Section 2.3

Function Notation and Making Predictions

Using an Equation of a Linear Model to Make Predictions

Example

Year	Average Salary (thousands of dollars)
1975	16.6
1980	22.1
1985	31.2
1990	41.9
1995	49.1
2000	57.7
2004	65.0

The table shows the average salaries of professors at four-year colleges and universities

Let *s* be the professors' average salary(in thousands of dollars) at *t* years since 1900. A possible model is

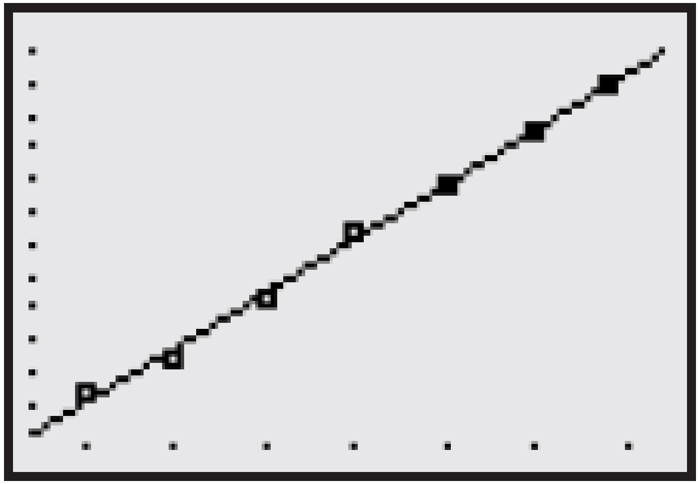
$$s = 1.71t - 113.12$$

1. Verify that the above function is the model.

Using an Equation of a Linear Model to Make Predictions

Solution

- Graph the model and the scattergram in the same viewing window
- Function seems to model the data well



Example Continued

2. Rewrite the equation $s = 1.71t - 113.12$ with the function name f.

Using an Equation of a Linear Model to Make Predictions

Solution

- t is the independent variable
- s is the dependent variable
- f is the function name, so we rewrite $s = f(t)$
- Substitute $f(t)$ for s: $f(t) = 1.71t - 113.12$

Example Continued

3. Predict the average salary in 2011.

Solution

- Represent the year 2011 by $t = 111$
- Substitute 111 for t into $f(t) = 1.71t - 113.12$

$$f(111) = 1.71(111) - 113.12 \quad \text{Substitute 111 for } t.$$
$$= 76.69 \quad \text{Simplify.}$$

Example Continued

4. Predict when the average salary will be $80,000.

Using an Equation of a Linear Model to Make Predictions

Solution

- Represent average salary of \$80,000 by $s = 80$
- Since $s = f(t)$, substitute 80 for $f(t)$ and solve for t

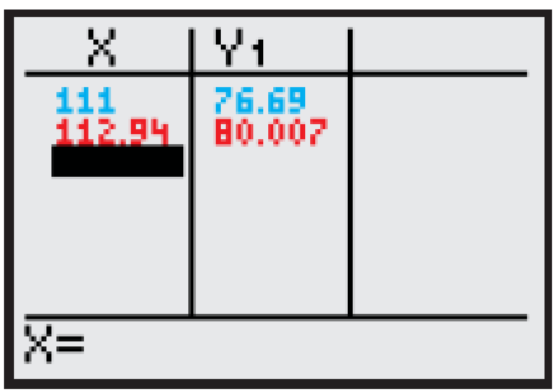
$80 = 1.71t - 113.12$	Substitute 80 for $f(t)$.
$80 + 113.12 = 1.71t - 113.12 + 113.12$	Add 113.12 to both sides.
$193.12 = 1.71t$	Combine like terms.
$\dfrac{193.12}{1.71} = \dfrac{1.71t}{1.71}$	Divide both sides by 1.71.
$112.94 \approx t$	Simplify.

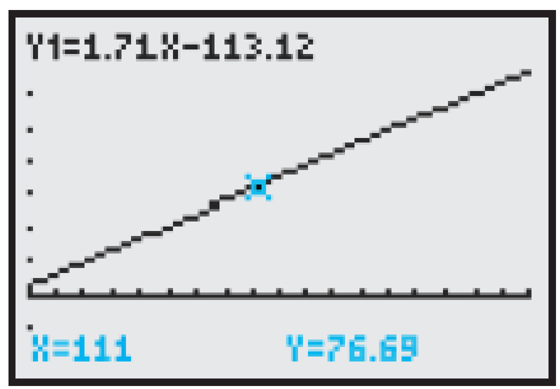
Using an Equation of a Linear Model to Make Predictions

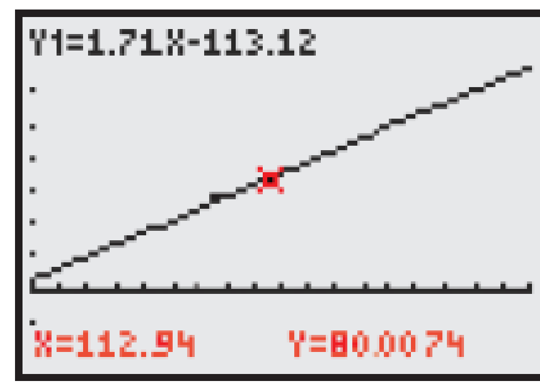
Using Function Notation with Models

Graphing Calculator

- According to model, average salary will be $80,000 in $1900 + 113 = 2013$
- Using TRACE verify the predictions







Using an Equation of a Linear Model to make Predictions

Summary

- When making a prediction about the dependent variable of a linear model, substitute a chosen value for the independent variable in the model. Then solve for the dependent variable.
- When making a prediction about the independent variable of a linear model, substitute a chosen value for the dependent variable in the model. Then solve for the independent variable.

Four-Step Modeling Process

Process

To find a linear model and make estimates and predictions,

1. Create a scattergram of data to determine whether there is a nonvertical line that comes close to the data points. If so, choose two points (not necessarily data points) that you can use to find the equation of a linear model.
2. Find an equation of your model.

Four-Step Modeling Process

Process

3. Verify your equation by checking that the graph of your model contains the two chosen points and comes close to all of the data points.
4. Use the equation of your model to make estimates, make predictions, and draw conclusions.

Example

In an example from Section 2.2, we found the equation $p = -0.53t + 74.50$, where p is the percentage of American adults who smoke and t years since 1990.

Year	Percent Who Smoke
1970	37.4
1980	33.2
1990	25.3
2000	23.1
2005	19.0

1. Rewrite the equation $p = -0.53t + 74.50$ with the function name g.

Solution

- To use the name g, substitute $g(t)$ for p:

$$g(t) = -0.53t + 74.50$$

Example Continued

2. Find $g(110)$. What does the result mean in this function?

Solution

- Substitute 110 for t in the equation $g(t) = -0.53t + 74.50$:

Solution Continued

$g(t) = -0.53t + 74.50$	Equation of g
$g(110) = -0.53(110) + 74.50$	Substitute 110 for t.
$= 16.2$	Simplify.

- When t is 110, p is 16.2. According to the model, 16.2% of American adults will smoke in 2010.

Example Continued

3. Find the value of t when $g(t) = 30$. What does is mean in this situation?

Using Function Notation; Finding Intercepts

Solution

- Substitute 30 for $g(t)$ in the equation and solve for t

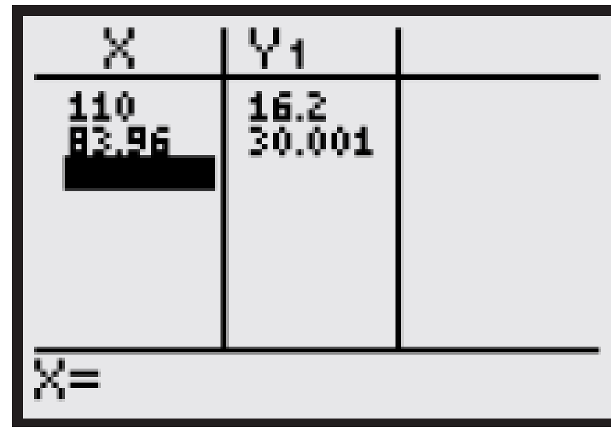
$g(t) = -0.53t + 74.50$	Equation of g
$30 = -0.53t + 74.50$	Substitute 30 for $g(t)$.
$30 - 74.50 = -0.53t + 74.50 - 74.50$	Subtract 74.50 from both sides.
$-44.5 = -0.53t$	Combine like terms.
$\dfrac{-44.5}{-0.53} = \dfrac{-0.53t}{-0.53}$	Divide both sides by -0.53.
$83.96 \approx t$	Simplify.

Solution Continued

- The model estimates that 30% of Americans smoked in

$$1900 + 83.96 \approx 1984$$

- Verify work on graphing calculator table



Example Continued

4. Find the *p*-intercept of the model. What does it mean in this situation?

Solution

- Since the model $g(t) = -0.53t + 74.50$ is in slope-intercept form the *p*-intercept is (0, 74.50)
- The model estimates that 74.5% of American adults smoked in 1900
- Research would show that this estimate is too high model breakdown has occurred

Example Continued

5. Find the *t*-intercept. What does it mean?

Solution

- To find the t-intercept, we substitute 0 for $g(t)$ and solve for t:

$$0 = -0.53t + 74.50 \qquad \text{Substitute 0 for } g(t).$$

$$0 + 0.53t = -0.53t + 74.50 + 0.53t \qquad \text{Add 0.53t to both sides.}$$

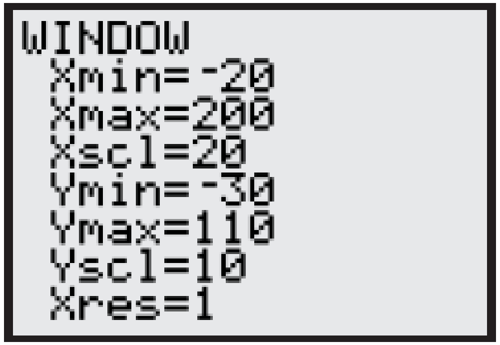
$$0.53t = 74.50 \qquad \text{Combine like terms.}$$

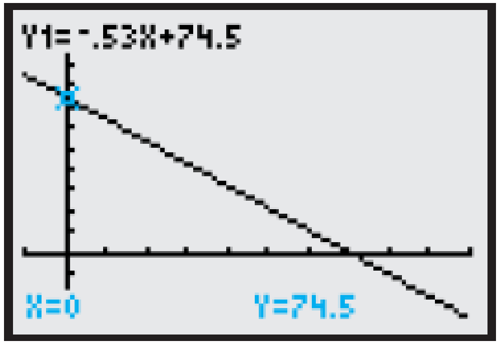
$$\frac{0.53t}{0.53} = \frac{74.50}{0.53} \qquad \text{Divide both sides by 0.53.}$$

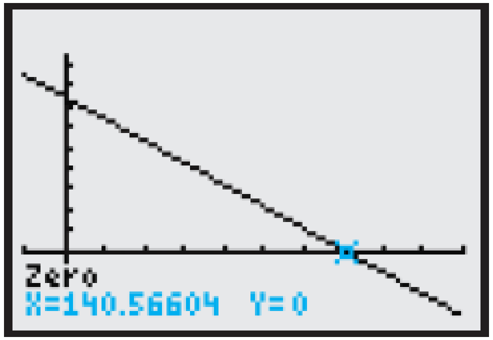
$$t \approx 140.57 \qquad \text{Simplify.}$$

Solution Continued

- The t-intercept is (140.57, 0)
- So, the model predicts that no Americans adults will smoke in $1900 + 140.57 \approx 2041$
- Common sense suggest this probably won't occur
- Use TRACE to verify the p- and i-intercepts.







Intercepts of Models

Finding Intercepts

Property

If a function of the form $p = mt + b$, where $m \neq 0$, is used to model a situation, then

- The *p*-intercept is $(0, b)$.
- To find the coordinate of the *t*-intercept, substitute 0 for *p* in the model's equation and solve for *t*.

Making a Prediction

Using Data Described in Words to Make Predictions

Example

Sales of bagged salads increased approximately linearly from \$0.9 billion in 1996 to \$2.7 billion in 2004. Predict in which year the sales will be \$4 billion.

Solution

- Let s be the sales (in billions of dollars)
- Let t be the years since 1990
- We want an equation of the form $s = mt + b$

Making a Prediction

Using Data Described in Words to Make Predictions

Solution Continued

Years Since 1990 t	Sales (billions of dollars) s
6	0.9
14	2.7

- First find the slope

$$m = \frac{2.7 - 0.9}{14 - 6} \approx 0.23$$

- Substitute 0.23 for *m*: $s = 0.23x + b$
- To find *b* we substitute 6 for *t* and 0.9 for *s*

$0.9 = 0.23(6) + b$	Substitute 6 for t and 0.9 for s.
$0.9 = 1.38 + b$	Multiply.
$0.9 - 1.38 = 1.38 + b - 1.38$	Subtract 1.38 from both sides.
$-0.48 = b$	Combine like terms.

Making a Prediction

Using Data Described in Words to Make Predictions

Solution Continued

- Then substitute –0.48 for *b*:

$$s = 0.23t - 0.48$$

- To predict when the sales will be $4 billion, we substitute 4 for *s* in the equation and solve for *t*:

$4 = 0.23t - 0.48$	Substitute 4 for *s*.
$4 + 0.48 = 0.23t - 0.48 + 0.48$	Add 0.48 to both sides.
$4.48 = 0.23t$	Combine like terms.
$19.48 \approx t$	Divide both sides by 0.23.

Solution Continued

- The model predicts that sales will be $4 billion in $1990 + 19 = 2009$
- Verify using a graphing calculator table