

Dr. Katiraie

(50 points + 15 Points Extra Credit) **Name** _____

Show all of your work on the test paper. Full credit is not given unless the answer follows from the work shown.

$$\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

7(a) Write a power series for $\frac{1}{5-x}$ and state the radius of convergence. (4 Points)

(b) Use this series to find a power series for $\ln(5-x)$ and state the radius of convergence (4 Points)
(Hint: Do not forget to find the value of C)

(c) Write a power series for $\frac{1}{5+x}$ and state the radius of convergence. (2 Points)

(d) Use this series to find a power series for $\ln(5+x)$ and state the radius of convergence.

(2 Points)

(Hint: Do not forget to find the value of C)

(e) Use your answers to (b) and (d) to find a power series for $\ln\left(\frac{5-x}{5+x}\right)$.

Hint: $\ln\left(\frac{A}{B}\right) = \ln A - \ln B$.

(2 Points)

8. Develop or write the Taylor series for $f(x) = e^{\frac{x}{10}}$ centered at $a = 5$ (8 Points)
Please write your answer in expanded form and in sigma form.

$$f(x) = \frac{f(a)}{0!} (x-a)^0 + \frac{f'(a)}{1!} (x-a)^1 + \frac{f''(a)}{2!} (x-a)^2 + \frac{f'''(a)}{3!} (x-a)^3 + \dots$$

9(a) Develop or write the Maclaurin series for $f(x) = \cos x$. (6 Points)

$$f(x) = \frac{f(a)}{0!} (x-a)^0 + \frac{f^1(a)}{1!} (x-a)^1 + \frac{f^2(a)}{2!} (x-a)^2 + \frac{f^3(a)}{3!} (x-a)^3 + \dots$$

(b) Use your answer to part (a) to write the Maclaurin series for $f(x) = \cos(x^2)$. (3 Points)

(c) Use your answer to part (b) to write the Maclaurin series for $f(x) = \frac{1 - \cos(x^2)}{x}$. (3 Points)

10. Write the first five terms of the Taylor series for $f(x) = \ln x$ centered at $a = 6$.

You do **not** need to find an expression for the general term.

(8 Points)

$$f(x) = \frac{f(a)}{0!} (x-a)^0 + \frac{f'(a)}{1!} (x-a)^1 + \frac{f''(a)}{2!} (x-a)^2 + \frac{f'''(a)}{3!} (x-a)^3 + \dots$$

11. Find the radius and interval of convergence for the power series $\sum_{n=2}^{\infty} (-1)^n \frac{x^n}{8^n \ln n}$

and **write your answers in the space provided.**

(8 Points)

Radius of convergence _____

Interval of convergence _____



Extra Credit

12) A ball is dropped from a height of 100 feet. Each time it hits the floor, the ball rebounds to $\frac{2}{5}$ of its previous height. Find the total distance the ball travels. (Must show procedure in order to get full credit). (5 Points)

13) Find a Cartesian equation for the curve $r = 2 \tan \theta \sec \theta$
(Hint: Rewrite $r = 2 \tan \theta \sec \theta$ into an equation containing x and y). (5 Points)
(Must show procedure in order to get full credit).



Extra Credit

13) Find the area of the region that lies inside the curve $r = 3 - 3 \sin \theta$ and outside the curve $r = 3$
(Must show procedure in order to get full credit).

(5 Points)