MONTGOMERY COLLEGE
Department of Mathematics
Rockville Campus

MATH 182 - REVIEW PROBLEMS

1. State whether each of the following can be integrated by partial fractions (PF), integration by parts (PI), u-substitution (U), or none of these (N). You do not have to evaluate the integrals.

a. $\int \frac{dx}{x^2-4}$ b. $\int \sqrt{\cos 2x}\, dx$ c. $\int \frac{6x\, dx}{x^2+8}$

Integrate:

2. $\int \frac{3}{\sqrt{kx}} dx$ 3. $\int \frac{(5x+3)}{x^3-2x^2-3x} dx$ 4. $\int 5\sin(3x)\, dx$

5. $\int \frac{6}{\cos^2 3x} dx$ 6. $\int \frac{x}{e^{x^2}} dx$ 7. $\int \arctan x\, dx$

8. $\int \sin 2x \ \cos^4 2x\, dx$ 9. $\int \frac{\cos x\, dx}{1 - \sin x}$ 10. $\int \frac{2x^3+x+3}{x^2} dx$

11. $\int \left(1-e^{-x}\right)^2 dx$

Integrate the following using the table of integrals on the inside rear book cover.

12. $\int \frac{dx}{25+16x^2}$ 13. $\int \cos^4 2x\, dx$

14. $\int \frac{1}{x\sqrt{3+9x^2}} dx$ 15. $\int x^2 \cos 3x\, dx$

16. Write $\frac{-2x-6}{(x^2+3)(x-1)}$ as the sum of two partial fractions.

17. The velocity V at time t of a point moving along a coordinate line is $V = te^{-3t}$ ft/sec. If the point is at the origin of $t=0$, find a formula for its position s at time t.

18. Food is placed in a freezer. After t hours, the temperature of the food is changing at a rate of $R = 10e^{-0.2t}$ where R is in degrees F/hr. How much has the temperature dropped in the first two hours?

19. a. Using the table below, show how to use n = 2 subintervals and trapezoids to approximate $\int_1^5 r(t)dt$

where r(t) is the population rate in thousands per year at a time t years after Jan. 1, 2009.

t	1	2	3	4	5
r(t)	3	1	2	2	4

b. What does the approximation in part (a) tell about the population? Be specific and include the correct units.

20. a. Give the correct four-decimal place approximation from a calculator program for $\int_1^4 e^{-x^2}dx$

with 10 midpoint rectangles (M10).

b. Find the correct answer to four decimal places for $\int_1^4 e^{-x^2}dx$ with your calculator.

c. What is the error in the approximation for M10 to four decimal places in part a?

21. Solve $\frac{dy}{dt} = \frac{t(\sin(t^2))}{y^2}$ assuming $y \neq 0$. Express *y* in terms of *t*.

22. Solve $\frac{dy}{dx} = 2xy$ where $y(0) = 3$ Express *y* in terms of *x*.

Evaluate using L'Hopital's Rule or other analytical methods.

23. $\lim_{x\to 0} \frac{\sin x - x\cos x}{x^3}$

24. $\lim_{x\to\infty} \frac{x^2 - 5}{2x^2 + 3x}$

25. $\lim_{x\to\infty} \frac{\sin\left(\frac{3}{x}\right)}{\frac{2}{x}}$

26. a. Tell why you cannot use L'Hopital's Rule to find $\lim_{x\to 0^+} \frac{\cos x}{x}$.

b. Evaluate the limit in part a and give evidence to support your answer.

Evaluate each limit below and give evidence to support your answer.

27. $\lim_{x\to\infty}\left(3 - e^{2x}\right)$

28. $\lim_{x\to 1} x^{\frac{1}{x-1}}$

Determine the convergence or divergence of each integral. If convergent, find the value.

29. $\int_{-1}^{1} \frac{dx}{x^{2/3}}$

30. $\int_1^\infty \frac{\ln x}{x}dx$

31. $\int_0^\infty \frac{\sin x}{e^x}dx$

32. Find the volume of the solid formed when the region bounded by the *x*-axis and the curve $y = 4 - x^2$ is revolved about the *x*-axis.

33. a. Write an integral equal to the arc length of $y = x^2$ from (1,1) to (3,9).

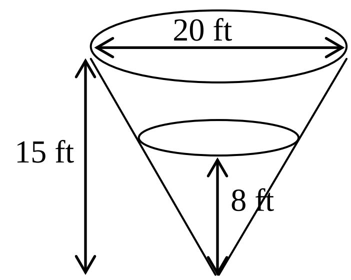
b. Approximate the arc length with a calculator program.

34. Find the area of the region enclosed by $y = x^2$ and $y = x + 6$.

35. If $\int_0^5 g(x)\,dx = 3$ find the average value of $g(x)$ on [0,5].

36. A spring has a natural length of 12 ft. A force of 80 lb stretches it to a length of 14 ft. Find the work done in stretching it from a length of 15 ft. to a length of 16 ft.

37. A tank in the shape of an inverted cone contains some water. The tank has diameter 20 feet at the top and is 15 feet deep (See figure). The water is 8 feet deep and has density $p = 62.5$ lb/ft^3. Write an integral that represents the work required to pump all the water over the top rim. Be sure to draw a picture showing how you are setting the problem up. Do not evaluate the integral.



38. Let $\frac{dy}{dx} = x + y^2$. Draw the direction field tangents at the points (-2,1) and (3,1) on an *xy*-graph.

39. Sketch a solution to $\frac{dy}{dx} = x + y^2$ with $y(1) = 0$ by first drawing the slope field on your calculator. Then sketch a solution curve using the slope field.

40. Consider the differential equation $\frac{dy}{dx} = x - 2y$. Show how to use Euler's Method (without a calculator program) to approximate *y*(2.3) by starting at (2,1) and using steps of $\Delta x = 0.1$.

41. Let $a_n = \frac{n+2}{2n-1}$

a. Write the first 4 terms of the sequence $\{a_n\}_{n=1}^{\infty}$

b. Does $\{a_n\}_{n=1}^{\infty}$ have a limit? If so, find it.

c. Does $\sum_{n=1}^{\infty} a_n$ converge or diverge? Give a reason for your answer.

42 100 g of a radioactive substance decays. After 10 days, 80 g remain. How much would remain after 14 days?

43. A ball is dropped from a height of 40 ft. Each time it hits the floor, the ball rebounds to $\frac{3}{4}$ of its previous height. Find the total distance the ball travels.

44. Determine the sum of $\sum_{n=2}^{\infty} \frac{3}{2^{n+2}}$

45. Determine the convergence or divergence of the following series. Justify your answer.

a. $\sum_{n=2}^{\infty} \frac{3n}{n!2^{n+1}}$ b. $\sum_{n=1}^{\infty} \frac{2n+1}{4n^2+n-1}$ c. $\sum_{n=2}^{\infty} \frac{3}{n\sqrt[3]{\ln n}}$

46. Determine if the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+2)}{2n^2+2n}$ converges or diverges. Justify your answer.

47. Find a power series representation of $\frac{3}{1+8x^3}$ and state its interval of convergence.

48. Determine the radius and interval of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n (x+1)^n}{n3^n}$.

49. Find the MacLaurin Series for $\ln(1+x)$ and determine how many terms must be used in this series to find $\ln 1.02$ accurate to 7 decimal places.

50. Let $f(x)=\ln(x+1)$. Find the Taylor Series for x near 0 by taking derivatives. Write the first 4 nonzero terms of the series.

51. a) Find the Taylor polynomial of degree 4 for y = cos(x) with center a = 0.
b) Use the result to approximate cos(0.2).
c) Estimate the magnitude of the error in your approximation in part b.

52. Find the Taylor Series for $e^{\frac{x}{2}}$ centered at $a = 2$ and use the Ratio Test to show that this series converges for all x.

53. Convert to polar coordinates. Use $r > 0$ and $0 \le \theta < 2\pi$.

a. (2, -2) b. $\left(-2,-2\sqrt{3}\right)$ c. $\left(-\sqrt{3},1\right)$

54. Convert the polar equation $r = \frac{6}{\sin\theta}$ to an equation in x and y.

55. Sketch the graph of $r = 2\cos\theta$ without using a graphing calculator.

56 a. Find the area inside of the region inside $r = 2\sin(3\theta)$.
b. Find the area of the region that is inside $r = 2\sin(3\theta)$ and outside $r = 1$.

REVIEW PROBLEMS - Answers.

1. a. PF b. N c. U

2. $\frac{6}{k}\sqrt{kx}+C$ 3. $-\ln|x|-\frac{1}{2}\ln|x+1|+\frac{3}{2}\ln|x-3|+C$

4. $-\frac{5}{3}\cos(3x)+C$ 5. $2\tan(3x)+C$

6. $-\frac{1}{2}e^{-x^2}+C$ 7. $x\arctan x-\frac{1}{2}\ln|1+x^2|+C$ 8. $-\frac{1}{10}\cos^5 2x+C$

9. $-\ln|1-\sin x|+C$ 10. $x^2+\ln|x|-\frac{3}{x}+C$

11. $x+2e^{-x}-\frac{1}{2}e^{-2x}+C$ 12. $\frac{1}{20}\arctan\left(\frac{4x}{5}\right)+C$

13. $\frac{1}{8}\cos^3 2x\sin 2x+\frac{3}{16}\cos 2x\sin 2x+\frac{3}{8}x+C$ 14. $-\frac{1}{\sqrt{3}}\ln\left|\frac{\sqrt{3+9x^2}+\sqrt{3}}{3x}\right|+C$

15. $\frac{1}{3}x^2\sin 3x+\frac{2}{9}x\cos 3x-\frac{2}{27}\sin 3x+C$ 16. $\frac{2x}{x^2+3}-\frac{2}{x-1}$

17. $S=\left(-\frac{t}{3}-\frac{1}{9}\right)e^{-3t}+\frac{1}{9}$ 18. 16.5^{o}F

19. a. $\frac{1}{2}\cdot 2(3+2\cdot 2+4)=11$

b. The population increases by about 11 thousand from Jan. 1, 2009 to Jan. 1, 2014.

20. a. 0.1366 b. 0.1394 c. -0.0028 21. $y=\sqrt[3]{-\frac{3}{2}\cos(t^2)+C}$

22. $y=3e^{x^2}$ 23. $\frac{1}{3}$ 24. $\frac{1}{2}$ 25. $\frac{3}{2}$

26. a. It is not in the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$. b. ∞ (look at the graph)

27. $-\infty$ 28. e

29. Converges to 6 30. Diverges

31. Converges to $\frac{1}{2}$ 32. $\frac{512\pi}{15}$

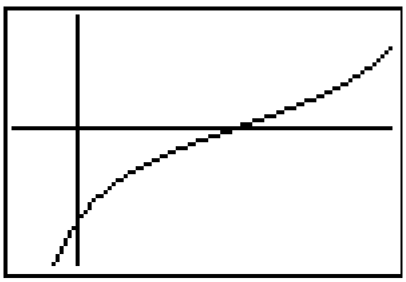
33. a. $\int_1^3 \sqrt{1+4x^2}\,dx$ b. 8.268

34. $\frac{125}{6}$ 35. 0.6 36. 140 ft - lb

37. $W=\int_0^8 \pi \frac{4}{9} y^2 (15-y)(62.5)\,dy = 27.78\pi \int_0^8 y^2 (15-y)\,dy$

38. Hint: slope at (-2,1) is -1 and slope at (3,1) is 4

39.



40.

x	2	2.1	2.2	2.3
y	1	1	1.01	1.028

41. a. $3, \frac{4}{3}, 1, \frac{6}{7}$ b. yes, $\frac{1}{2}$ c. diverges by the Test for Divergence

42. 73.2 g 43. 280 ft 44. $3/8$

45. a. C (Ratio Test) b. D (compare to $\frac{1}{n}$) c. D (integral test)

46. C 47. $\sum_{n=1}^{\infty} 3(-8x^3)^{n-1}$ for $-\frac{1}{2} < x < \frac{1}{2}$ 48. $\left(-\frac{5}{2}, \frac{1}{2}\right]$ and $R=\frac{3}{2}$

49. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$ and first 3 terms must be used.

50. $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}$

51. a) $1-\frac{x^2}{2}+\frac{x^4}{24}$ b) .9800666667 c) approximately $\frac{(.2)^6}{6!} = 8.9\times 10^{-8}$

52. $e\sum_{n=0}^{\infty}\frac{1}{2^n}\frac{(x-2)^n}{n!}$

53. a. $\left(2\sqrt{2},\frac{7\pi}{4}\right)$ b. $\left(4,\frac{4\pi}{3}\right)$ c. $\left(2,\frac{5\pi}{6}\right)$

54. $y=6$

55. a circle with radius 1 and center at (1,0)

56. a. π b. $\frac{\pi}{3}+\frac{\sqrt{3}}{2}$ or 1.91