Name: Solutions

Calculus I Dr. Katiraie Sections 5.2 and 5.3

1) Find the general indefinite integrals.

a) $\int \frac{\sin x}{1-\sin^2 x}dx$ hint: $\sin^2 x+\cos^2 x=1$

$\cos^2 x = 1-\sin^2 x$

$\int \frac{\sin x}{\cos^2 x}dx = \int \frac{1}{\cos x}\cdot\frac{\sin x}{\cos x}dx = \int \sec x \tan x \, dx$

$= \boxed{\sec x + C}$

b) $\int\left(-5e^{7x}+\frac{8}{x}\right)dx = \boxed{-\frac{5e^{7x}}{7} + 8\ln|x| + C}$

c) $\int\left((1-\frac{1}{\sqrt{x}})(1+\frac{1}{\sqrt{x}})\right)dx = \int \left(1-\frac{1}{x}\right)dx = \boxed{x - \ln|x| + C}$

d) $\int\frac{t^3+2t^2}{\sqrt{t}}dt = \int\left(t^{3-\frac{1}{2}} + 2t^{2-\frac{1}{2}}\right)dt$

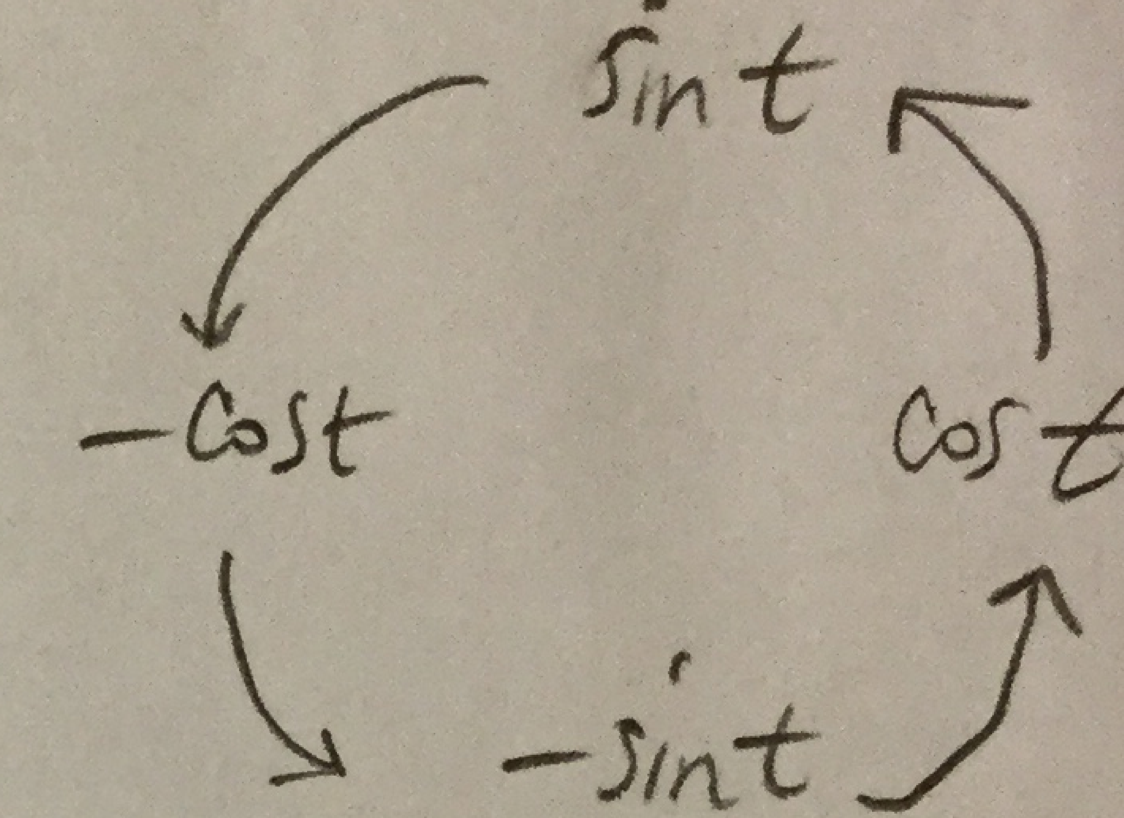
$= \int\left(t^{\frac{5}{2}} + 2t^{\frac{3}{2}}\right)dt = \frac{t^{\frac{5}{2}+1}}{\frac{5}{2}+1} + \frac{2t^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C$

$= \frac{t^{\frac{7}{2}}}{\frac{7}{2}} + \frac{2t^{\frac{5}{2}}}{\frac{5}{2}} + C$

$= \boxed{\frac{2}{7}t^{\frac{7}{2}} + \frac{4}{5}t^{\frac{5}{2}} + C}$

e) $\int \pi^3 dx = \boxed{\pi^3 x + c}$

f) $\int \pi^3 x dx = \boxed{\frac{\pi^3 x^2}{2} + c}$



g) $\int(\sin(t) + \cos(t) - \csc t \cot t)dt =$

$= \boxed{-\cos t + \sin t + \csc t + c}$

h) $\int(\sec^2 t + t^2 + 2)dt$

$= \boxed{\tan t + \frac{t^3}{3} + 2t + c}$

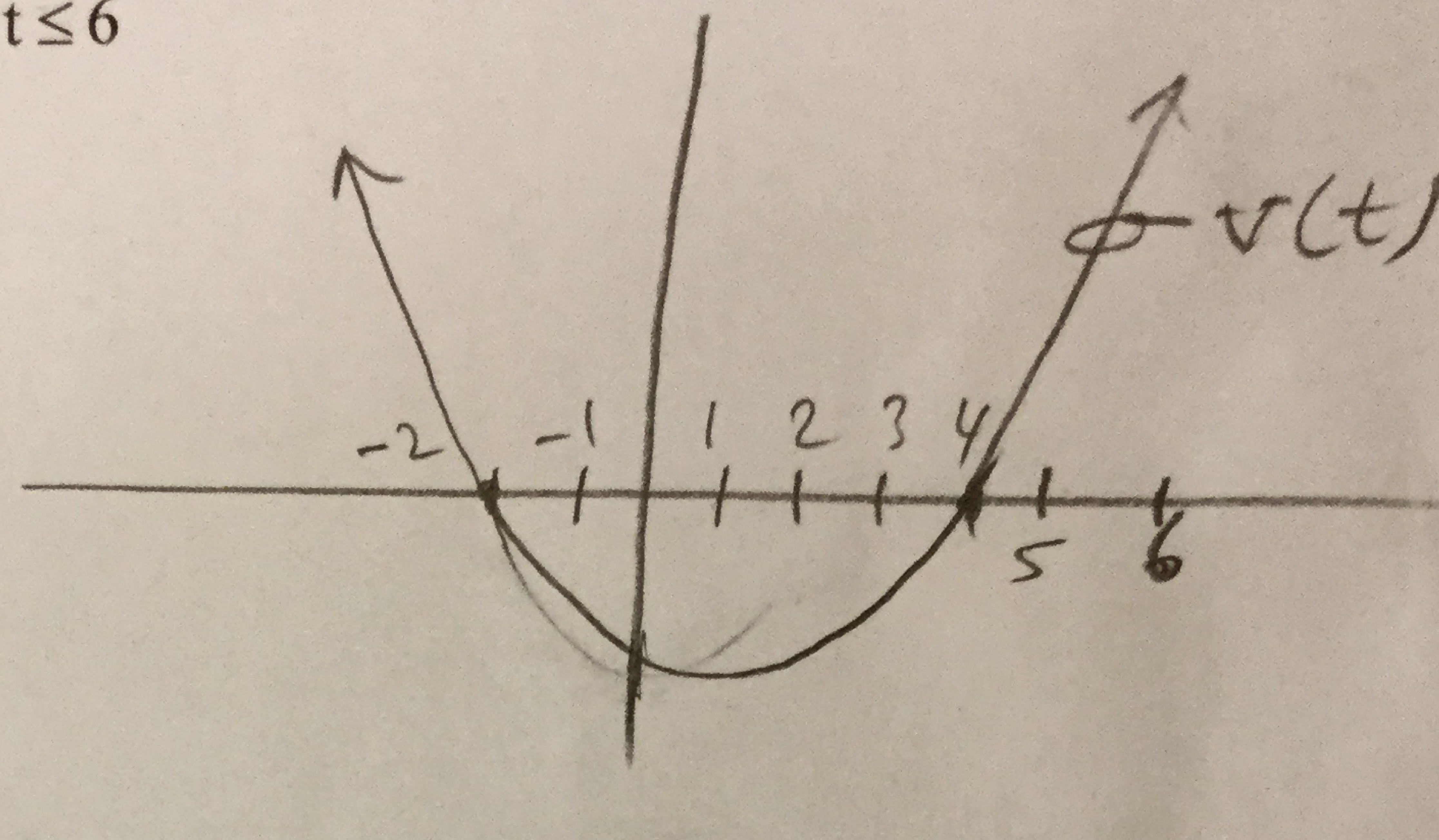
i) $\int \frac{\sin 2x}{\sin x} dx$ hint: sin2x = 2 sinx cosx (2 point)

$= \int \frac{2\cancel{\sin x} \cos x}{\cancel{\sin x}} dx = \int 2\cos x \, dx = \boxed{2 \sin x + c}$

2) Given the velocity function (in meters per second) for a particle along a line is

$v(t) = t^2 - 2t - 8 \ , \quad 1 \le t \le 6$

a) Graph the velocity function,



b) Find the displacement of the particle during the time interval $1 \le t \le 6$ seconds

$$\int_1^6 (t^2 - 2t - 8)\,dt = \frac{t^3}{3} - \frac{2t^2}{2} - 8t \Big|_1^6$$

$$= \left(\frac{6^3}{3} - 6^2 - 8(6)\right) - \left(\frac{1^3}{3} - 1^2 - 8(1)\right) = -3\frac{1}{3} \text{ meters}$$

The object is $3\frac{1}{3}$ meters to the left of the starting point

c) Find the distance traveled by the particle during the time interval $1 \le t \le 6$ seconds

$$\left|\int_1^4 (t^2 - 2t - 8)\,dt\right| + \left|\int_4^6 (t^2 - 2t - 8)\,dt\right|$$

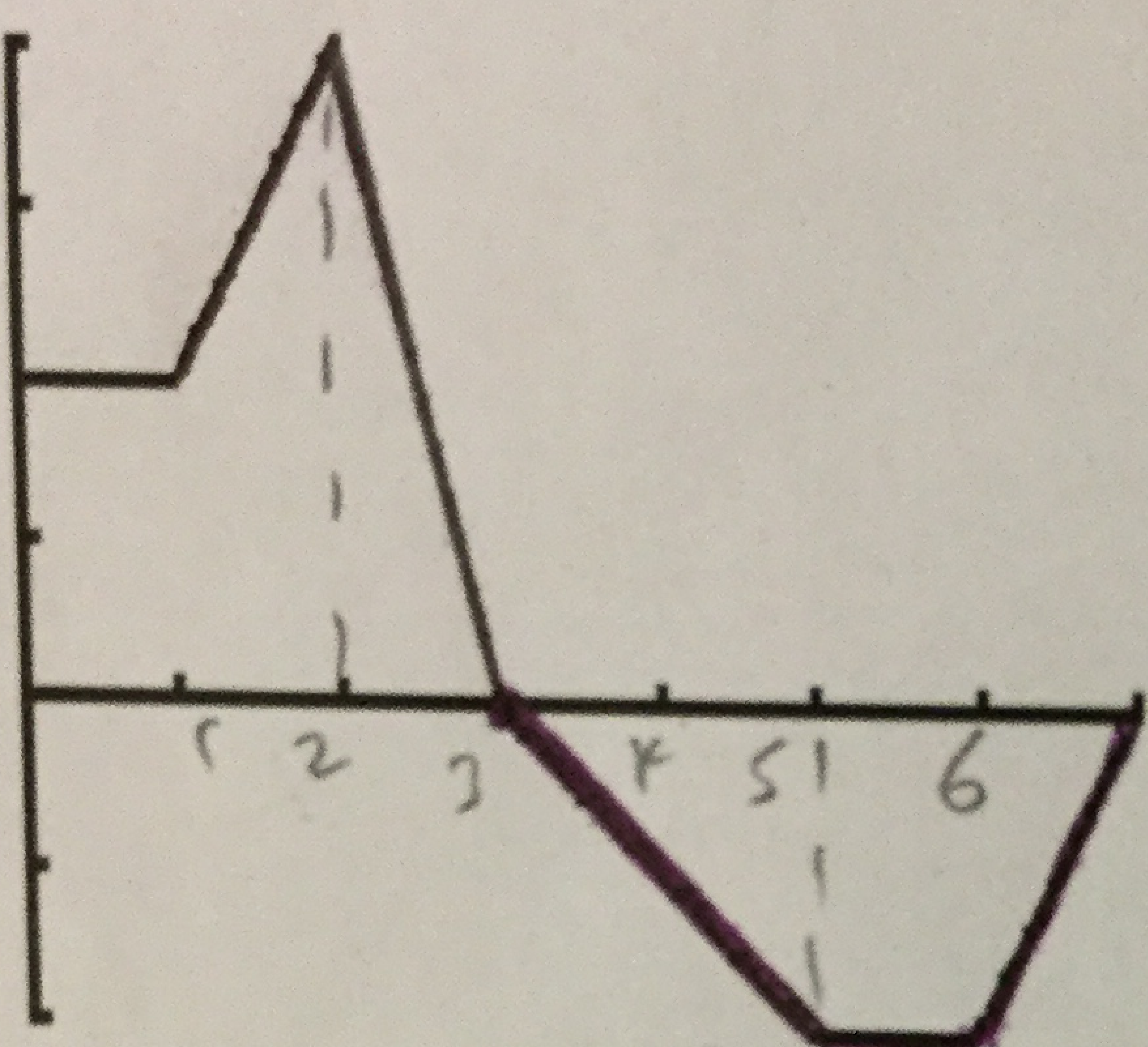
$$= |-18| + |14.67| = 18 + 14.67 = 32.67 \text{ meters}$$

3) Let

$g(x)=\int_0^x f(t)\,dt$ *where,* x is in the closed interval 0 to 7, namely $0 \le x \le 7$

,and f(t) is the function whose graph is shown below.

(Note: $g'(x)=f(x)$ for $0<x<7$



a) Evaluate g(0) $= \int_0^0 f(t)\,dt = \boxed{0}$

b) Evaluate g(1) $= \int_0^1 f(t)\,dt = (1)(2) = \boxed{2}$

c) Evaluate g(2) $= \int_0^2 f(t)\,dt = (2)(2) + \frac{1}{2}(1)(2) = 4 + 1 = \boxed{5}$

d) Evaluate g(3) $= \int_0^3 f(t)\,dt = (2)(2) + \frac{1}{2}(1)(2) + \frac{1}{2}(1)(4) =$
$= 4 + 1 + 2 = \boxed{7}$

e) Evaluate g(5) $= 7 + \frac{1}{2}(2)(-2) = \boxed{5}$

f) Evaluate g(6) $= 5 + 1(-2) = \boxed{3}$

g) Evaluate g(7) $= 3 + \frac{1}{2}(1)(-2) = 3 - 1 = \boxed{2}$

b) On what interval is g(x) increasing?

$0 < t < 3$

c) On what interval is g(x) decreasing?

$3 < t < 7$

c) For what value does g(x) have a maximum value?

$t = 3$

d) On what interval is g(x) concave upward?

$g''(x)$ is positive $1 < t < 2$ or $6 < t < 7$

e) On what interval is g(x) concave downward?

$g''(x)$ is Negative $2 < t < 3$ OR $3 < t < 5$