MA 182 Test #4 Part I Fall 2013

Dr. Katiraie (50 points + 5 Points Extra Credit) **Name** ______________________________

Show all of your work on the test paper. Full credit is not given unless the answer follows from the work shown.

$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$ $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ $\sin 2\theta = 2\sin\theta\cos\theta$

1. A tank contains 1000 liters of pure water. Brine that contains 0.07 kg of salt per liter of water enters the tank at a rate of 8 liters per minute. In addition, brine that contains 0.05 kg of salt per liter enters the tank at a rate of 5 liters per minute. The well-mixed solution drains from the tank at a rate of 12 liters per minute. Let y be the amount of salt in the tank at time t.

   Hint: $\frac{dy}{dt} = \text{ rate in } - \text{ rate out}$

(a) Set up the differential equation that models this situation and state the initial condition.

(3 points)

(b) Solve this differential equation.

(3 points)

(c) How much salt will be in the tank after half an hour?

(1 point)

(d) How much salt will be in the tank after a very long time?

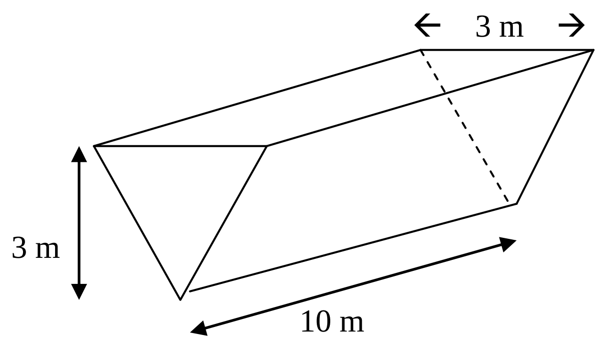
(1 point)

2. Use Calculus to evaluate the following limit. (5 points)

$$\lim_{x\to\infty}\left(1+\frac{5}{x}\right)^{x}$$

3. The ends of a 10 m. long water trough are triangles having top side of length 3 m.
If the trough is filled with water to a depth of 2 meters, (density = 1000 kg/m^3.)
Set up, but do not integrate or evaluate, an integral to
Find the work required to pump all of the water (6 points)

(a) Over the top rim of the container



(b) Through a pipe that rises to a height of 4 m above the top of the container.

4) Solve the initial value problem $\frac{dy}{dt} + 3yt = y$ y(0)=5 (6 points)

5) (Parts a -- f) Determine whether each of the following series converges or diverges. Tell which test you used. If you used the comparison test or limit comparison test, give the comparison series. **If the series is a geometric series, give r, and if it is a convergent geometric series, find the sum**. (24 Points, 4 Points Each)

a) $\sum_{n=1}^{\infty} (-1)^n \frac{2}{2n^3 + n}$

b) $\sum_{n=1}^{\infty} \frac{n}{3^n (n+1)}$

c) $\sum_{n=1}^{\infty} 5\left(\frac{2}{3}\right)^{n-1}$

d) $\sum_{n=1}^{\infty} \frac{n+1}{3+2n}$

e) $\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$

f) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

Extra Credit (5 Points)

6. A roast is taken from the refrigerator (where the temperature is 40^{o} F) and put in a 360^{o} F oven. One hour later, the meat thermometer shows a temperature of 110^{o} F. Assuming that the roast heats at a rate which obeys Newton's Law of Heating and Cooling, that is, that the rate of heating or cooling of an object is proportional to the difference between the temperature of the object and the temperature of its surroundings, the differential equation that models this situation is

$$\frac{dT}{dt} = k(360 - T)$$

where T is the temperature of the object at time t.

a) Solve this differential equation subject to the conditions given. Find the value of all constants.

Clearly State the value of C

Clearly State the value of K

(b) What is the temperature of the roast after 120 minutes?
(Round your answer to the nearest whole number.)