MA 182 **USING SERIES TO EVALUATE NATURAL LOGARITHMS**
Section 8.6

(1) Write a series for $\dfrac{1}{1-x}$ centered at x = 0 and state the radius of convergence.

(2) Use this series to find a series for $\ln(1-x)$ and state the radius of convergence.

(3) Write a series for $\dfrac{1}{1+x}$ centered at x = 0 and state the radius of convergence.

(4) Use this series to find a series for $\ln(1+x)$ and state the radius of convergence.

(5) Use your answers to (2) and (4) to find a series for $\ln\left(\dfrac{1+x}{1-x}\right)$. Hint: $\ln\left(\dfrac{A}{B}\right) = \ln A - \ln B$.

(6) Find the value of x so that $\dfrac{1+x}{1-x} = 2$.

(7) Use the value you just found to evaluate $\ln 2$ by substituting this value into the series in (5). Use enough terms of the series so that your answer seems to be accurate to four decimal places. To do this, use the summation technique that we have used before on your calculator. Use it more than once, increasing the number of terms to a high enough number so that the fourth decimal place does not change.

Compare the answer you got to the value given by your calculator for $\ln 2$. Are the values the same?

(8) To continue the process of finding natural logarithms with small values of x (which will give good results quickly), find the value of x so that $\frac{1+x}{1-x} = \frac{3}{2}$.

Use this value to get $\ln\frac{3}{2}$. Again, use enough terms so that the answer appears to be accurate to four decimal places.

Since $\ln\frac{3}{2} = \ln 3 - \ln 2$, use your answer for $\ln\frac{3}{2}$ to find $\ln 3$.

(9) Find $\ln 4$ (accurate to 4 decimal places) by first finding $\ln\frac{4}{3}$, and then using your answer for $\ln 3$ and the fact that $\ln\frac{4}{3} = \ln 4 - \ln 3$.

(10) Using the same pattern, find $\ln 5$ (accurate to 4 decimal places).

(11) To show that the x-values to be used in this method will always fall within the interval of convergence, suppose we wanted to find $\ln k$, where k is a positive number. To find the x needed for the series, solve the equation $\frac{1+x}{1-x} = k$ for x.

Since k is positive, what can we say about the value of x, that is, will the series converge for any x-value obtained in this way? Why or why not?