MA 182 Test #1 – Spring 2009

Dr. Katiraie (100 points) Name (1 point)

Show all of your work on the test paper. Full credit is not given unless the answer follows from the work shown.

1. Use the Fundamental Theorem of Calculus to evaluate the following definite integral. (15 points)

a) $\displaystyle\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx$

let $u = 2x+1$

$du = 2\,dx$

$= \frac{1}{2}\int (2x+1)^{-\frac{1}{2}}\, 2\,dx$

$= \frac{1}{2}\int u^{-1/2}\,du = \frac{1}{2}\,\frac{u^{1/2}}{1/2} = (2x+1)^{1/2}\Big|_0^4 = 3 - 1 = \boxed{2}$

b) $\displaystyle\int_1^{64} \frac{1+\sqrt[3]{x}}{\sqrt{x}}\,dx = \int_1^{64}\left(x^{-1/2} + x^{-\frac{1}{6}}\right)dx = \frac{x^{\frac{1}{2}}}{1/2} + \frac{x^{\frac{5}{6}}}{\frac{5}{6}}\Bigg|_1^{64}$

$= 2x^{\frac{1}{2}} + \frac{6}{5}x^{\frac{5}{6}}\Bigg|_1^{64}$

$= \boxed{51.2}$

c) $\displaystyle\int_0^{\pi/4} \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta = \int_0^{\pi/4}\left(\sec^2\theta + 1\right)d\theta$

$= \tan\theta + \theta\Big|_0^{\frac{\pi}{4}} = \left(\tan\frac{\pi}{4} + \frac{\pi}{4}\right) - (0+0)$

$= \boxed{1 + \frac{\pi}{4}}$

2. Evaluate 8 of the following 9 indefinite integrals. You may do all 9 for up to 9 points extra credit. Indicate which one you are doing for extra credit. (72 points)

(a) $\int \frac{3}{\sqrt{4-x^2}}\,dx \quad = \boxed{3\sin^{-1}\left(\frac{x}{2}\right)+c}$

Please: Recall $\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\left(\frac{u}{a}\right)+c$

(b) $\int \sin^3 x\,dx = \int \sin^2 x \sin x\,dx$

$= \int (1-\cos^2 x)\sin x\,dx$

$= \int \sin x\,dx + \int \cos^2 x(-\sin x\,dx)$

$u = \cos x$

$du = -\sin x\,dx$

$= \boxed{-\cos x + \frac{\cos^3 x}{3} + c}$

(c) $\int x^2 \cos x\,dx$

$u = x^2 \quad dv = \cos x\,dx$

$du = 2x\,dx \quad v = \sin x$

$= x^2\sin x - 2\int x\sin x\,dx$

$u = x \quad dv = \sin x\,dx$

$du = dx \quad v = -\cos x$

$= x^2\sin x - 2\left[-x\cos x - \int -\cos x\,dx\right]$

$= \boxed{x^2\sin x + 2x\cos x - 2\sin x + c}$

OVER

(d) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

let $u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2} x^{-1/2} dx$

$du = \frac{1}{2\sqrt{x}} dx$

$$= 2\int e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} dx$$

$$= 2\int e^{u} du = 2e^{u} + c = \boxed{2e^{\sqrt{x}} + c}$$

(e) $\int \frac{x+5}{x^2+9} dx = \int \frac{x}{x^2+9} dx + \int \frac{5}{x^2+9} dx$

let $u = x^2+9$

$du = 2x\, dx$

$$= \frac{1}{2} \int \frac{du}{u}$$

$$= \frac{1}{2} \ln|x^2+9| + 5 \cdot \frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right) + c$$

$$= \boxed{\frac{1}{2} \ln|x^2+9| + \frac{5}{3} \tan^{-1}\left(\frac{x}{3}\right) + c}$$

(f) $\int x^3 \ln x\, dx$

let $u = \ln x$ $\qquad dv = x^3 dx$

$du = \frac{1}{x} dx$ $\qquad v = \frac{x^4}{4}$

$$\int x^3 \ln x\, dx = \frac{1}{4} x^4 \ln x - \int \frac{1}{x} \frac{x^4}{4} dx$$

$$= \frac{x^4}{4} \ln x - \frac{1}{4} \int x^3 dx$$

$$= \boxed{\frac{x^4}{4} \ln x - \frac{1}{16} x^4 + c}$$

OVER →

(g) $\int \frac{x+11}{x^2+2x-3}dx$

$$\frac{x+11}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$$

$$A\Big|_{x=-3} = \frac{-3+11}{-3-1} = \frac{8}{-4} = -2$$

$$= \int \left(\frac{-2}{x+3} + \frac{3}{x-1} \right) dx \qquad B\Big|_{x=1} = \frac{1+11}{1+3} = \frac{12}{4} = 3$$

$$\boxed{= -2\ln|x+3| + 3\ln|x-1| + c}$$

OR

$$\boxed{= \ln\left|\frac{(x-1)^3}{(x+3)^2}\right| + c}$$

(h) $\int \frac{(x^3-4)^2}{x^4}dx$ $= \int \frac{x^6 - 8x^3 + 16}{x^4} dx$

$$= \int \left(x^2 - 8x^{-1} + 16x^{-4} \right) dx$$

$$= \frac{x^3}{3} - 8\ln|x| + \frac{16x^{-3}}{-3} + c$$

$$\boxed{= \frac{1}{3}x^3 - 8\ln|x| - \frac{16}{3x^3} + c}$$

OVER →

(i) $\int \sqrt{16-x^2}\,dx$ HINT: Use the trigonometric substitution $x = 4\sin\theta$, where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ and Recall the following trig. identities

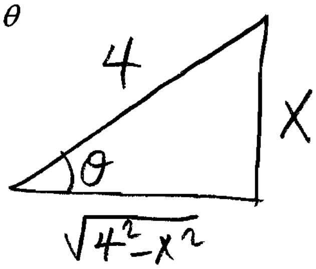
$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$X = 4\sin\theta$

$dx = 4\cos\theta\, d\theta$



$$= \int \sqrt{16 - 16\sin^2\theta}\ \ 4\cos\theta\, d\theta$$

$$= \int \sqrt{16(1-\sin^2\theta)}\ \ 4\cos\theta\, d\theta$$

$$= \int \sqrt{16\cos^2\theta}\ \ 4\cos\theta\, d\theta = \int 4\cos\theta \cdot 4\cos\theta\, d\theta$$

$$= 16\int \cos^2\theta\, d\theta$$

$$= 16\int \frac{1}{2}(1+\cos 2\theta)\, d\theta$$

$$= 8\left[\theta + \frac{1}{2}\sin(2\theta)\right] + c$$

$$= 8\left[\sin^{-1}\left(\frac{X}{4}\right) + \frac{1}{2}(2\sin\theta\cos\theta)\right] + c$$

OVER

$$= 8\left[\sin^{-1}\left(\frac{X}{4}\right) + \frac{X}{4}\,\frac{\sqrt{16-X^2}}{4}\right] + c$$

$$= 8\sin^{-1}\left(\frac{X}{4}\right) + \frac{X\sqrt{16-X^2}}{2} + c$$

3. Using the following formula from the Table of Integrals

$$\int \frac{du}{u^2\sqrt{a^2+u^2}} = -\frac{\sqrt{a^2+u^2}}{a^2 u} + C$$

Evaluate the following integral. (12 points)

$$\int \frac{1}{x^2\sqrt{4x^2+9}}\,dx$$

$a^2 = 9$

$a = 3$

$u^2 = 4x^2$

$u = 2x \implies \boxed{x = \frac{u}{2}}$

$\boxed{dx = \frac{1}{2}du}$

$$= \int \frac{1}{\left(\frac{u}{2}\right)^2\sqrt{u^2+a^2}}\frac{1}{2}du$$

$$= \int \frac{\frac{1}{2}}{\frac{1}{4}u^2\sqrt{u^2+a^2}}du$$

$$= 2\int \frac{du}{u^2\sqrt{a^2+u^2}} = 2\left[-\frac{\sqrt{a^2+u^2}}{a^2 u}\right] + c$$

$$= -2\,\frac{\sqrt{9+4x^2}}{9(2x)} + c$$

$$= \boxed{-\frac{\sqrt{9+4x^2}}{9x} + c}$$