Q(x)}$ if $m = n$.

$y = \frac{a_m}{b_n}$; HA

(b) Find $\lim_{x\to\infty} \frac{P(x)}{Q(x)}$ if $m < n$.

$y = 0$; H.A

(c) Find $\lim_{x\to\infty} \frac{P(x)}{Q(x)}$ if $m > n$.

H.A.; None

Group Work 2, Section 2.5
I Am the Greatest

1. Show that $\lim_{x\to 0^+} \frac{[\![x]\!]}{x} = 0$

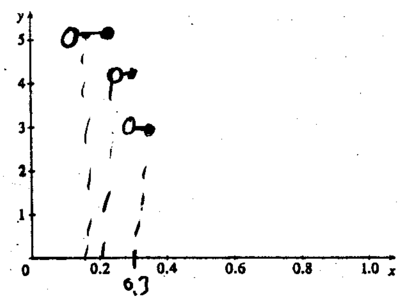
Graph $Y_1 = \text{int}(X) \div X$

and Evaluate Y_1 at $X = 0.0000001$

and values close to 0.

2. (a) Sketch a graph of $\left[\!\!\left[\frac{1}{x}\right]\!\!\right]$ on the axes below.

X	Y
$\frac{1}{6}$	5
$\frac{1}{5}$	5
$\frac{1}{4}$	4
$\frac{1}{3}$	3
$\frac{1}{2}$	2
0.6	1
0.7	1
0.9	1



(b) If $\frac{1}{n+1} < x < \frac{1}{n}$ find upper and lower bounds for the expression $x\left[\!\!\left[\frac{1}{x}\right]\!\!\right]$.

lower bound $\frac{n}{n+1}$

upper bound 1

(c) Use the estimates above to show that $\lim_{x\to 0^+} x\left[\!\!\left[\frac{1}{x}\right]\!\!\right] = 1$.

Just Graph

3. Compute $\lim\limits_{x \to 0^+} x^2 \left[\!\left[\frac{1}{x} \right]\!\right]$. $= 0$

4. Show that $\lim\limits_{x \to \infty} x \left[\!\left[\frac{1}{x} \right]\!\right] = 0$.

As x increases the $x \left[\!\left[\frac{1}{x} \right]\!\right] = 0$

5. Compute $\lim\limits_{x \to \infty} \frac{1}{x} [\![x]\!]$ Justify your reasoning.