

If the function f has an infinite discontinuity anywhere in [a,b], then  $\int_a^b f(x) dx$  is an improper integral. To evaluate such an integral, *replace* the value at which the function is discontinuous by a letter, say t, and then take the limit as t approaches the value of x which is causing the problem. More formally:

(1) If f is continuous on (a,b] and is discontinuous at a, then  $\int_a^b f(x) dx = \lim_{t \rightarrow a^+} \int_t^b f(x) dx$  if this limit exists.

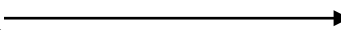
(2) If f is continuous on [a,b) and is discontinuous at b, then  $\int_a^b f(x) dx = \lim_{t \rightarrow b^-} \int_a^t f(x) dx$  if this limit exists.

(3) If f is discontinuous at  $x = c$ , where  $a < c < b$ , and if both  $\int_a^c f(x) dx$  and  $\int_c^b f(x) dx$  are convergent, then we define  $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$ . If either of the integrals on the right is divergent, then  $\int_a^b f(x) dx$  is said to be divergent.

**Exercises:**

Determine whether each improper integral is convergent or divergent. Evaluate those that are convergent. You may use integral tables if appropriate.

1.  $\int_0^1 \frac{1}{x^2} dx$

**OVER** 

2.  $\int_0^1 \frac{1}{x} dx$

3.  $\int_0^1 \frac{1}{\sqrt{x}} dx$

4.  $\int_0^3 \frac{1}{x^2 - 9} dx$

5.  $\int_{-4}^0 \frac{1}{\sqrt{16 - x^2}} dx$

6.  $\int_0^4 \frac{1}{(x-1)^3} dx$