Section 7.8

Modeling with Quadratic Functions

Using a Quadratic Function to Make Predictions

Example

Airlines originated frequent-flier programs in 1981. The cumulative unredeemed miles are the total number of frequent-flier miles that members have not redeemed (spent) from 1981 through a specified year (see table). Let c be the cumulative unredeemed miles (in trillions of miles) at t years since 1980.

Year	Cumulative Unredeemed Miles (trillions of miles)
1986	0.1
1988	0.4
1990	0.9
1992	1.5
1994	2.2
1996	3.2
1998	4.6
2000	6.6
2002	9.1
2004	12.4
2005	14.2

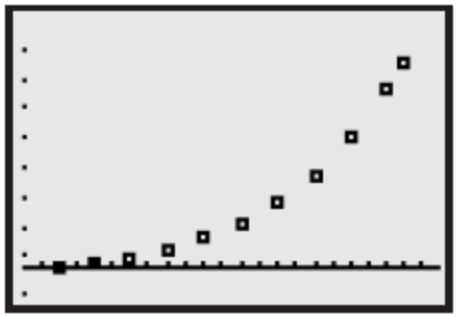
Using a Quadratic Function to Make Predictions

Example Continued

1. Find a model to describe the situation.
2. In what years is there model breakdown for certain?
3. Predict the cumulative unredeemed miles in 2010.
4. Predict when the cumulative unredeemed miles will be 25 trillion miles.

Solution

- Use a graphing Calculator to plot a scattergram



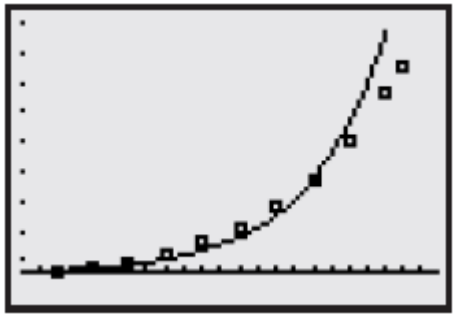
Using a Quadratic Function to Make Predictions

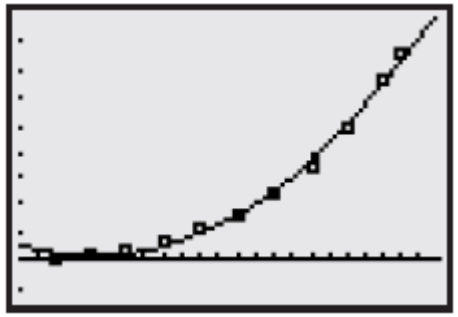
Solution

1. The curve "bend" so not linear

$E(t) = 0.064(1.26)^t$ Exponential regression equation

$Q(t) = 0.047t^2 - 0.77t + 3.5$ Quadratic regression equation





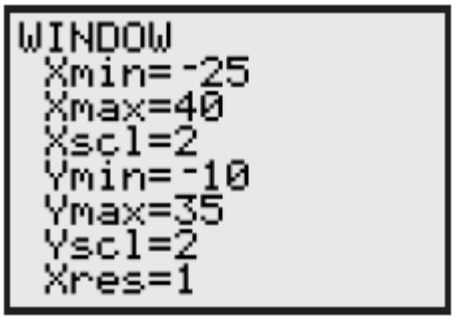
- Quadratic Model is a better fit
- $Q(t) = 0.047t^2 - 0.77t + 3.5$ is a better model

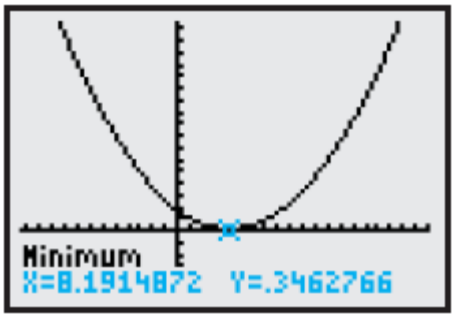
Using a Quadratic Function to Make Predictions

Solution Continued

2. Find the vertex: $t = -\dfrac{-0.77}{2(0.047)} \approx 8.19$

$$Q(8.19) = 0.047(8.19)^2 - 0.77(8.19) + 3.5 \approx 0.35$$





3. Find miles in 2010, so $t = 30$

$$Q(30) = 0.047(30)^2 - 0.77(30) + 3.5 = 22.7$$

Using a Quadratic Function to Make Predictions

Solution Continued

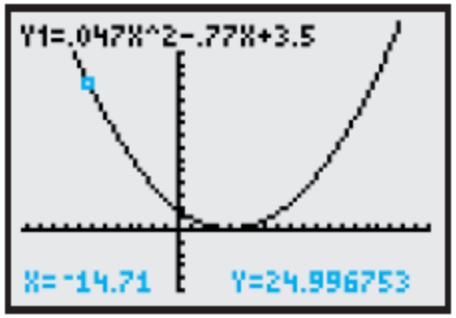
4. $0.047t^2 - 0.77t + 3.5 = 25$ Substitute 25 for $Q(t)$.

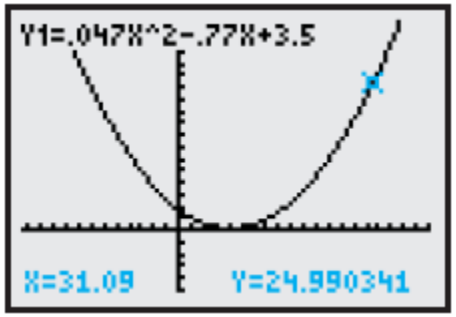
$0.047t^2 - 0.77t - 21.5 = 0$ Subtract 25 from both sides.

$$t = \frac{-(-0.77) \pm \sqrt{(-0.77)^2 - 4(0.047)(-21.5)}}{2(0.047)}$$

Substitute $a = 0.047$, $b = -0.77$, and $c = -21.5$ in quadratic formula.

$t \approx -14.71$ or $t \approx 31.09$ Compute.





Finding the Maximum or Minimum Value of a Quantity

Example

The Table lists the percentages of workers who use computers on the job by age groups. Let $p = f(t)$ be the percentage of workers who use computers at age t years.

Age Group (years)	Age Used to Represent Age Group (years)	Percent
18–25	21.5	34.4
25–29	27.0	48.3
30–39	34.5	50.7
40–49	44.5	51.3
50–59	54.5	43.9
over 59	62.5	27.2

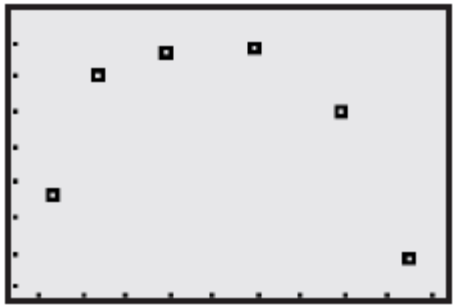
1. Find a formula of a function that provides a reasonable model of the computer data.

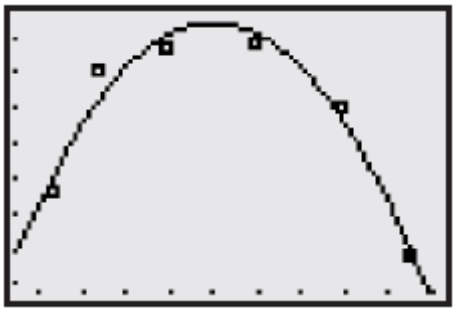
Example Continued

2. Use f to estimate the percentage of 22-year-old workers who use computers on the job.
3. Estimate the age(s) at which half of workers use computers on the job.
4. Estimate the age of workers who are most likely to use computers on the job (maximum percentage). What is that maximum percentage?
5. Find the t-intercepts. What do they mean in this situation?

Solution

1. Use a graphing calculator to find quadratic regression





$$f(t) = -0.051t^2 + 4.09t - 28.14$$

Solution Continued

2. About 37.2% of 22-year-old workers use computers at work

$$f(22) = -0.051(22)^2 + 4.09(22) - 28.14 \approx 37.16$$

3. Half of all workers is 50%
   - Substitute 50 for $f(t)$

$$50 = -0.051t^2 + 4.09t - 28.14$$

$$t = \frac{-4.09 \pm \sqrt{4.09^2 - 4(-0.051)(-78.14)}}{2(-0.051)}$$

Substitute $a = -0.051$, $b = 4.09$, and $c = -78.14$ in quadratic formula.

$t \approx 48.80$ or $t \approx 31.40$

Compute.

Solution Continued

4. t coordinate is about 40

$$t = -\frac{4.09}{2(-0.051)} \approx 40.10$$

- Evaluate $f(40)$

$$f(40) = -0.051(40)^2 + 4.09(40) - 28.14 = 53.86$$

5. Find t-intercepts

$$0 = -0.051t^2 + 4.09t - 28.14$$

Substitute 0 for $f(t)$.

$$t = \frac{-4.09 \pm \sqrt{4.09^2 - 4(-0.051)(-28.14)}}{2(-0.051)}$$

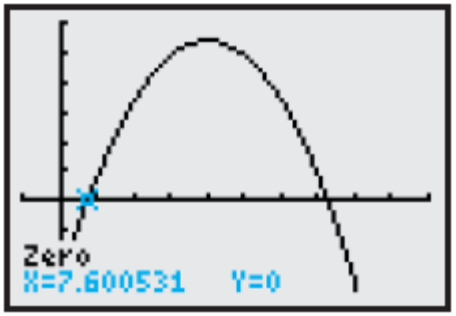
Substitute $a = -0.051$, $b = 4.09$, and $c = -28.14$ in quadratic formula.

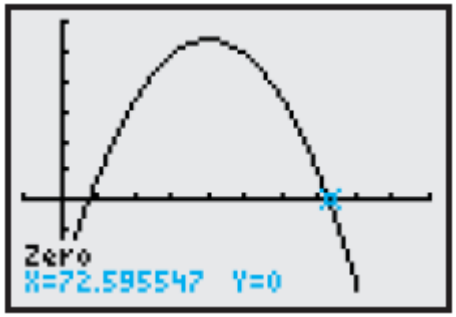
$$t \approx 72.60 \quad \text{or} \quad t \approx 7.60$$

Compute.

Solution Continued

- Verify using a graphing calculator





Modeling with a System of Quadratic Equations

Example

The numbers of videocassettes and DVDs bought by U.S. dealers to sell as rentals are shown in the table for various years.

Year	Number of Videocassettes (millions of units)	Number of DVDs (millions of units)
1998	57.0	1.6
1999	86.2	8.6
2000	99.4	13.9
2001	86.2	37.1
2002	73.6	79.3
2003	53.2	110.9

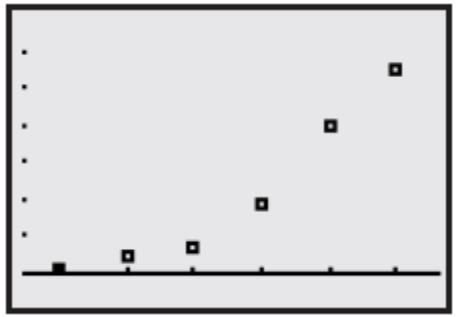
Estimate when sales of DVDs to dealers overtook sales of videocassettes to dealers.

Modeling with a System of Quadratic Equations

Solution

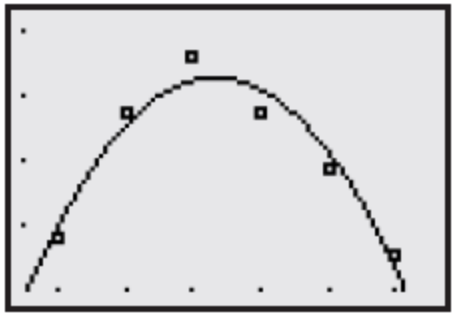
Videocassette Scattergram

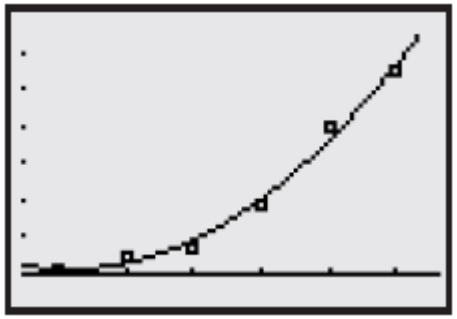
DVD Scattergram



$$S = v(t) = -6.27t^2 + 129.7t - 576.2$$

$$S = d(t) = 4.83t^2 - 79.1t + 326.0$$





Modeling with a System of Quadratic Equations

Solution Continued

- Find when they are equal

$$-6.27t^2 + 129.7t - 576.2 = 4.83t^2 - 79.1t + 326.0$$
$$-11.1t^2 + 208.8t - 902.2 = 0$$

$$t = \frac{-208.8 \pm \sqrt{208.8^2 - 4(-11.1)(-902.2)}}{2(-11.1)}$$

$$t \approx 12.09 \quad \text{or} \quad t \approx 6.73$$