MONTGOMERY COLLEGE
Department of Mathematics
Rockville Campus

**MA 182 - REVIEW PROBLEMS** **REVISED SPRING 2008**

1. State whether each of the following can be integrated by partial fractions (PF), integration by parts (PI), u-substitution (U), or none of these (N). You do not have to evaluate the integrals.

   a. $\int \frac{dx}{x^2-4}$ b. $\int \sqrt{\cos 2x}\, dx$ c. $\int \frac{6x\,dx}{x^2+8}$

Integrate:

2. $\int \frac{3}{\sqrt{kx}} dx$

3. $\int \frac{(5x+3)}{x^3-2x^2-3x} dx$

4. $\int 5\sin(3x)\, dx$

5. $\int \frac{6}{\cos^2 3x} dx$

6. $\int \frac{x}{e^{x^2}} dx$

7. $\int \arctan x\, dx$

8. $\int \sin 2x \ \cos^4 2x\, dx$

9. $\int \frac{\cos x\, dx}{1 - \sin x}$

10. $\int \frac{2x^3+x+3}{x^2} dx$

11. $\int \left(1-e^{-x}\right)^2 dx$

Integrate the following using the table of integrals on the inside rear book cover.

12. $\int \frac{dx}{25+16x^2}$

13. $\int \cos^4 2x\, dx$

14. $\int \frac{1}{x\sqrt{3+9x^2}} dx$

15. $\int x^2 \cos 3x\, dx$

16. Write $\frac{-2x-6}{(x^2+3)(x-1)}$ as the sum of two partial fractions.

17. The velocity $V$ at time $t$ of a point moving along a coordinate line is $V = te^{-3t}$ ft/sec. If the point is at the origin of $t=0$, find a formula for its position $s$ at time $t$.

18. Food is placed in a freezer. After $t$ hours, the temperature of the food is changing at a rate of $R = 10e^{-0.2t}$ where $R$ is in degrees F/hr. How much has the temperature dropped in the first two hours?

19. Solve $\frac{dy}{dt} = ky^2$ assuming $y \neq 0$. Express $y$ in terms of $t$.

20. Solve $\frac{dy}{dx} = 2xy$ where $y(0) = 3$ Express $y$ in terms of $x$.

Determine the convergence or divergence of each integral. If convergent, find the value.

21. $\int_{-1}^{1} \frac{dx}{x^{2/3}}$

22. $\int_{1}^{\infty} \frac{\ln x}{x} dx$

23. $\int_{0}^{\infty} \frac{\sin x}{e^x} dx$

24. Find the volume of the solid formed when the region bounded by the $x$-axis and the curve $y = 4 - x^2$ is revolved about the $x$-axis.

25. a. Write an integral equal to the arc length of $y = x^2$ from (1,1) to (3,9).

b. Approximate the arc length with a calculator program.

Evaluate using L'Hopital's Rule or other analytical methods.

26. $\lim_{x \to 0} \frac{\sin x - x\cos x}{x^3}$

27. $\lim_{x \to \infty} \frac{x^2 - 5}{2x^2 + 3x}$

28. $\lim_{x \to \infty} \frac{\sin\left(\frac{3}{x}\right)}{\frac{2}{x}}$

29. a. Tell why you cannot use L'Hopital's Rule to find $\lim_{x \to 0^+} \frac{\cos x}{x}$.

b. Evaluate the limit in part a and give evidence to support your answer.

Evaluate each limit below and give evidence to support your answer.

30. $\lim_{x \to \infty} \left(3 - e^{2x}\right)$

31. $\lim_{x \to 1} x^{\frac{1}{x-1}}$

32. An object is thrown vertically upward from a height of 8 ft with a velocity of 48 ft/sec. Use $a = -32$ ft/sec$^2$.

a. When will it be 40 ft above the ground?

b. Show how to find the maximum height of the object analytically.

c. Show how to find when the object hits the ground using information from your graphing calculator.

33. If $\int_{0}^{5} g(x)\,dx = 3$ find the average value of $g(x)$ on [0,5].

34. A tank is filling with water. The volume of water in a tank has a rate of change *R(t)* in ft$^3$/min after t minutes.
What does $\int_4^{10} R(t)dt$ represent about the water? Be specific and include the correct units.

35. Let $\frac{dy}{dx} = x + y^2$. Draw the direction field tangents at the points (-2,1) and (3,1) on an *xy*-graph.

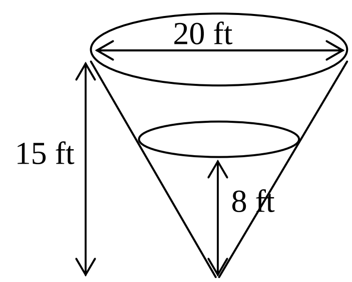
36. Using the table below, show how to use n = 2 subintervals and trapezoids to approximate $\int_1^5 g(x)\,dx$.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $g(x)$ | 3 | 1 | 2 | 2 | 4 |

37. Find the area of the region enclosed by $y = x^2$ and $y = x + 6$.

38. A spring has a natural length of 12 ft. A force of 80 lb stretches it to a length of 14 ft. Find the work done in stretching it from a length of 15 ft. to a length of 16 ft.

39. A tank in the shape of an inverted cone contains some water. The tank has diameter 20 feet at the top and is 15 feet deep (See figure). The water is 8 feet deep and has density $p = 62.5$ lb/ft$^3$. Write an integral that represents the work required to pump all the water over the top rim. Be sure to draw a picture showing how you are setting the problem up. Do not evaluate the integral.




40. Sketch a solution to $\dfrac{dy}{dx} = x + y^2$ with $y(1) = 0$ by first drawing the slope field on your calculator. Then sketch a solution curve using the slope field.

41. Consider the differential equation $\dfrac{dy}{dx} = x - 2y$. Show how to use Euler's Method (without a calculator program) to approximate $y(2.3)$ by starting at (2,1) and using steps of $\Delta x = 0.1$.

42. Let $a_n = \dfrac{n+2}{2n-1}$

a. Write the first 4 terms of the sequence $\{a_n\}_{n=1}^{\infty}$

b. Does $\{a_n\}_{n=1}^{\infty}$ have a limit? If so, find it.

c. Does $\sum_{n=1}^{\infty} a_n$ converge or diverge? Give a reason for your answer.

43. Let $f(x) = \ln(x+1)$. Find the Taylor Series for $x$ near 0 by taking derivatives. Write the first 4 nonzero

terms of the series.

44. a) Find the Taylor polynomial of degree 4 for y = cos(x) with center a = 0.
b) Use the result to approximate cos(.2).
c) Estimate the magnitude of the error in your approximation in part b.

45. Find the Taylor Series for $e^{\frac{x}{2}}$ centered at $a = 2$ and use the Ratio Test to show that this series converges for all $x$.

46 100 g of a radioactive substance decays. After 10 days, 80 g remain. How much would remain after 14 days?

47. A ball is dropped from a height of 40 ft. Each time it hits the floor, the ball rebounds to $3/4$ of its previous height. Find the total distance the ball travels.

48. Determine the sum of $\sum_{n=2}^{\infty} \frac{3}{2^{n+2}}$

49. Find a power series representation of $\frac{3}{1+8x^3}$ and state its interval of convergence.

50. Determine the convergence or divergence of the following series. Justify your answer.

a. $\sum_{n=2}^{\infty} \frac{3n}{n!2^{n+1}}$ b. $\sum_{n=1}^{\infty} \frac{2n+1}{4n^2+n-1}$ c. $\sum_{n=2}^{\infty} \frac{3}{n\sqrt[3]{\ln n}}$

51. Determine if the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+2)}{2n^2+2n}$ converges or diverges. Justify your answer.

52. Determine the radius and interval of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n (x+1)^n}{n3^n}$.

53. Find the MacLaurin Series for $\ln(1+x)$ and determine how many terms must be used in this series to find $\ln 1.02$ accurate to 7 decimal places.

54. Convert to polar coordinates. Use $r > 0$ and $0 \le \theta < 2\pi$.

a. (2, -2) b. $\left(-2,-2\sqrt{3}\right)$ c. $\left(-\sqrt{3},1\right)$

55. Convert the polar equation $r = \frac{6}{\sin\theta}$ to an equation in $x$ and $y$.

56. Sketch the graph of $r = 2\cos\theta$ without using a graphing calculator.

57. Find the area of the region that is inside $r = 2\sin(3\theta)$ and outside $r = 1$.

**REVIEW PROBLEMS - Answers.**

1. a. PF b. N c. U

2. $\frac{6}{k}\sqrt{kx}+C$

3. $-\ln|x|-\frac{1}{2}\ln|x+1|+\frac{3}{2}\ln|x-3|+C$

4. $-\frac{5}{3}\cos(3x)+C$

5. $2\tan(3x)+C$

6. $-\frac{1}{2}e^{-x^2}+C$

7. $x\arctan x-\frac{1}{2}\ln|1+x^2|+C$

8. $-\frac{1}{10}\cos^5 2x+C$

9. $-\ln|1-\sin x|+C$

10. $x^2+\ln|x|-\frac{3}{x}+C$

11. $x+2e^{-x}-\frac{1}{2}e^{-2x}+C$

12. $\frac{1}{20}\arctan\left(\frac{4x}{5}\right)+C$

13. $\frac{1}{8}\cos^3 2x\sin 2x+\frac{3}{16}\cos 2x\sin 2x+\frac{3}{8}x+C$

14. $-\frac{1}{\sqrt{3}}\ln\left|\frac{\sqrt{3+9x^2}+\sqrt{3}}{3x}\right|+C$

15. $\frac{1}{3}x^2\sin 3x+\frac{2}{9}x\cos 3x-\frac{2}{27}\sin 3x+C$

16. $\frac{2x}{x^2+3}-\frac{2}{x-1}$

17. $S=\left(-\frac{t}{3}-\frac{1}{9}\right)e^{-3t}+\frac{1}{9}$

18. 16.5°F

19. $y=-\frac{1}{Kt+C}$

20. $y=3e^{x^2}$

21. Converges to 6

22. Diverges

23. Converges to $\frac{1}{2}$

24. $\frac{512\pi}{15}$

25. a. $\int_1^3 \sqrt{1+4x^2}\,dx$

b. 8.268

26. $\frac{1}{3}$

27. $\frac{1}{2}$

28. $\frac{3}{2}$

29. a. It is not in the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

b. $\infty$ (look at the graph)

30. $-\infty$

31. $e$

32. a. after 1 second and after 2 seconds

b. $y = -32t + 48 = 0$
$t = 1.5$ sec
$s = -16(1.5)^2 + 48(1.5) + 8 = 44$ ft

c. Graph the position function and find the *x*-intercept.
$t \approx 3.16$ sec

33. 0.6

34. The integral represents the total increase in the volume of water in $\text{ft}^3$ from t = 4 minutes to t = 10 minutes.

35. Hint: slope at (-2,1) is -1 and slope at (3,1) is 4

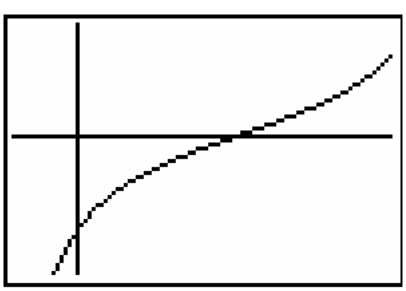
36. $\frac{1}{2} \cdot 2(3+2\cdot 2+4) = 11$

37. $\frac{125}{6}$

38. 140 ft - lb

39. $W = \int_0^8 \pi \frac{4}{9} y^2 (15-y)(62.5)\,dy = 27.78\pi \int_0^8 y^2 (15-y)\,dy$

40.



41.

| $x$ | 2 | 2.1 | 2.2 | 2.3 |
|---|---|---|---|---|
| $y$ | 1 | 1 | 1.01 | 1.028 |

42. a. $3, \frac{4}{3}, 1, \frac{6}{7}$ b. yes, $\frac{1}{2}$ c. diverges by the Test for Divergence

43. $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$

44. a) $1 - \frac{x^2}{2} + \frac{x^4}{24}$ b) .9800666667 c) approximately $\frac{(.2)^6}{6!} = 8.9 \times 10^{-8}$

45. $e \sum_{n=0}^{\infty} \frac{1}{2^n} \frac{(x-2)^n}{n!}$

46. 73.2 g

47. 280 ft

48. $\frac{3}{8}$

49. $\sum_{n=1}^{\infty} 3 \bullet (-8x^3)^{n-1}$ for $-\frac{1}{2} < x < \frac{1}{2}$

50. a. C (Ratio Test) b. D (compare to $\frac{1}{n}$ ) c. D (integral test)

51. C

52. $\left( -\frac{5}{2}, \frac{1}{2} \right]$ and $R = \frac{3}{2}$

53. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ and first 3 terms must be used.

54. a. $\left( 2\sqrt{2}, \frac{7\pi}{4} \right)$ b. $\left( 4, \frac{4\pi}{3} \right)$ c. $\left( 2, \frac{5\pi}{6} \right)$

55. $y = 6$

56. a circle with radius 1 and center at (1,0)

57. $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$ or 1.91