Section 5.6 lecture Material

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(Ex2) $\int \ln x \, dx$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x} dx \qquad v = x$

$\int \ln x \, dx = uv - \int v \, du$

$= x \ln x - \int x \frac{1}{x} dx$

$= \boxed{x \ln x - x + c}$

(Ex1) $\int x \sin x \, dx$

$u = x \qquad dv = \sin x \, dx$

$du = dx \qquad v = -\cos x$

$= uv - \int v \, du$

$= -x\cos x - \int -\cos x \, dx = -x\cos x + \int \cos x \, dx$

$= \boxed{-x\cos x + \sin x + c}$

Ex3) $\int t^2 e^t \, dt$

$u = t^2 \quad dv = e^t dt$

$du = 2t\,dt \quad v = e^t$

$= t^2 e^t - \int 2t e^t \, dt$

$= t^2 e^t - 2\int t e^t \, dt$

$u = t \quad dv = e^t$

$du = dt \quad v = e^t$

$= t^2 e^t - 2\left[t e^t - \int e^t \, dt\right]$

$\boxed{= t^2 e^t - 2te^t + 2e^t + C}$

Ex4) $\int e^x \sin x \, dx$

$u = e^x \quad dv = \sin x \, dx$

$du = e^x \, dx \quad v = -\cos x$

$\int e^x \sin x \, dx = -e^x \cos x + \int e^x \cos x \, dx$

$u = e^x \quad dv = \cos x \, dx$

$du = e^x dx \quad v = \sin x$

$\int e^x \sin x \, dx = -e^x \cos x + e^x \sin x - \int \sin x \, e^x \, dx$

$2\int e^x \sin x \, dx = -e^x \cos x + e^x \sin x + C$

$\int e^x \sin x \, dx = \boxed{-\frac{1}{2} e^x \cos x + \frac{1}{2} e^x \sin x + C}$

Section 5.6 lecture Notes

$\boxed{\text{Ex 5}}$ $\int_0^1 \tan^{-1}(x)\,dx$

$$\begin{cases} u = \tan^{-1}(x) \\ du = \dfrac{1}{1+x^2}\,dx \end{cases} \qquad dv = dx \qquad v = x$$

$$= x\tan^{-1}(x) - \int \frac{x}{1+x^2}\,dx$$

$$\begin{cases} u = 1+x^2 \\ du = 2x\,dx \end{cases}$$

$$= x\tan^{-1}(x) - \frac{1}{2}\int \frac{du}{u}$$

$$= x\tan^{-1}(x) - \frac{1}{2}\ln|u|$$

$$= x\tan^{-1}(x) - \frac{1}{2}\ln|1+x^2| \Big|_0^1$$

$$= \left[1\tan^{-1}(1) - \frac{1}{2}\ln(2)\right] - \left[0\tan^{-1}(0) - \frac{1}{2}\ln(1)\right]$$

$$= \frac{\pi}{4} - \frac{\ln 2}{2} - 0 + 0$$

$$= \boxed{\frac{\pi}{4} - \frac{\ln 2}{2}}$$