MA 182 Infinite Series - Solutions

Sections 8.2 – 8.4

1. $\sum_{n=1}^{\infty} \frac{2}{3n}$ Diverges; multiple of harmonic series.

2. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2}{3n}$ Converges by Alt. Series Test, but not absolutely convergent.

3. $\sum_{n=1}^{\infty} \frac{n}{2^n(n+1)}$ Converges by Ratio Test; t $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{2} < 1$.

4. $\sum_{n=1}^{\infty} \frac{2n+1}{3n+2}$ Diverges by nth-term test because $\lim_{n\to\infty}\frac{2n+1}{3n+2} = \frac{2}{3} \neq 0$

5. $\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$ Converges by Basic Comp. Test because $\ln n < n,\ so\ \frac{\ln n}{n^3} < \frac{n}{n^3} = \frac{1}{n^2}$

6. $\sum_{n=1}^{\infty} \frac{n}{e^{n^2}}$ Converges by Ratio Test; $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 0 < 1$. Integral Test can also be used.

7. $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ Converges by Ratio Test; $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 0 < 1$

8. $\sum_{n=1}^{\infty} \frac{(2n+3)^2}{(n+1)^3}$ Diverges; use Limit Comp. Test with $\sum_{n=1}^{\infty} \frac{1}{n}$

9. $\sum_{n=1}^{\infty} 5(-1)^n \left(\frac{2}{3}\right)^{n-1}$ Absolutely convergent Geometric Series with r = -2/3, a = -5, S = -3

10. $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{2n^3+n}$ Converges by Alt. Series Test; also absolutely convergent by Limit Comp. Test with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

11. $\sum_{n=1}^{\infty} \frac{n^{100}}{n!}$ Converges by Ratio Test; $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 0 < 1$

12. $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$ Diverges by Ratio Test, $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 4 > 1$