MA 182 Improper Integrals: f is Discontinuous in [a,b]
Section 5.10

If the function f has an infinite discontinuity anywhere in [a,b], then $\int_a^b f(x)\,dx$ is an improper integral. To evaluate such an integral, *replace* the value at which the function is discontinuous by a letter, say t, and then take the limit as t approaches the value of x which is causing the problem. More formally:

(1) If f is continuous on (a,b] and is discontinuous at a, then $\int_a^b f(x)\,dx = \lim_{t\to a^+}\int_t^b f(x)\,dx$ if this limit exists.

(2) If f is continuous on [a,b) and is discontinuous at b, then $\int_a^b f(x)\,dx = \lim_{t\to b^-}\int_a^t f(x)\,dx$ if this limit exists.

(3) If f is discontinuous at x = c, where a < c < b, and if both $\int_a^c f(x)\,dx$ and $\int_c^b f(x)\,dx$ are convergent, then we define $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$. If either of the integrals on the right is divergent, then $\int_a^b f(x)\,dx$ is said to be divergent.

Exercises:

Determine whether each improper integral is convergent or divergent. Evaluate those that are convergent. You may use integral tables if appropriate.

1. $\int_0^1 \frac{1}{x^2}\,dx$

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2. $\int_0^1 \frac{1}{x}\,dx$

3. $\int_0^1 \frac{1}{\sqrt{x}}\,dx$

4. $\int_0^3 \frac{1}{x^2-9}\,dx$

5. $\int_{-4}^0 \frac{1}{\sqrt{16-x^2}}\,dx$

6. $\int_0^4 \frac{1}{(x-1)^3}\,dx$