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MA 182 **Some Problems Involving Integration by Parts** Solutions By Dr. K

Section 5.6

The integration by parts formula $\int u\,dv = uv - \int v\,du$ sometimes has to be applied more than once in order to find an antiderivative.

Repeated integration by parts

1. Find $\int x^2 e^x dx$

To do this problem, apply integration by parts with $u = x^2$ and $dv = e^x dx$. Then apply integration by parts again to the integral you got for $\int v\,du$.

$u = x^2 \qquad dv = e^x dx$

$du = 2x\,dx \qquad v = e^x$

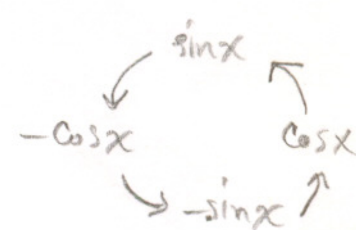
$u = 2x \qquad dv = e^x dx$

$du = 2\,dx \qquad v = e^x$

$$\int x^2 e^x dx = x^2 e^x - \int e^x 2x\,dx$$

$$= x^2 e^x - \left[2xe^x - \int e^x 2\,dx\right]$$

$$\boxed{= x^2 e^x - 2xe^x + 2e^x + C}$$



2. Use the above technique to find $\int x^2 \sin 5x\,dx$.

$u = x^2 \qquad dv = \sin 5x\,dx$

$du = 2x\,dx \qquad v = -\frac{1}{5}\cos 5x$

$$\int x^2 \sin 5x\,dx = -\frac{1}{5}x^2\cos 5x - \int -\frac{1}{5}\cos(5x)\,2x\,dx$$

$u = \frac{2}{5}x \qquad dv = \cos(5x)\,dx$

$du = \frac{2}{5}dx \qquad v = \frac{1}{5}\sin(5x)$

$$= -\frac{1}{5}x^2\cos 5x + \int \frac{2}{5}x\cos(5x)\,dx$$

$$= -\frac{1}{5}x^2\cos(5x) + \left[+\frac{2}{25}x\sin 5x - \int \frac{+2}{25}\sin 5x\,dx\right]$$

$$\boxed{= -\frac{1}{5}x^2\cos(5x) + \frac{2}{25}x\sin 5x + \frac{2}{125}\cos(5x) + C}$$

3. How many times do you think you would have to use integration by parts to find $\int x^4 \cos x\,dx$? (You don't have to do it!) (4 times)

OVER ⟶

$u = \cos x \quad dv = e^x dx$

$du = -\sin x\, dx \quad v = e^x$

Repeated integration by parts and then solving

4. To find $\int e^x \cos x\, dx$,

(a) Apply integration by parts with $u = \cos x$ and $dv = e^x dx$.

$\int e^x \cos x\, dx = e^x \cos x - \int -\sin x\, e^x\, dx$

(b) Apply integration by parts to the integral you got for $\int v\, du$ with $u = \sin x$ and $dv = e^x dx$

$\boxed{du = \cos x\, dx \quad dv = \int e^x dx \quad v = e^x}$

$\int e^x \cos x\, dx = e^x \cos x + \int \sin x\, e^x\, dx =$

$= e^x \cos x + e^x \sin x - \int e^x \cos x\, dx$

(c) If we call the original integral I, you should now have an expression in which I has reappeared on the right-hand side. Since I is equal to this expression, "collect" the I terms on the left and then solve for I. Make sure you add a constant of integration C to your final answer for the antiderivative.

$2\int e^x \cos x\, dx = e^x \cos x + e^x \sin x$

$$\boxed{\int e^x \cos x\, dx = \frac{e^x \cos x + e^x \sin x}{2} + C}$$

Do the remaining problems on another sheet of paper.

5. Use this technique to find $\int e^{2x} \sin 4x\, dx$.

Using a substitution to transform the integrand before using integration by parts

6. Sometimes a substitution will transform the integrand so that integration by parts can be used.

Example: $\int e^{\sqrt{x}} dx$ Let t = $\sqrt{x}$. Square each side to get $t^2 = x$, and differentiate to get $2t\, dt = dx$. The integral becomes $2\int t\, e^t dt$. Now use integration by parts to finish the problem.

W.S 5.6

#5

$\sin x$ $\quad -\cos x \quad \cos x \quad -\sin x$

$$\int e^{2x} \sin 4x \, dx$$

$u = e^{2x}$ $\qquad dv = \sin 4x \, dx$

$du = 2e^{2x} dx$ $\qquad v = \frac{-1}{4}\cos(4x)$

$$\int e^{2x} \sin 4x \, dx = -\frac{1}{4} e^{2x} \cos(4x) - \int \frac{-2}{4} e^{2x} \cos(4x)\, dx$$

$$= \frac{-1}{4} e^{2x} \cos 4x + \frac{1}{2} \int e^{2x} \cos(4x)\, dx$$

$$= \frac{-1}{4} e^{2x} \cos 4x + \frac{1}{2} \int e^{2x} \cos(4x)\, dx$$

$u = e^{2x}$ $\qquad dv = \cos 4x \, dx$

$du = 2e^{2x} dx$ $\qquad v = \frac{1}{4}\sin 4x$

$$\int e^{2x} \sin 4x\, dx = \frac{-1}{4} e^{2x} \cos 4x + \frac{1}{2}\left[\frac{1}{4} e^{2x} \sin 4x - \int \frac{2}{4} e^{2x} \sin 4x \, dx\right]$$

$$\int e^{2x} \sin 4x\, dx = \frac{-1}{4} e^{2x} \cos 4x + \frac{1}{8} e^{2x} \sin 4x - \frac{1}{4}\int e^{2x} \sin 4x \, dx$$

$$+\frac{1}{4}\int e^{2x} \sin 4x\, dx \qquad\qquad +\frac{1}{4}\int e^{2x} \sin 4x\, dx$$

$$\frac{5}{4}\int e^{2x} \sin 4x\, dx = \frac{-1}{4} e^{2x} \cos 4x + \frac{1}{8} e^{2x} \sin 4x$$

$$\Longrightarrow \int e^{2x} \sin(4x)\, dx = \frac{4}{5}\left[\frac{-1}{4} e^{2x} \cos 4x + \frac{1}{8} e^{2x} \sin 4x\right]$$

$$\int e^{2x} \sin(4x)\, dx = \frac{-1}{5} e^{2x} \cos(4x) + \frac{1}{10} e^{2x} (\sin 4x) + C$$

Over ⇒

$$\int e^{2x}\sin(4x)\,dx \overset{?}{=} -\frac{1}{5}e^{2x}\cos(4x)+\frac{1}{10}e^{2x}\sin(4x)+c$$

$$\left(-\frac{1}{5}e^{2x}\cos(4x)+\frac{1}{10}e^{2x}\sin(4x)+c\right)'$$

$$=-\frac{2}{5}e^{2x}\cos(4x)+\frac{4}{5}e^{2x}\sin(4x)+\frac{2}{10}e^{2x}\sin 4x+\frac{4}{10}e^{2x}\cos(4x)+0$$

$$=\cancel{-\frac{2}{5}e^{2x}\cos(4x)}+\underbrace{\frac{4}{5}e^{2x}\sin(4x)+\frac{1}{5}e^{2x}\sin 4x}+\cancel{\frac{2}{5}e^{2x}\cos(4x)}+0$$

these are Like Terms
so, we add them

$$=\frac{5}{5}e^{2x}\sin(4x)$$

$$=e^{2x}\sin(4x)$$

$$\therefore\ \boxed{\int e^{2x}\sin(4x)\,dx=-\frac{1}{5}e^{2x}\cos(4x)+\frac{1}{10}e^{2x}\sin(4x)+c}$$

section 5.6

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$\int e^{\sqrt{x}}\,dx$ let $t=\sqrt{x}$

$t^2 = x$.

differentiate

$2t\,dt = dx$

$$\int e^{\sqrt{x}}\,dx = \int e^{t}\,2t\,dt$$

let $u = 2t$ $\quad dv = e^t\,dt$

$du = 2dt$ $\quad v = e^t$

Then $\int e^{\sqrt{x}}\,dx = \int e^t\,2t\,dt = 2te^t - \int 2e^t\,dt$

$= 2te^t - 2e^t + c$

But $t=\sqrt{x}$,

then

$$\int e^{\sqrt{x}}\,dx = 2\sqrt{x}\,e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$$

Prove that $\int e^{\sqrt{x}}\,dx = 2\sqrt{x}\,e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$

Derivative of the Right side $= \cancel{2\,\frac{1}{2}x^{-\frac{1}{2}}e^{\sqrt{x}}} + 2\sqrt{x}\cdot\frac{1}{2}x^{-\frac{1}{2}}\cdot e^{\sqrt{x}} \cancel{- 2\cdot\frac{1}{2}x^{-\frac{1}{2}}e^{\sqrt{x}}} + 0$

$$= 2x^{\frac{1}{2}}\cdot\frac{1}{2}x^{-\frac{1}{2}}\cdot e^{\sqrt{x}}$$

$$= e^{\sqrt{x}}$$