MA 182 **Some Problems Involving Integration by Parts**
Section 5.6

The integration by parts formula $\int u\,dv = uv - \int v\,du$ sometimes has to be applied more than once in order to find an antiderivative.

Repeated integration by parts

1. Find $\int x^2 e^x dx$

To do this problem, apply integration by parts with $u = x^2$ and $dv = e^x dx$. Then apply integration by parts again to the integral you got for $\int v\,du$.

2. Use the above technique to find $\int x^2 \sin 5x\,dx$.

3. How many times do you think you would have to use integration by parts to find $\int x^4 \cos x\,dx$? (You don't have to do it!)

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Repeated integration by parts and then solving

4. To find $\int e^x \cos x \, dx$,

(a) Apply integration by parts with $u = \cos x$ and $dv = e^x dx$.

(b) Apply integration by parts to the integral you got for $\int v \, du$ with u = sin x and $dv = e^x dx$

(c) If we call the original integral I, you should now have an expression in which I has reappeared on the right-hand side. Since I is equal to this expression, "collect" the I terms on the left and then solve for I. Make sure you add a constant of integration C to your final answer for the antiderivative.

Do the remaining problems on another sheet of paper.

5. Use this technique to find $\int e^{2x} \sin 4x \, dx$.

Using a substitution to transform the integrand before using integration by parts

6. Sometimes a substitution will transform the integrand so that integration by parts can be used.

Example: $\int e^{\sqrt{x}} dx$ Let t = $\sqrt{x}$. Square each side to get $t^2 = x$, and differentiate to get $2t \, dt = dx$. The integral becomes $2\int t e^t dt$. Now use integration by parts to finish the problem.