MA 182 Test #2 – Spring 2009

Dr. Katiraie (100 points) **Name** (1 Point)

Show all of your work on the test paper. Full credit is not given unless the answer follows from the work shown.

$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$ $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ $\sin 2\theta = 2\sin\theta\cos\theta$

1. Evaluate the following indefinite integrals. **(5 Points Each)**

(a) $\int x^2 \cos x\,dx = x^2\sin x - \int 2x\sin x\,dx$

$u = x^2 \quad dv = \cos x\,dx$
$du = 2x\,dx \quad v = \sin x$

$= x^2\sin x + 2x\cos x - 2\int \cos x\,dx$

$u = 2x \quad dv = -\sin x\,dx$
$du = 2\,dx \quad v = \cos x$

$= x^2\sin x + 2x\cos x - 2\sin x + C$

(b) $\int \sin^2(7x)\,dx = \int \frac{1}{2}(1-\cos(14x))\,dx$

$= \frac{1}{2}x - \frac{1}{28}\sin(14x) + C$

(c) $\int \frac{x+6}{x^2+25}\,dx = \int \frac{x}{x^2+25}\,dx + \int \frac{6}{x^2+25}\,dx$

$u = x^2+25$
$du = 2x\,dx$

$= \frac{1}{2}\ln|x^2+25| + \frac{6}{5}\tan^{-1}\left(\frac{x}{5}\right) + C$

OVER

(#1 Continued) Evaluate the following indefinite integrals. (5 Points Each)

(d) $\int \frac{3e^{\sqrt{x}}}{\sqrt{x}}dx$

$u = \sqrt{x}$

$du = \frac{1}{2} x^{-1/2} dx$

$= (2)3\int \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx$

$= 6\int e^{u}\,du$

$= 6e^{u} + c = \boxed{6e^{\sqrt{x}} + c}$

(e) $\int \frac{x+10}{x^2+5x-6}dx$ $\Rightarrow$ $\frac{x+10}{(x+6)(x-1)} = \frac{A}{x+6} + \frac{B}{x-1}$

$A\Big|_{x=-6} = \frac{-6+10}{-6-1} = \frac{4}{-7} = \frac{-4}{7}$

$B\Big|_{x=+1} = \frac{1+10}{1+6} = 11/7$

$\int \frac{x+10}{x^2+5x-6}\,dx = \int \frac{-4/7}{x+6}\,dx + \int \frac{11/7}{x-1}\,dx$

$= \frac{-4}{7}\ln|x+6| + 11/7\,\ln|x-1| + c$

OR $= \ln\left|\frac{(x-1)^{11/7}}{(x+6)^{4/7}}\right| + c$

OVER

2. Given the integral $\int_1^4 \frac{1}{\sqrt{1+x^3}}dx$,

(a) Use the program NUMINT to approximate the value of this integral by using the Trapezoidal Rule with n = 50. Write down your answer with at least 6 decimal places. (2 points)

0.896014

(b) If E_T is the error which occurs when the Trapezoidal Rule is used with n subintervals to evaluate the integral $\int_a^b f(x)dx$, then $|E_T| \le \frac{K(b-a)^3}{12n^2}$, where $|f''(x)| \le K$ for $a \le x \le b$.

i) Please Show Your Work and Find $f''(x)$ (4 points)

$f(x) = (1+x^3)^{-1/2}$

$f'(x) = -\frac{1}{2}(1+x^3)^{-3/2}(3x^2) = -\frac{3}{2}x^2(1+x^3)^{-3/2}$

$f''(x) = -3x(1+x^3)^{-3/2} - \frac{3}{2}x^2\left(-\frac{3}{2}\right)(1+x^3)^{-\frac{5}{2}} \cdot 3x^2$

$f''(x) = -3x(1+x^3)^{-\frac{3}{2}} + \frac{27}{4}x^4(1+x^3)^{-5/2}$

ii) Using your calculator, find K so that $|f''(x)| \le K$ (2 points)

$|f''(x)| \le 0.37721646$

iii) How large should we take n in order to guarantee that the Trapezoidal Rule approximation for $\int_1^4 \frac{1}{\sqrt{1+x^3}}dx$ is accurate to within 0.00001? (4 points)

$$\frac{0.37721646(4-1)^3}{12n^2} < 0.00001$$

$291.33 < n$

$\boxed{n \geq 292}$

3. You will need to use Simpson's Rule in this problem. (6 points)

Simpson's Rule:

$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}[f(x_o) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)],$$

where n is even and $\Delta x = (b-a)/n$

The speed of an object was observed at two-second intervals and recorded in the chart below. Use Simpson's Rule to estimate the distance traveled by the object.

t (sec)	0	2	4	6	8	10	12
v (ft/sec)	20	25	28	26	30	32	34

Distance $= \frac{2}{3}\left[20 + 4(25) + 2(28) + 4(26) + 2(30) + 4(32) + 34\right]$

$= 334.67$ feet or $\frac{1004}{3}$ feet

4. Use Calculus to evaluate the limit. (8 points)
Show all of your work on the test paper. Full credit is not given unless the answer follows from the work shown.

$$\lim_{x \to 0^+} (1-x)^{1/x} = 1^\infty$$

let $y = (1-x)^{1/x}$

$\ln y = \frac{1}{x}\ln(1-x)$

$\lim_{x\to 0^+} \ln y = \frac{\ln(1-x)}{x} = \frac{0}{0}$ Indeterminate

L.H Rule $= \lim_{x \to 0} \frac{\frac{1}{1-x} \cdot -1}{1} = -1$

$\lim_{x\to 0^+} \ln y = -1 \Rightarrow \lim_{x\to 0^+} y = e^{-1} = \frac{1}{e}$

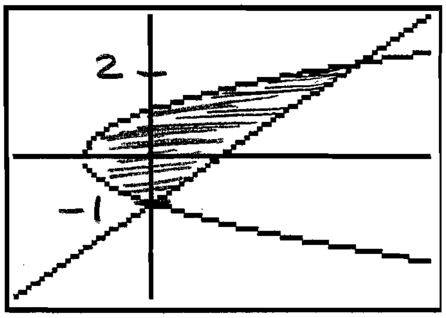
5. Set up, but do not integrate or evaluate, an integral which represents the area of the region between the graphs of $x = y + 1$ and $x = y^2 - 1$. The region is shown below. (7 points)

$y^2 - 1 = y + 1$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 2 \quad y = -1$



$$\int_{y=-1}^{y=2} \left(y+1-(y^2-1)\right) dy = \int_{y=-1}^{y=2} \left(y+2-y^2\right) dy$$

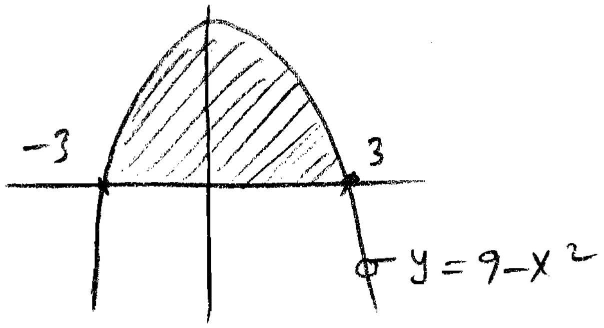
6.
Set up, but do not evaluate, an integral which represents the volume of the solid of revolution obtained by revolving the region bounded by the curve $y = 9 - x^2$ and the x-axis about the x-axis.

(7 points)

$9 - x^2 = 0$

$9 = x^2$

$x = \pm 3$



$9 - x^2 = 0$

$9 = x^2$

$x = \pm 3$

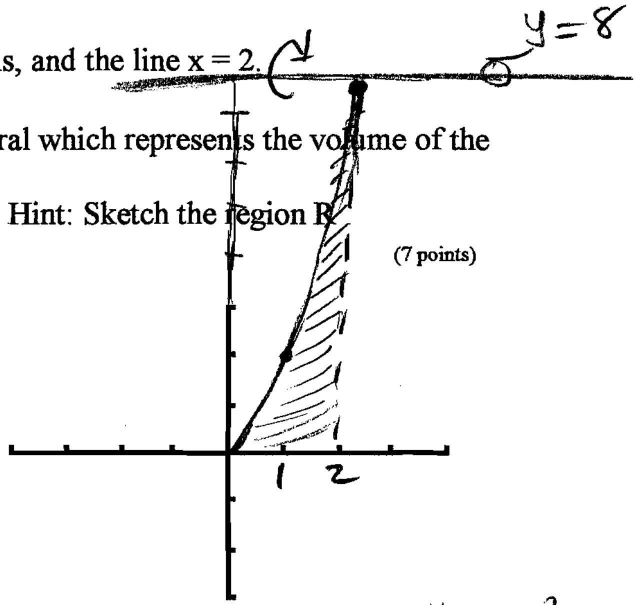
$$\int_{x=-3}^{x=3} \pi (9-x^2)^2 \, dx$$

$$= \int_{x=-3}^{x=3} \pi (81 - 18x^2 + x^4) \, dx$$

7. Let R be the region bounded by $y = 2x^2$, the x-axis, and the line x = 2.

(a) **Set up, but do not integrate or evaluate**, the integral which represents the volume of the solid of revolution obtained by rotating region R

Hint: Sketch the region R

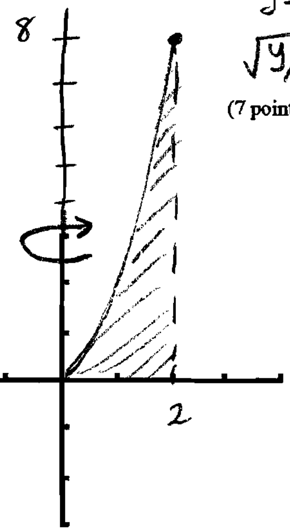


(i) about the line y = 8 (7 points)

$R_{out} = 8 \Rightarrow A_{out} = \pi(8)^2$

$r_{inner} = 8 - 2x^2 \Rightarrow A_{inner} = \pi(8-2x^2)^2$

$$\boxed{V = \int_{x=0}^{x=2} \left(\pi(8)^2 - \pi(8-2x^2)^2\right)dx}$$



$y = 2x^2$

$\sqrt{y/2} = x$

(ii) about the y-axis (7 points)

$R_{out} = 2 \Rightarrow A_{out} = \pi(2)^2 = 4\pi$

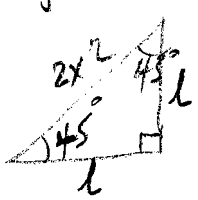
$r_{inner} = \sqrt{\frac{y}{2}} \Rightarrow A_{inner} = \pi\left(\sqrt{\frac{y}{2}}\right)^2$

$= \pi \cdot y/2 = \frac{\pi y}{2}$

$$\boxed{V = \int_{y=0}^{y=8} \left(4\pi - \frac{\pi}{2}y\right)dy}$$

(b) The base of a solid is region R described above. Parallel Cross-sections perpendicular to the base ~~are squares~~ are isosceles right triangles with hypothenuse lying along the base. **Set up, but do not integrate or evaluate**, the integral which represents the volume of the solid. (8 points)

hypothenuse $= 2x^2$

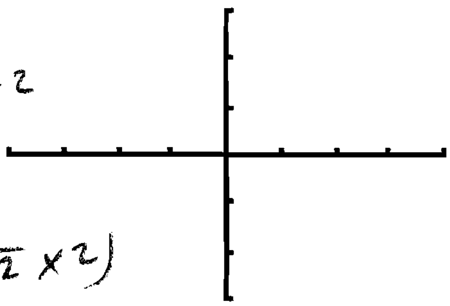


$\sin 45° = \frac{l}{2x^2} \Rightarrow l = \frac{\sqrt{2}}{2} \cdot 2x^2$

$l = \sqrt{2}x^2$

$A = \frac{1}{2} b \cdot h = \frac{1}{2}(\sqrt{2}x^2)(\sqrt{2}x^2)$

$A = x^4$



$$V = \int_0^2 x^4\,dx$$

8. Evaluate each improper integral or show that it diverges.
Show all of your work on the test paper. Full credit is not given unless the answer follows from the work shown.

(a) $\int_{-2}^{3} \frac{1}{x^3}\,dx = \int_{-2}^{0} \frac{1}{x^3}\,dx + \int_{0}^{3} \frac{1}{x^3}\,dx$ (6 points)

$\lim_{t\to 0^-} \int_{-2}^{t} x^{-3}\,dx + \lim_{t\to 0^+} \int_{t}^{3} x^{-3}\,dx$

$\lim_{t\to 0^-} \frac{x^{-2}}{-2}\Big|_{-2}^{t} + \lim_{t\to 0^+} \frac{x^{-2}}{-2}\Big|_{t}^{3}$

$\frac{-1}{2x^2}\Big|_{-2}^{t\to 0^-} + \frac{-1}{2x^2}\Big|_{t\to 0^+}^{3}$

$\left(\frac{-1}{2(0)} - \frac{-1}{4}\right) + \left(\frac{-1}{18} - \frac{-1}{0^+}\right) = \infty + \infty = \boxed{\infty \quad \text{Diverges}}$

$+\infty$

(b) $\int_{2}^{\infty} e^{-x}\,dx = \lim_{t\to\infty} \int_{2}^{t} e^{-x}\,dx$ (6 points)

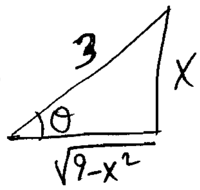
$\lim_{t\to\infty} -e^{-x}\Big|_{2}^{t} = \lim_{t\to\infty} \frac{-1}{e^x}\Big|_{2}^{t}$

$= \frac{-1}{e^t} - \frac{-1}{e^2} = \frac{-1}{\infty} + \frac{1}{e^2} = \boxed{\frac{1}{e^2}}$

0

☺☺☺☺ Extra Credit ☺☺☺☺

9) Evaluate the following indefinite integrals. (4 points) $\sin\theta = \frac{X}{3}$



$\int \sqrt{9-x^2}\,dx$

HINT: Use the trigonometric substitution $x = 3\sin\theta$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

$X = 3\sin\theta$

$\sqrt{9-x^2} = \sqrt{9-9\sin^2\theta}$

$= 3\sqrt{1-\sin^2\theta} = 3\cos\theta$

$dx = 3\cos\theta\, d\theta$

$= \int 3\cos\theta \cdot 3\cos\theta\, d\theta$

$= 9\int \cos^2\theta\, d\theta$

$= 9\left[\int \frac{1}{2}(1+\cos 2\theta)\,d\theta\right]$

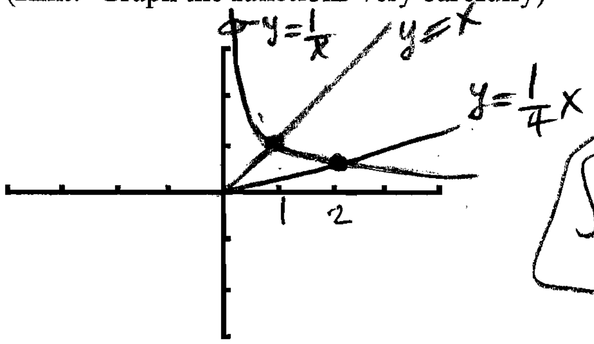
$= 9\left(\frac{1}{2}\theta + \frac{1}{4}\sin(2\theta)\right) + c = \frac{9}{2}\theta + \frac{9}{4}\sin(2\theta) + c$

$= \frac{9}{2}\theta + \frac{9}{4}2\sin\theta\cos\theta + c = \frac{9}{2}\left[\sin^{-1}\left(\frac{X}{3}\right) + \frac{X}{3}\cdot\frac{\sqrt{9-X^2}}{3}\right] + c$

10. A) Set up, but do not integrate or evaluate, integral(s) which represents the area of the region R between the graphs of $y = \frac{1}{x}$, $y = x$, $y = \frac{1}{4}x$ (3 points)

assume $X > 0$

(Hint: Graph the functions very carefully)



$\frac{1}{4}X = \frac{1}{X}$

$X^2 = 4 \Rightarrow X = 2$

$$\boxed{\int_{X=0}^{X=1}\left(X - \frac{1}{4}X\right)dx + \int_{X=1}^{X=2}\left(\frac{1}{X} - \frac{1}{4}X\right)dx}$$

10. B) Set up, but do not integrate or evaluate, the integral(s) which represents the volume of the solid of revolution obtained by rotating region R about the x axis. and $X > 0$ (3 points)

$$\int_{X=0}^{X=1}\left(\pi X^2 - \pi\left(\frac{1}{4}X\right)^2\right)dX + \int_{X=1}^{2}\left(\pi\left(\frac{1}{X}\right)^2 - \pi\left(\frac{1}{4}X\right)^2\right)dX$$

class Avg 85.2

14 A
4 B
4 C
3 D
2 F