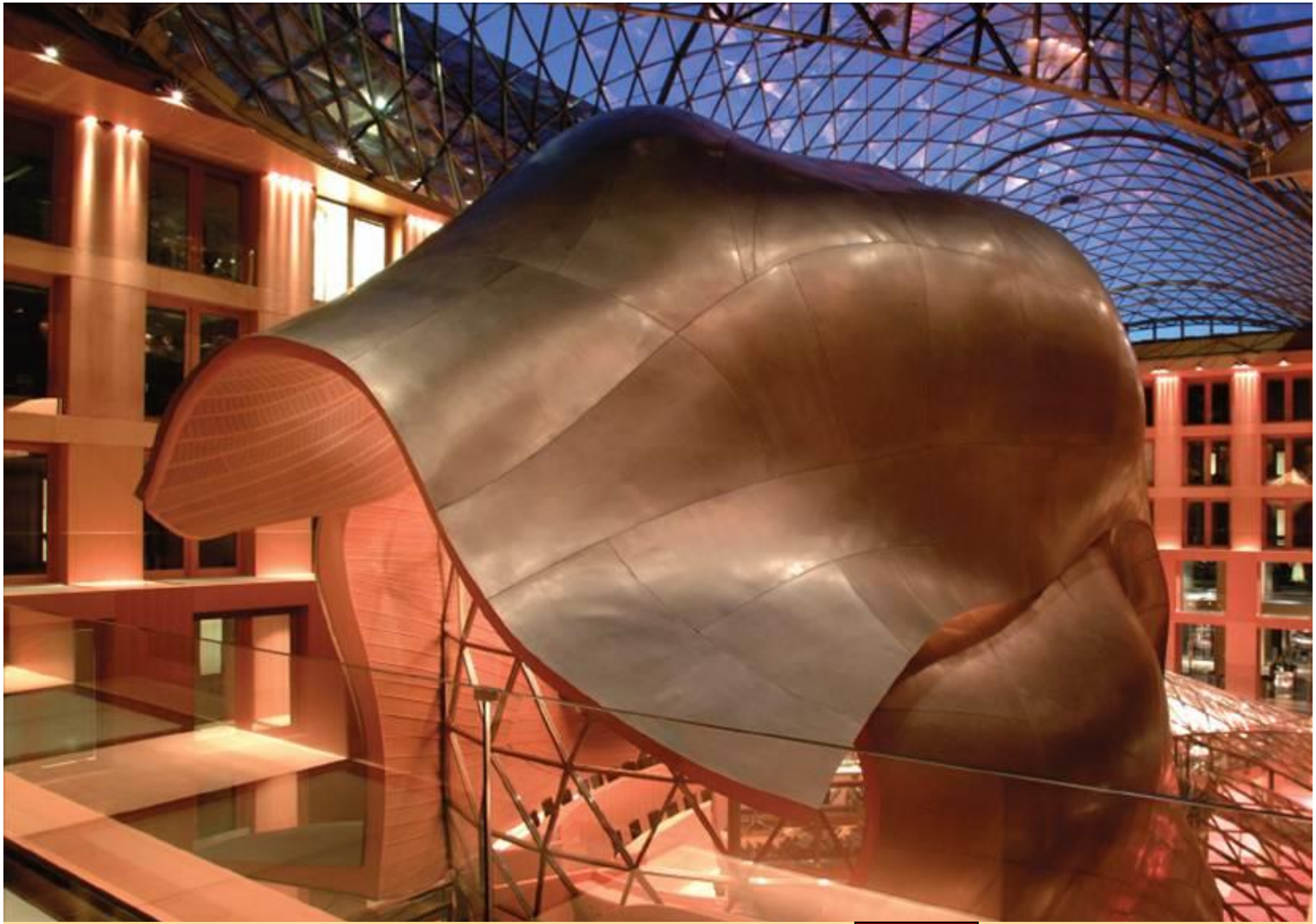

# Integrals

5

# 5.8 Integration Using Tables and Computer Algebra Systems

# Tables of Integrals

# Tables of Integrals

Tables of indefinite integrals are very useful when we are confronted by an integral that is difficult to evaluate by hand and we don’t have access to a computer algebra system.

Usually we need to use the Substitution Rule or algebraic manipulation to transform a given integral into one of the forms in the table.

# Example 1

Use the Table of Integrals to evaluate $\int_0^2 \frac{x^2+12}{x^2+4}\,dx.$

Solution:

The only formula that resembles our given integral is:

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

If we perform long division, we get

$$\frac{x^2+12}{x^2+4} = 1 + \frac{8}{x^2+4}$$

# Example 1 – *Solution*

cont'd

Now we can use Formula 17 with $a = 2$:

$$\int_0^2 \frac{x^2 + 12}{x^2 + 4}\,dx = \int_0^2 \left(1 + \frac{8}{x^2 + 4}\right) dx$$

$$= x + 8 \cdot \tfrac{1}{2} \tan^{-1} \frac{x}{2} \Big]_0^2$$

$$= 2 + 4 \tan^{-1} 1$$

$$= 2 + \pi$$

# Computer Algebra Systems

# Computer Algebra Systems

Computers are particularly good at matching patterns. And just as we used substitutions in conjunction with tables, a CAS can perform substitutions that transform a given integral into one that occurs in its stored formulas.

So it isn't surprising that computer algebra systems excel at integration.

To begin, let's see what happens when we ask a machine to integrate the relatively simple function $y = 1/(3x - 2)$.

# Computer Algebra Systems

Using the substitution $u = 3x - 2$, an easy calculation by hand gives

$$\int \frac{1}{3x - 2}\,dx = \tfrac{1}{3}\ln|3x - 2| + C$$

whereas Derive, Mathematica, and Maple all return the answer

$$\tfrac{1}{3}\ln(3x - 2)$$

The first thing to notice is that computer algebra systems omit the constant of integration.

# Computer Algebra Systems

In other words, they produce a *particular* antiderivative, not the most general one.

Therefore, when making use of a machine integration, we might have to add a constant.

Second, the absolute value signs are omitted in the machine answer. That is fine if our problem is concerned only with values of $x$ greater than $\frac{2}{3}$.

But if we are interested in other values of $x$, then we need to insert the absolute value symbol.

# Example 5

Use a computer algebra system to find $\int x\sqrt{x^2 + 2x + 4}\, dx$.

Solution:

Maple responds with the answer

$$\tfrac{1}{3}(x^2 + 2x + 4)^{3/2} - \tfrac{1}{4}(2x + 2)\sqrt{x^2 + 2x + 4} - \frac{3}{2}\operatorname{arcsinh}\frac{\sqrt{3}}{3}(1 + x)$$

The third term can be rewritten using the identity

$$\operatorname{arcsinh} x = \ln(x + \sqrt{x^2 + 1})$$

# Example 5 – *Solution*

cont'd

Thus

$$\operatorname{arcsinh}\frac{\sqrt{3}}{3}(1 + x) = \ln\left[\frac{\sqrt{3}}{3}(1 + x) + \sqrt{\tfrac{1}{3}(1 + x)^2 + 1}\right]$$

$$= \ln\frac{1}{\sqrt{3}}\left[1 + x + \sqrt{(1 + x)^2 + 3}\right]$$

$$= \ln\frac{1}{\sqrt{3}} + \ln\left(x + 1 + \sqrt{x^2 + 2x + 4}\right)$$

The resulting extra term $-\frac{3}{2}$ ln(1/ $\sqrt{3}$ ) can be absorbed into the constant of integration.

# Example 5 – *Solution*

cont'd

Mathematica gives the answer

$$\left(\frac{5}{6} + \frac{x}{6} + \frac{x^2}{3}\right)\sqrt{x^2 + 2x + 4} - \frac{3}{2}\operatorname{arcsinh}\left(\frac{1 + x}{\sqrt{3}}\right)$$

Mathematica combined the first two terms of the Maple result into a single term by factoring.

Derive gives the answer

$$\tfrac{1}{6}\sqrt{x^2 + 2x + 4}\,(2x^2 + x + 5) - \tfrac{3}{2}\ln\left(\sqrt{x^2 + 2x + 4} + x + 1\right)$$

# Can We Integrate All Continuous Functions?

# Can We Integrate All Continuous Functions?

The question arises: Will our basic integration formulas, together with the Substitution Rule, integration by parts, tables of integrals, and computer algebra systems, enable us to find the integral of every continuous function?

In particular, can we use these techniques to evaluate $\int e^{x^2}\,dx$? The answer is No, at least not in terms of the functions that we are familiar with.

Most of the functions that we have been dealing with in this book are what are called **elementary functions**.

# Can We Integrate All Continuous Functions?

These are the polynomials, rational functions, power functions ($x^a$), exponential functions ($a^x$), logarithmic functions, trigonometric and inverse trigonometric functions, and all functions that can be obtained from these by the five operations of addition, subtraction, multiplication, division, and composition.

For instance, the function

$$f(x) = \sqrt{\frac{x^2 - 1}{x^3 + 2x - 1}} + \ln(\cos x) - xe^{\sin 2x}$$

is an elementary function.

# Can We Integrate All Continuous Functions?

If $f$ is an elementary function, then $f'$ is an elementary function but $\int f(x)\,dx$ need not be an elementary function.

Consider $f(x) = e^{x^2}$. Since $f$ is continuous, its integral exists, and if we define the function $F$ by

$$F(x) = \int_0^x e^{t^2}\,dt$$

then we know from Part 1 of the Fundamental Theorem of Calculus that

$$F'(x) = e^{x^2}$$

# Can We Integrate All Continuous Functions?

Thus $f(x) = e^{x^2}$ has an antiderivative $F$, but it has been proved that $F$ is not an elementary function. This means that no matter how hard we try, we will never succeed in evaluating $\int e^{x^2}\,dx$ in terms of the functions we know.

The same can be said of the following integrals:

$$\int \frac{e^x}{x}\,dx \qquad \int \sin(x^2)\,dx \qquad \int \cos(e^x)\,dx$$

$$\int \sqrt{x^3 + 1}\,dx \qquad \int \frac{1}{\ln x}\,dx \qquad \int \frac{\sin x}{x}\,dx$$

In fact, the majority of elementary functions don’t have elementary antiderivatives.