Note: Adjustments may be made to this review sheet if we deviate from the class outline.

TEST POLICIES (REMINDERS)

If you know that you will be absent on the day of a test, it may be possible to make arrangements with me to take the test on an earlier day. This alternative is not automatic!! Each case will be considered individually. You should notify me as soon as possible regarding planned absences. There are no make-up exams provided for unexpected absences. If you miss an exam for an unexpected absence, the average that you earn on the final will make-up the score for your missed exam. There are no exceptions – do not ask!!

HONOR POLICY: You must observe the following rules during any in class exam or quiz.

(1) Be prepared to move to a designated seat if requested.
(2) You are not permitted to leave the room.
(3) You must not look anywhere in the room other than at your own test paper.
(4) You may not use or even touch a cell phone. Remember your cell phone is to be silenced in class.
(5) You may not speak to another student.
(6) You may not share materials with another student.
(7) Have all your materials ready. You may not retrieve items from your backpack etc.

Failure to observe all of the policies will result in a zero score for the test or quiz.

When you complete a test, you will hand it to me personally. You may leave the room at this time and return (if necessary) at the planned time. The only questions permitted during an exam is in reference to a misprint, omission, or illegible text. You are responsible for being prepared for the exam. You do not have the time to figure out problems on exam day. You must turn in your test paper when time is called. I will give a five minute warning before collecting exams. If you do not hand in your paper at that time, I will not accept it later. It is your responsibility to keep track of time during the exam.

TEST REVIEW PROBLEMS

This test review will give you an idea of the difficulty level of the problems that will be on the exam. This test review contains a sample problem from every topic that will be covered on the exam. Test problems will be similar, but not identical. if you have done all the homework, asked for help as needed, and reviewed the material diligently, you should now find this review easy to moderate to complete. If you struggle with the review, we will go over some solutions in class. However, please be aware that your struggle is an indication that you should go back to the homework sets and work additional problems similar to those on this review in order to perform well on the exam.

The problems begin on the next page. The answers are included at the end of the review.

**** Also review group work exercises, quizzes, and class examples. ****

1. Find the solution of the initial-value problem

A. $y' = \frac{\ln x}{xy},\ y(1) = 2$

B. $\frac{dy}{dt} = 2ty^2 + 3y^2,\ y(0) = 1$

2. A roasted turkey is taken from the oven which its temperature has reached 185° F and placed on a table in a room where the temperature is 75° F. One half-hour later, the temperature of the turkey is 150° F. Assuming that the turkey cools at a rate which is proportional to the difference between the temperature of the object and the temperature of its surroundings, the differential equation that models this situation is $\frac{dy}{dt} = k(75 - y)$, where y is the temperature(in ° F) of the object at time t (in hours).

A. Solve this differential equation subject to the conditions given. Find the value of all constants.

B. What temperature is the turkey after 45 minutes?

C. When will the turkey have cooled to 100° F?

3. For each of the following sequences:

i. State whether the sequence is convergent or divergent

ii. If convergent, state limit. If divergent, explain why the limit does not exist.

A. $a_n = 0.9999^n$

B. $a_n = 1.0001^n$

C. $a_n = \frac{(-2)^n}{n}$

D. $a_n = (-1)^n\left(1 - \frac{1}{\sqrt{n}}\right)$

E. $a_n = \frac{2 + n^3}{1 + 2n^3}$

F. $a_n = \frac{n^3}{1 + n^2}$

4. Find the sum of the following series:

A. 0.9 + 0.09 + 0.009 + 0.00009 + …

B. $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$

C. $\sum_{n=1}^{\infty} \frac{1}{(3n - 1)(3n + 2)}$

D. $\sum_{n=1}^{\infty} \frac{3 + (-1)^n}{3^n}$

5. Which of the following series are divergent?

A. $\sum_{n=1}^{\infty} \frac{1}{e^{2n}}$

B. $\sum_{n=1}^{\infty} (0.9999)^n$

C. $\sum_{n=1}^{\infty} (1.0001)^n$

D. $\sum_{n=1}^{\infty} \frac{n^2}{3(n + 1)(n + 2)}$

6. A series $\sum_{k=1}^{\infty} a_k$ has partial sums, s_n, given by $s_n = \dfrac{7n - 2}{n}$.

A. Is $\sum_{k=1}^{\infty} a_k$ convergent? If it is, find the sum.

B. Find $\lim_{k \to \infty} a_k$.

C. Find $\sum_{k=1}^{200} a_k$

7. Determine whether the given series is convergent or divergent. Indicate the test you use and show any necessary computation.

A. $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2 + 1}{5n^3 - n + 4}$

B. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1 + \sin n}{n}\right)^2$

C. $\displaystyle\sum_{n=1}^{\infty} n \bullet \sin\left(\frac{1}{n}\right)$

D. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2}{n\sqrt{n}} + \frac{3}{n^3}\right)$

E. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

8. Use the Limit Comparison Test to test the convergence of the series

A. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n\sqrt{n} + 7}$

B. $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

C. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3 - 1}$

9. Which of the following are alternating series?

A. $\dfrac{(-1)^{2n}}{n}$

B. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

C. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n\pi}{n}$

10. Which of the following series converges?

A. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n + 1)}$

B. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \ln(n + 1)$

C. $1 - \dfrac{1}{2} + \dfrac{2}{3} - \dfrac{3}{4} + \dfrac{4}{5} - \dfrac{5}{6} + \ldots.$

11. Use the Alternating Series or the Ratio Test (as appropriate) to test the convergence of the following series

A. $\displaystyle\sum_{n=1}^{\infty} \frac{n^{100}}{n!}$

B. $\displaystyle\sum_{n=1}^{\infty} \frac{n - 2}{n\,2^n}$

C. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n + 1}{\ln(n + 1)}$

D. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\ln(n + 1)}{n + 1}$

12. How many terms of the alternating series $\sum_{n=1}^{\infty}(-1)^{n+1}n^{-2}$ must be added to be sure that the partial sum s_n is within 0.0001 of the sum s?

13. Estimate $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^4+1}$ to within 0.01.

14. Find the radius and interval of convergence of the series.

A. $\sum_{n=0}^{\infty}\frac{x^n}{3n+1}$ B. $\sum_{n=0}^{\infty}\frac{(-3x)^n}{3n+1}$ C. $\sum_{n=0}^{\infty}\frac{n}{4^n}(x+3)^n$

15. Evaluate $\int\frac{1}{1+x^5}\,dx$ as a power series.

16. Approximate $\int_0^{0.1}\frac{1}{1+x^5}\,dx$ accurate to six decimal places.

17. Find a power series representation for $\ln(1+x^2)$.

18. Find the coefficient of x^3 in the Maclaurin series for $f(x) = \sin 2x$.

19. Find the radius of convergence of the Maclaurin series for $f(x)=\frac{1}{4+x^2}$.

20. If the Maclaurin series for f(x) is $1 - 9x + 16x^2 - 25x^3 + \ldots$ find $f^{(3)}(0)$.

21. Find the Taylor series expansion of $f(x) = \sin x$ about the point $c=\frac{\pi}{4}$.

22. Use a Maclaurin series (given in section 8.7) to find the Maclaurin series for

A. $f(x) = \cos 2x$ B. $f(x) = x\cos 2x$ C. $f(x)=e^{-\frac{x}{5}}$

23. Evaluate the indefinite integral as an infinite series

A. $\int\frac{\sin x}{x}\,dx$ B. $\int\frac{e^x-1}{x}\,dx$

24. Find the sum of the series

A. $\sum_{n=0}^{\infty}\frac{(-1)^n\pi^{2n+1}}{3^{2n}(2n+1)!}$ B. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}$ C. $\sum_{n=0}^{\infty}\frac{3^n}{5^n n!}$

SOLUTIONS:

1. A. $\frac{dy}{dx} = \frac{\ln x}{xy} \rightarrow y\,dy = \frac{\ln x}{x}dx \rightarrow \int y\,dy = \int \frac{\ln x}{x}dx \rightarrow \frac{y^2}{2} = \frac{(\ln x)^2}{2} + C$

When x = 1, y = 2 ➔ $2^2/2 = (\ln 1)^2/2 + C$ ➔ 2 = C

$\frac{y^2}{2} = \frac{(\ln x)^2}{2} + 2 \rightarrow y = \sqrt{(\ln x)^2 + 4}$

B. $\frac{dy}{dt} = 2ty^2 + 3y^2 = y^2(2t + 3) \rightarrow \frac{dy}{y^2} = (2t + 3)dt$

$\int \frac{dy}{y^2} = \int (2t + 3)dt \rightarrow -\frac{1}{y} = t^2 + 3t + C$, y(0) = 1, -1 = C,

$-\frac{1}{y} = t^2 + 3t - 1 \rightarrow \frac{1}{y} = 1 - 3t - t^2 \rightarrow y = \frac{1}{1 - 3t - t^2}$

2. A. $\frac{dy}{75 - y} = k\,dt$ ➔ $\int \frac{dy}{75 - y} = \int k\,dt$ ➔ ln |75 – y| = kt + C

y(0) = 185 ➔ ln|75 – 20| = k(0) + C ➔ C = ln 110

Now we have, ln |75 – y| = kt + ln 110

y(0.5) = 150 ➔ ln |75 – 150| = 0.5k + ln 110

➔ k = [ln 75 – ln 110]/0.5 ~ -0.7660

Thus, ln |75 – y| = -0.766t + ln 110

$e^{\ln|75 - y|} = e^{-0.766t + \ln 110}$ ➔ $y - 75 = 110e^{-0.766t}$

$y = 75 + 110e^{-0.766t}$

B. $y(3/4) = 75 + 110e^{-0.766(3/4)}$ = 136.93 degrees

C. $110 = 75 + 110e^{-0.766t}$ ➔ $25 = 110e^{-0.766t}$ ➔ t = 1.93 hours

3. A. Converges, 0

B. Diverges, Form r^n, with r > 1

C. Diverges, infinite limit

D. Diverges, $|a_n|$ ➔ 1, but $(-1)^n$ makes the sequence oscillate between 1 and -1.

E. Converges, ½

F. Diverges, infinite limit

4. A. a = 0.9, r = 0.1 S = 0.9/(1 - .1) = 1

B. a = ¼, r = -3/4, S = 1/4/(1 - -3/4) = 1/7

4. C. $S_n = \sum_{i=1}^{n} \frac{1}{(3i-1)(3i+2)} = \sum_{i=1}^{n} \frac{1}{3}\left[\frac{1}{3i-1} - \frac{1}{3i+2}\right] =$

$$\frac{1}{3}\left[\underbrace{\frac{1}{2} - \frac{1}{5}}_{i=1} + \underbrace{\frac{1}{5} - \frac{1}{8}}_{i=2} + \underbrace{\frac{1}{8} - \frac{1}{11}}_{i=3} + \ldots + \underbrace{\frac{1}{3n-4} - \frac{1}{3n-1}}_{i=n-1} + \underbrace{\frac{1}{3n-1} - \frac{1}{3n+2}}_{i=n}\right]$$

$Sn = \frac{1}{3}\left[\frac{1}{2} - \frac{1}{3n+2}\right]$ and $\lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{1}{3}\left[\frac{1}{2} - \frac{1}{3n+2}\right] = \frac{1}{6}$

D. $\sum_{n=1}^{\infty} \frac{3+(-1)^n}{3^n} = \sum_{n=1}^{\infty} \frac{3}{3^n} + \sum_{n=1}^{\infty} \frac{(-1)^n}{3^n}$ a = 1, r = 1/3 and a = -1/3, r = -1/3

S = (1/(1 – 1/3)) + (-1/3/(1 - -1/3)) = 3/2 + -1/4 = 5/4

5. Which of the following series are divergent?

C. $r \geq 1$ D. limit of n^{th} term is 1/3

6. A. Yes, the limit of the partial sums is 7, thus the sum is 7.

B. Since it is known that the series is convergent, this limit must be zero.

C. S_{200} = (7*200 – 2)/200 = 1398/200 = 699/100 = 6.99

7. A. $\frac{2n^2}{5n^3} = \frac{1}{n} \to \frac{2n^2+1}{5n^3} \geq \frac{1}{n} \to \frac{2n^2+1}{5n^3-n} \geq \frac{1}{n}$

Since, $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, the original series diverges by the comparison test

B. $1 + \sin n \leq 1 \to \frac{1+\sin n}{n} \leq \frac{1}{n} \to \left(\frac{1+\sin n}{n}\right)^2 \leq \frac{1}{n^2}$

Since, $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, the original series converges by the comparison test

C. Diverges, by the nth term test. The limit as n ➔ infinity of $n \bullet \sin\left(\frac{1}{n}\right) = 1 \neq 0$.

$\lim_{n\to\infty} n \bullet \sin\left(\frac{1}{n}\right)$ has the form "infinity x zero". $\lim_{n\to\infty} n \bullet \sin\left(\frac{1}{n}\right) = \lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}}$

Which has the form "0/0". $\lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} \underset{=}{H} \lim_{n\to\infty} \frac{\cos\left(\frac{1}{n}\right) \bullet \frac{-1}{n^2}}{\frac{-1}{n^2}}$

$= \lim_{n\to\infty} \cos\left(\frac{1}{n}\right) = \cos\text{"}0\text{"} = 1$

7. D. $\sum_{n=1}^{\infty}\left(\frac{2}{n\sqrt{n}} + \frac{3}{n^3}\right) = \sum_{n=1}^{\infty}\frac{2}{n\sqrt{n}} + \sum_{n=1}^{\infty}\frac{3}{n^3}$ which both converge by the comparison test with $n^{-3/2}$ and n^{-3}. So the original series converges.

E. See example 4 on page 581 of the textbook.

8. A. $\frac{n}{n\sqrt{n} + 7}$ appears to resemble $\frac{1}{\sqrt{n}}$. However, if you attempt the comparison test, since $n\sqrt{n} + 7 > n\sqrt{n}$, we can't claim that $\frac{n}{n\sqrt{n} + 7} > \frac{n}{n\sqrt{n}} = \frac{1}{\sqrt{n}}$

The Limit Comparison Test examines $\lim_{n\to\infty}\frac{\frac{1}{\sqrt{n}}}{\frac{n}{n\sqrt{n} + 7}} = \lim_{n\to\infty}\frac{n\sqrt{n} + 7}{n\sqrt{n}} = 1$

So $\sum_{n=1}^{\infty}\frac{n}{n\sqrt{n} + 7}$ diverges as does $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$

B. $\sum_{n=1}^{\infty}\sin\left(\frac{1}{n}\right)$ Note: $\sin\left(\frac{1}{n}\right)$ has a zero limit, thus the nth term test is inconclusive. Apply the Limit Comparison Test with 1/n.

$$\lim_{n\to\infty}\frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} \underset{=}{H} \lim_{n\to\infty}\frac{\frac{-1}{n^2}\cos\left(\frac{1}{n}\right)}{\frac{-1}{n^2}} = \lim_{n\to\infty}\frac{\cos\left(\frac{1}{n}\right)}{1} = 1$$

So $\sum_{n=1}^{\infty}\sin\left(\frac{1}{n}\right)$ diverges as does $\sum_{n=1}^{\infty}\frac{1}{n}$

C. $\sum_{n=1}^{\infty}\frac{n}{n^3 - 1}$

$\sum_{n=1}^{\infty}\frac{n}{n^3 - 1}$ appears to resemble $\sum_{n=1}^{\infty}\frac{1}{n^2}$. However, if you attempt the comparison test, since $n^3 - 1 < n^3$, we can't claim that $\frac{n}{n^3 - 1} < \frac{n}{n^3} = \frac{1}{n^2}$

The Limit Comparison Test examines $\lim_{n\to\infty}\frac{\frac{1}{n^2}}{\frac{n}{n^3 - 1}} = \lim_{n\to\infty}\frac{n^3 - 1}{n^3} = 1$

So $\sum_{n=1}^{\infty}\frac{n}{n^3 - 1}$ converges as does $\sum_{n=1}^{\infty}\frac{1}{n^2}$

9. A. No, (-1) is always raised to an even power, so each term is positive.

9. B. Yes, the terms alternate between positive and negative.

C. Yes, the cosine of odd multiples of pi is -1 and the cosine of even multiples of pi is 1. The numerators of this series are cos 1(pi), cos 2(pi), cos 3(pi), ….

10. A. $\ln((n+1)+1) \geq \ln(n+1)$ for all n.

so $\dfrac{1}{\ln\left((n+1)+1\right)} \leq \dfrac{1}{\ln(n+1)}$ AND $\lim\limits_{n\to\infty} \dfrac{1}{\ln(n+1)} = 0$

Both conditions are satisfied. This series is convergent.

B. $\lim\limits_{n\to\infty} \ln(n+1) = \infty$, so condition (ii) of the Alternating series test is not satisfied. This series is divergent.

C. $\dfrac{2}{3} < \dfrac{3}{4} < \dfrac{4}{5} < \dfrac{5}{6} < \ldots$. Condition (i) of the alternating series test is not satisfied. This series is divergent.

11. A. $$\lim_{n\to\infty}\left|\frac{\frac{(n+1)^{100}}{(n+1)!}}{\frac{n^{100}}{n!}}\right| = \lim_{n\to\infty}\left|\frac{n!(n+1)^{100}}{(n+1)!\,n^{100}}\right| = \lim_{n\to\infty}\left|\frac{(n+1)^{100}}{(n+1)\,n^{100}}\right| = \text{"}\lim_{n\to\infty}\frac{1}{n}\text{"} = 0 < 1$$

The series converges by the ratio test.

B. $$\lim_{n\to\infty}\left|\frac{\frac{(n+1-2)}{(n+1)2^{n+1}}}{\frac{(n-2)}{n\,2^{n}}}\right| = \lim_{n\to\infty}\left|\frac{(n+1-2)}{(n+1)2^{n+1}} \bullet \frac{n2^{n}}{(n-2)}\right|$$

$$= \lim_{n\to\infty}\left|\frac{n(n-1)}{2(n+1)(n-2)}\right| = \frac{1}{2} < 1.$$ The series converges by the ratio test.

C. $$\lim_{n\to\infty}\left|(-1)^{n-1}\frac{n+1}{\ln(n+1)}\right| = \lim_{n\to\infty}\left|\frac{n+1}{\ln(n+1)}\right| \overset{H}{=} \lim_{n\to\infty}\left|\frac{1}{1/(n+1)}\right|$$
$$= \lim_{n\to\infty}|n+1| = \infty$$

So, condition (ii) of the alternating series test is not satisfied.
This series diverges.

D. $$\lim_{n\to\infty}\left|(-1)^{n-1}\frac{\ln(n+1)}{n+1}\right| = \lim_{n\to\infty}\frac{\ln(n+1)}{n+1} \overset{H}{=} \lim_{n\to\infty}\frac{1/(n+1)}{1} = 0.$$

$$f(x) = \frac{\ln(x+1)}{x+1} \rightarrow f'(x) = \frac{\frac{1}{(x+1)}\bullet(x+1) - 1\bullet\ln(x+1)}{(x+1)^2}$$

$$f'(x) = \frac{1 - \ln(x+1)}{(x+1)^2} < 0 \text{ for } x \geq e - 1$$

So, while $a_1 < a_2$, $a_2 > a_3 > a_4 > \ldots a_{n+1} < a_n$ for $n \geq 2$
Thus, condition (i) is satisfied.

$$\lim_{n \to \infty} \left| (-1)^{n-1} \frac{\ln(n+1)}{n+1} \right| = \lim_{n \to \infty} \left| \frac{\ln(n+1)}{n+1} \right| \underset{=}{H} \lim_{n \to \infty} \left| \frac{1/(n+1)}{1} \right| = 0$$

So condition (ii) is satisfied. This series is convergent.

12. You can guess and check, $1/50^2 = 0.0004$, $1/100^2 = 0.0001$, thus 100 terms.
Or you can solve for n algebraically: $1/n^2 = 0.0001 \Rightarrow n^2 = 1/0.0001 \Rightarrow n^2 = 10{,}000$
n = 100 terms.

13. $a_1 = 1/2 = 0.5$, $a_2 = 1/17 \sim 0.0588$, $a_3 = 1/82 \sim 0.01219$ $a_4 = 1/257 \sim 0.004 < 0.01$
Use $S_4 = 1/2 - 1/17 + 1/82 - 1/257 \sim 0.449$
Note: Three terms is sufficient as $a_3 = 0.01219$ does not contribute excess to the first two decimal places.

Solving for n: $1/(n^4 + 1) = 0.01 \Rightarrow n^4 + 1 = 100 \Rightarrow n^4 = 99 \Rightarrow n \sim 3.15$ use n = 4

14. Find the radius and interval of convergence of the series.

A. A term of the series $a_n = x^n/(3n + 1)$, so

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{\frac{x^{n+1}}{3(n+1)+1}}{\frac{x^n}{3n+1}} \right| = \lim_{n\to\infty} \left| \frac{x(3n+4)}{(3n+1)} \right| = |x|$$

By the Ratio Test, this series converges if $|x| < 1$. The radius of convergence is R = 1.

When x = -1, $\sum_{n=0}^{\infty} \frac{x^n}{3n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{3n+1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{3n+1}$ which converges by A.S.T.

When x = 1, $\sum_{n=0}^{\infty} \frac{x^n}{3n+1} = \sum_{n=0}^{\infty} \frac{(1)^n}{3n+1} = \sum_{n=0}^{\infty} \frac{1}{3n+1}$ which diverges by limit comp. with 1/n

The interval of convergence is [-1, 1).

B. $$\sum_{n=0}^{\infty} \frac{(-3x)^n}{3n+1}$$

A term of the series $a_n = (-3x)^n/(3n + 1)$, so

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\dfrac{(-3x)^{n+1}}{3(n+1)+1}}{\dfrac{(-3x)^n}{3n+1}}\right| = \lim_{n\to\infty}\left|\frac{-3x(3n+4)}{(3n+1)}\right| = |-3x|$$

By the Ratio Test, this series converges if $|-3x| < 1$. The radius of convergence is $R = 1/3$.

When $x = -1/3$, $\sum_{n=0}^{\infty}\frac{(-3x)^n}{3n+1} = \sum_{n=0}^{\infty}\frac{(1)^n}{3n+1}$ which diverges by limit comp.

When $x = 1/3$, $\sum_{n=0}^{\infty}\frac{(-3x)^n}{3n+1} = \sum_{n=0}^{\infty}\frac{(-1)^n}{3n+1}$ which converges by A.S.T.

The interval of convergence is (-1/3, 1/3].

C. $\sum_{n=0}^{\infty}\frac{n}{4^n}(x+3)^n$

A term of the series $a_n = n(x + 3)^n/4^n$, so

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\dfrac{(n+1)(x+3)^{n+1}}{4^{n+1}}}{\dfrac{(n)(x+3)^n}{4^n}}\right| = \lim_{n\to\infty}\left|\frac{(x+3)(n+1)}{4n}\right| = \left|\frac{(x+3)}{4}\right|$$

By the Ratio Test, this series converges if $\left|\frac{(x+3)}{4}\right| < 1$.

The radius of convergence is $R = 4$.

When $x = -7$, $\sum_{n=0}^{\infty}\frac{n(x+3)^n}{4^n} = \sum_{n=0}^{\infty}\frac{n(-4)^n}{4^n}$ which diverges.

When $x = 1$, $\sum_{n=0}^{\infty}\frac{n(x+3)^n}{4^n} = \sum_{n=0}^{\infty}\frac{n(4)^n}{4^n}$ which diverges.

The interval of convergence is (-7, 1).

15. $$\int\frac{1}{1+x^5}\,dx = \int\frac{1}{1-(-x^5)}\,dx = \int\sum_{n=0}^{\infty}(-1)^n x^{5n}\,dx = \sum_{n=0}^{\infty}\int(-1)^n x^{5n}\,dx$$

$$= C + \sum_{n=0}^{\infty}(-1)^n\frac{x^{5n+1}}{5n+1}$$

16. $$\int_0^{0.1}\frac{1}{1+x^5}\,dx = \int_0^{0.1}\sum_{n=0}^{\infty}(-1)^n x^{5n}\,dx = \left[\sum_{n=0}^{\infty}(-1)^n\frac{x^{5n+1}}{5n+1}\right]_0^{0.1}$$

$$= \left[x - \frac{x^6}{6} + \frac{x^{11}}{11} - \frac{x^{16}}{16} + \ldots.\right]_0^{0.1} = \left[0.1 - \frac{(0.1)^6}{6} + \frac{(0.1)^{11}}{11} - \frac{(0.1)^{16}}{16} + \ldots.\right]$$

The series is alternating, the 2nd term $(0.1)^6/6 \sim 2.0 \times 10^{-7}$. Using the first term to sum the series, the error is less than 2.0×10^{-7}.

$I \sim 0.100000$

17. $$\frac{d}{dx}\ln\left(1 + x^2\right) = \frac{2x}{1 + x^2}$$

$$\ln(1 + x^2) = \int \frac{2x}{1 + x^2}\,dx = \int \frac{2x}{1 - \left(-x^2\right)}\,dx$$

$$= \int \sum_{n=0}^{\infty} (-1)^n 2x^{2n+1}\,dx = C + \sum_{n=0}^{\infty} (-1)^n \frac{2x^{2n+2}}{2n + 2} = \sum_{n=0}^{\infty} (-1)^n \frac{2x^{2(n+1)}}{2(n + 1)}$$

note: $x = 0$, $C = \ln(1 + x^2) = \ln(1) = 0$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2(n+1)}}{(n + 1)} = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{n}$$

18. $$C_3 = \frac{f^{(3)}(0)}{3!} = \frac{-8\cos(2 \bullet 0)}{6} = -\frac{8}{6} = -\frac{4}{3}$$

19. $$\frac{1}{1 - x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \ldots \quad R = 1,$$

$$\frac{1}{4 + x^2} = \frac{1/4}{1 + x^2/4} = \frac{1/4}{1 - \left(-x^2/4\right)} = \frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{-x^2}{4}\right)^n$$

This series will converge when $\left|-x^2/4\right| < 1,\ \left|x^2\right| < 4,\ |x| < 2$, $R = 2$

20. $$C_3 = \frac{f^{(3)}(0)}{3!} = -25,\ f^{(3)}(0) = -25 \bullet 6 = -150$$

21. $$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{2} \bullet \frac{1}{2!}\left(x - \frac{\pi}{4}\right)^2 - \frac{\sqrt{2}}{2} \bullet \frac{1}{3!}\left(x - \frac{\pi}{4}\right)^3 + \ldots$$

22. A. Given $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$

$$\cos(2x) = \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n}}{(2n)!} = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \ldots$$

B. Using the result from part A,

$$x\cos(2x) = \sum_{n=0}^{\infty} (-1)^n \frac{(2)^{2n}(x)^{2n+1}}{(2n)!} = 1 - \frac{2^2x^3}{2!} + \frac{2^4x^5}{4!} - \frac{2^6x^7}{6!} + \ldots$$

C. Given $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

$$e^{-\frac{x}{5}} = \sum_{n=0}^{\infty} \frac{\left(-1/5\right)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n}{5^n n!} = 1 + \frac{-x}{5 \bullet 1!} + \frac{x^2}{5^2 \bullet 2!} + \frac{-x^3}{5^3 \bullet 3!} + \ldots$$

23. A. $$\frac{\sin x}{x} = \frac{1}{x} \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n+1)!}$$

$$\int \frac{\sin x}{x} dx = \int \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n+1)!} dx = C + \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)(2n+1)!}$$

B. $$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!},\ e^x - 1 = \sum_{n=1}^{\infty} \frac{x^n}{n!},\ \frac{e^x - 1}{x} = \sum_{n=1}^{\infty} \frac{x^{n-1}}{n!}$$

$$\int \frac{e^x - 1}{x} dx = \int \sum_{n=1}^{\infty} \frac{x^{n-1}}{n!} dx = C + \sum_{n=1}^{\infty} \frac{x^n}{n \bullet n!}$$

24. A. $$3 \bullet \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{3^{2n+1}(2n+1)!} = 3\sin\left(\frac{\pi}{3}\right) = 3 \bullet \frac{\sqrt{3}}{2}$$

B. $$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} = -1 + 1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} = -1 + \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = -1 + e^{-1}$$

C. $$\sum_{n=0}^{\infty} \frac{3^n}{5^n n!} = e^{3/5}$$