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MA 182
Section 5.5

Integrals Involving Exponential Functions

1. Often an integral involving an exponential function can be transformed into a much simpler integral of the type $\int e^u du$ by using an appropriate u-substitution for the exponent.

(a) $\int e^{5x} dx$ Let u = 5x $du = 5dx$

$\frac{1}{5}\int e^{5x} 5dx = \frac{1}{5}\int e^u du = \frac{1}{5}e^u + c$

$\boxed{= \frac{1}{5}e^{5x} + c}$

(b) $\int e^{kx} dx$ Let u = Kx $du = Kdx$

$\frac{1}{K}\int e^{Kx} Kdx = \frac{1}{K}\int e^u du = \frac{1}{K}e^u + c$

$\boxed{= \frac{1}{K}e^{Kx} + c}$

(c) $\int \frac{1}{e^x} dx$ Hint: Rewrite the integral as $\int e^{-x} dx$ and make use of your answer to part (b).

$= \int e^{-x} dx$ let $u = -x$; $du = -1dx$

$= -\int e^{-x}(-dx) = -\int e^u du = -e^u + c$

$\boxed{= -e^{-x} + c}$

(d) $\int x^2 e^{x^3} dx$ Let $u = x^3$

$du = 3x^2 dx$

$\frac{1}{3}\int e^{x^3} 3x^2 dx = \frac{1}{3}\int e^u du$

$= \frac{1}{3}e^u + c = \boxed{\frac{1}{3}e^{x^3} + c}$

OVER →

(e) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$ Let u = $\sqrt{x} = x^{\frac{1}{2}}$ $du = \frac{1}{2} x^{-\frac{1}{2}} dx$

$= 2\int e^{\sqrt{x}} \frac{1}{2} x^{-\frac{1}{2}} dx$

$= 2\int e^{u} du = 2e^{u} + C = \boxed{2e^{\sqrt{x}} + C}$

2. Not every integral involving an exponential function is best integrated by substituting u for the exponent. Often, if the integrand involves more than one exponential function, it is best to substitute u for more than just the exponent.

(a) $\int \frac{e^x}{e^x + 1} dx$ Let $u = e^x + 1$

$du = e^x dx$

$= \int \frac{du}{u}$

$= \ln|u| + C = \boxed{\ln|e^x + 1| + C}$

(b) $\int e^{2x}\sqrt{e^{2x}+5}\, dx$ Let u = $e^{2x} + 5$

$du = 2e^{2x} dx$

$\frac{1}{2}\int \sqrt{e^{2x}+5}\,(2e^{2x} dx)$

$= \frac{1}{2}\int u^{\frac{1}{2}} du = \frac{1}{2} \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \frac{1}{2} \cdot \frac{2}{3} u^{\frac{3}{2}} + C = \boxed{\frac{1}{3}(e^{2x}+5)^{\frac{3}{2}} + C}$