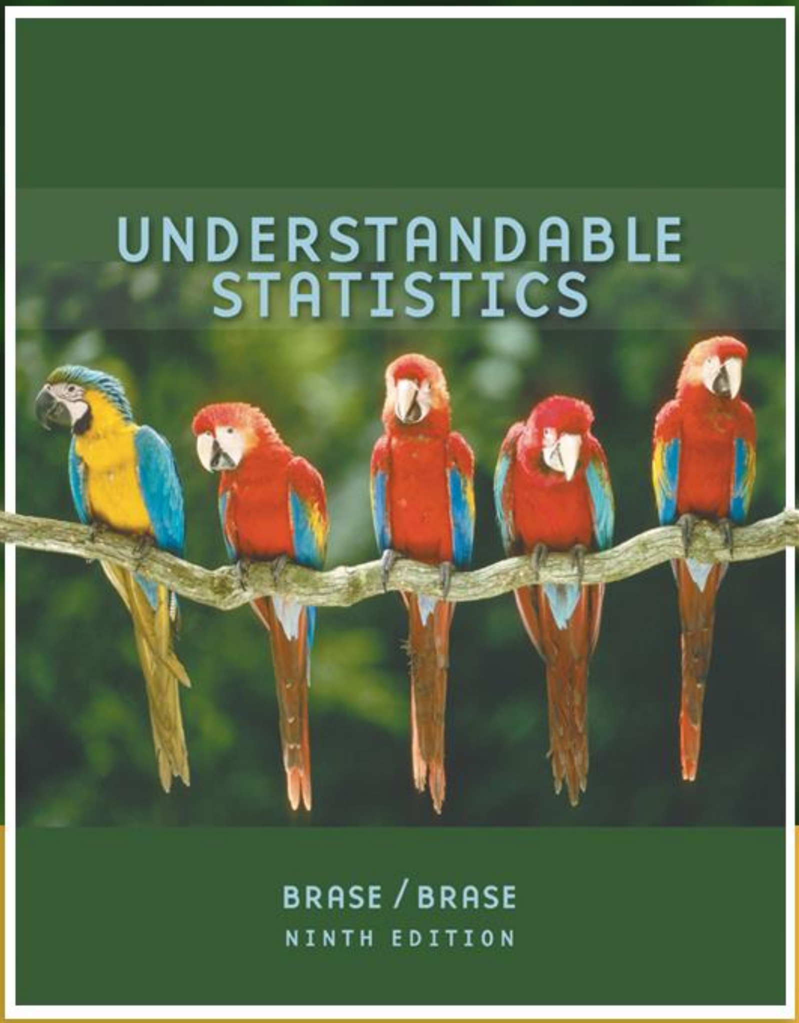

Chapter 4
Elementary Probability Theory

Understandable Statistics
Ninth Edition
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Probability

- Probability is a numerical measure that indicates the likelihood of an event.
- All probabilities are between 0 and 1, inclusive.
- A probability of 0 means the event is impossible.
- A probability of 1 means the event is certain to occur.
- Events with probabilities near 1 are likely to occur.

Probability

- Events can be named with capital letters: A, B, C…
- $P(A)$ means the probability of A occurring.
  - $P(A)$ is read "P of A"
  - $0 \leq P(A) \leq 1$

Probability Assignment

- Assignment by intuition – based on intuition, experience, or judgment.
- Assignment by relative frequency –

  $P(A)$ = Relative Frequency = $\frac{f}{n}$
- Assignment for equally likely outcomes

$$P(A) = \frac{Number\ of\ Outcomes\ Favorable\ to\ Event\ A}{Total\ Number\ of\ Outcomes}$$

Law of Large Numbers

- In the long run, as the sample size increases, the relative frequency will get closer and closer to the theoretical probability.
  - Example: We repeat the penny experiment, and the relative frequency gets closer and closer to P(head) = 0.50

Relative Frequency	**0.52**	**0.518**	**0.495**	**0.503**	**0.4996**
***f* = number of flips**	**104**	**259**	**495**	**1006**	**2498**
***n* = number of heads**	**200**	**500**	**1000**	**2000**	**5000**

Probability Definitions

- Statistical Experiment: Any random activity that results in a definite outcome.
- Event: A collection of one or more outcomes in a statistical experiment.
- Simple Event: An event that consists of exactly one outcome in a statistical experiment.
- Sample Space: The set of all simple events.

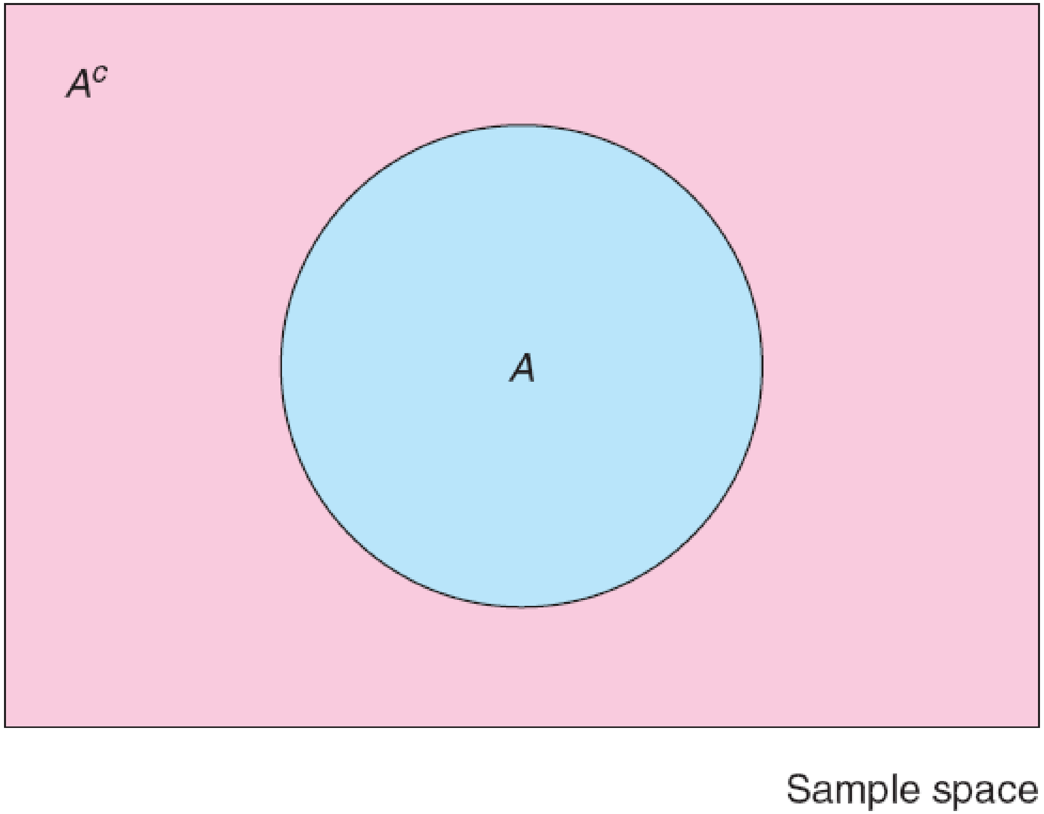
The Sum Rule and The Complement Rule

- The sum of the probabilities of all the simple events in the sample space must equal 1.
- The complement of event A is the event that A *does not occur,* denoted by A^c
- $P(A^c) = 1 - P(A)$

The Complement Rule

FIGURE 4-1

The Event A and Its Complement A^c



Probability versus Statistics

- Probability is the field of study that makes statements about what will occur when a sample is drawn from a known population.

- Statistics is the field of study that describes how samples are to be obtained and how inferences are to be made about unknown populations.

Independent Events

- Two events are independent if the occurrence or nonoccurrence of one event does *not* change the probability of the other event.

- Multiplication Rule for Independent Events

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

- General Multiplication Rule – For all events (independent or not):

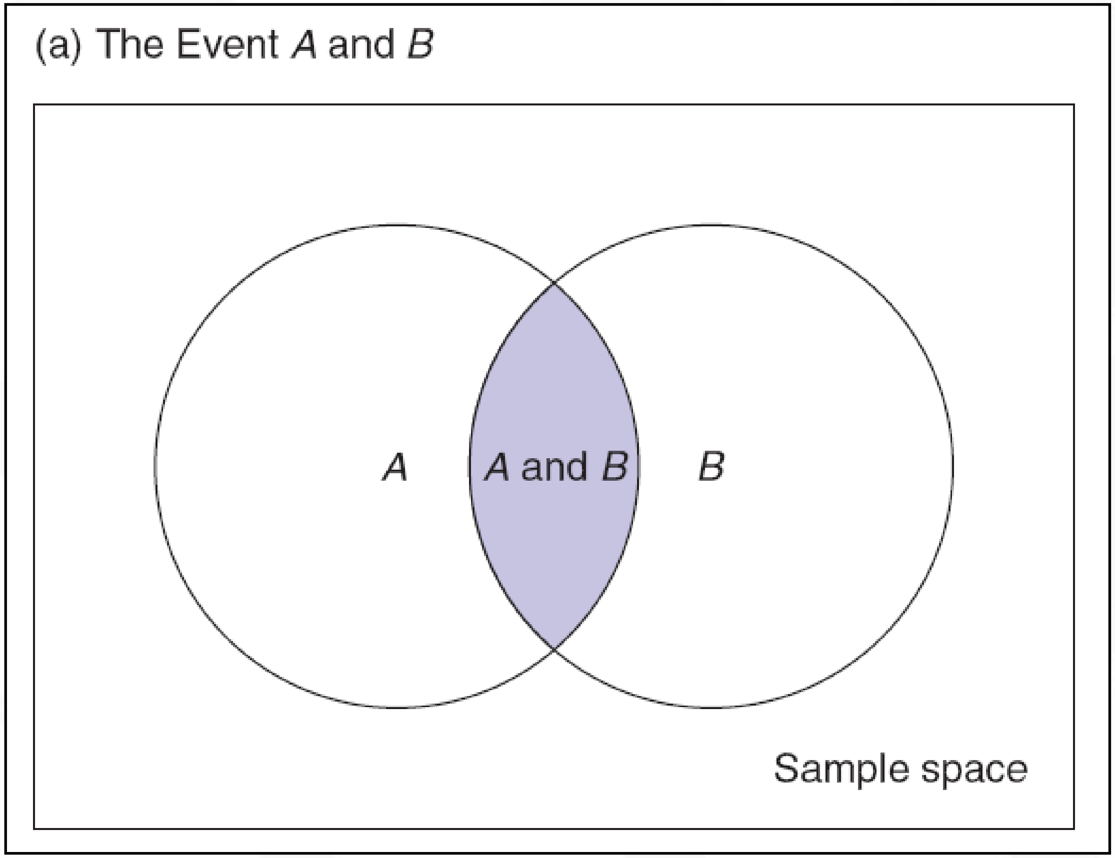
$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

$$P(A \text{ and } B) = P(B) \cdot P(A \mid B)$$

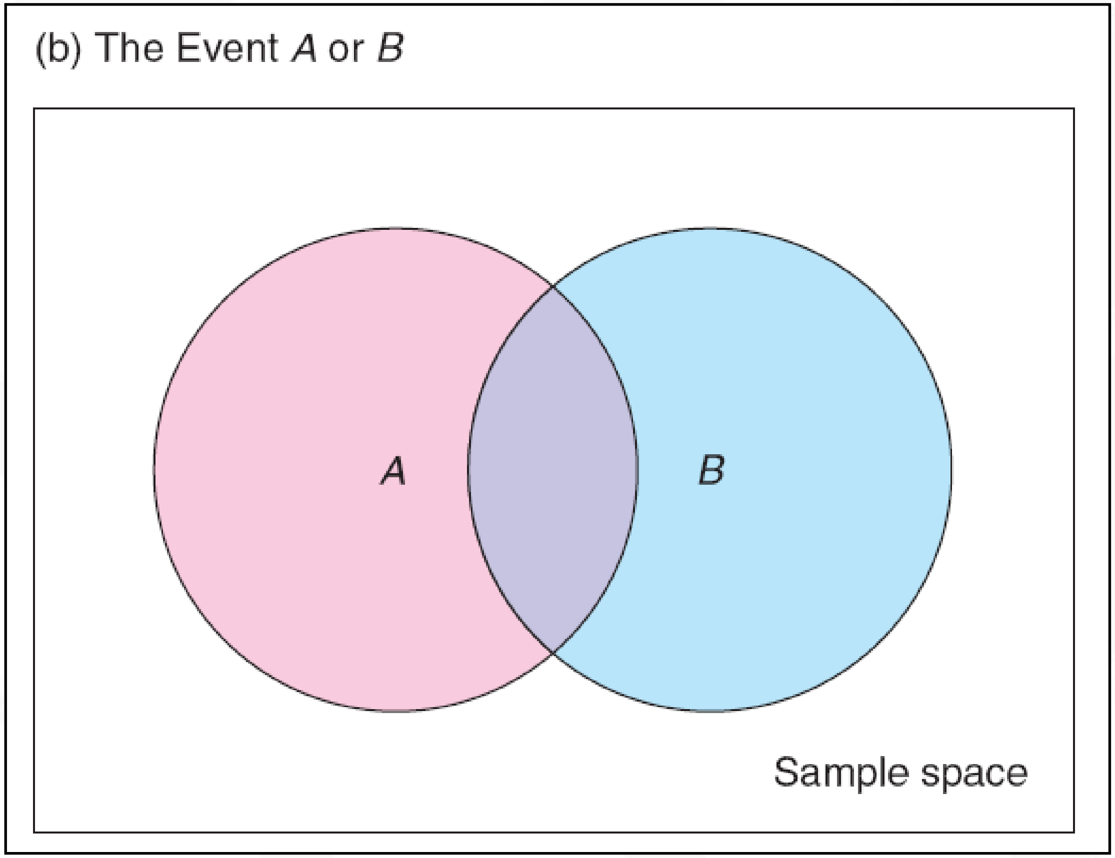
- Conditional Probability (when $P(B) \neq 0$):

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

Two Events Occurring Together



Either or Both of Two Events Occurring



Mutually Exclusive Events

- Two events are mutually exclusive if they cannot occur at the same time.
- Mutually Exclusive = Disjoint
- If A and B are mutually exclusive, then

$$P(A \text{ and } B) = 0$$

Addition Rules

- If A and B are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$.
- If A and B are not mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Critical Thinking

- Pay attention to translating events described by common English phrases into events described using *and, or, complement*, or *given.*

- Rules and definitions of probabilities have extensive applications in everyday lives.

Multiplication Rule for Counting

Multiplication rule of counting

If there are n possible outcomes for event E_1 and m possible outcomes for event E_2, then there are a total of $n \times m$ or nm possible outcomes for the series of events E_1 followed by E_2.

This rule extends to outcomes involving three, four, or more series of events.

Tree Diagrams

- Displays the outcomes of an experiment consisting of a sequence of activities.
  - The total number of branches equals the total number of outcomes.
  - Each unique outcome is represented by following a branch from start to finish.

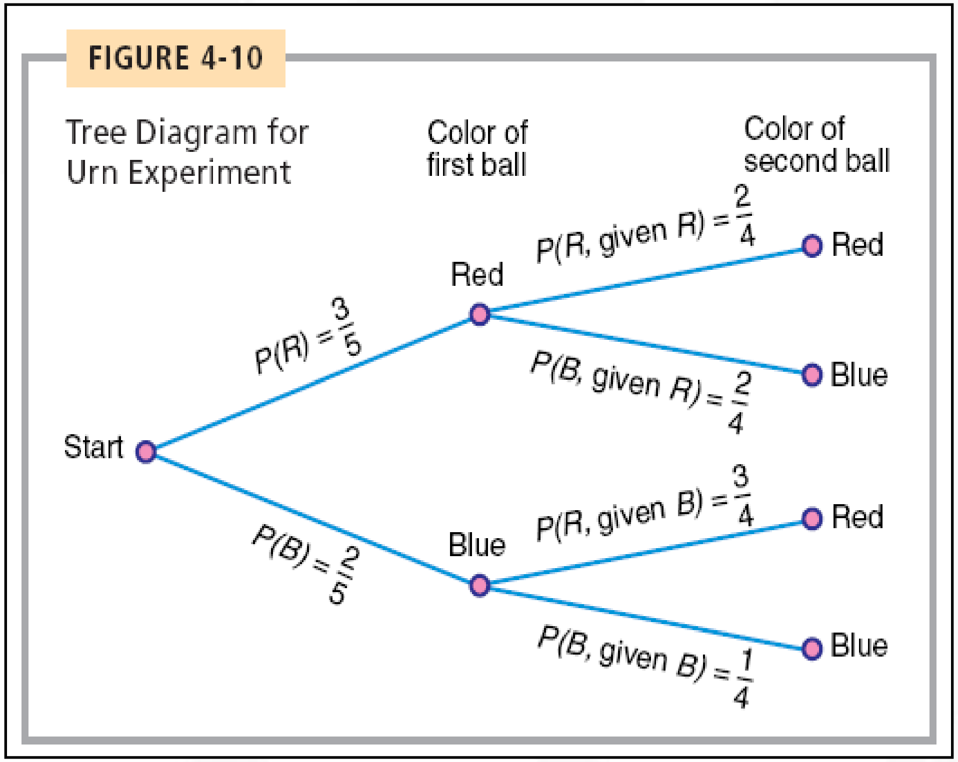
Tree Diagrams with Probability

- We can also label each branch of the tree with its respective probability.

- To obtain the probability of the events, we can multiply the probabilities as we work down a particular branch.

Urn Example

- Suppose there are five balls in an urn. Three are red and two are blue. We will select a ball, note the color, and, without replacing the first ball, select a second ball.



There are four possible outcomes:
Red, Red
Red, Blue
Blue, Red
Blue, Blue

We can find the probabilities of the outcomes by using the multiplication rule for dependent events.

Factorials

- For counting numbers 1, 2, 3, …
- ! is read “factorial”
  - So for example, 5! is read “five factorial”
- $n! = n * (n-1) * (n-2) * \ldots * 3 * 2 * 1$
  - So for example, 5! = 5 * 4 * 3 * 2 * 1 = 120
- 1! = 1
- 0! = 1

Permutations

- Permutation: ordered grouping of objects.
- Counting Rule for Permutations

Counting rule for permutations

The number of ways to *arrange in order n* distinct objects, taking them r at a time, is

$$P_{n,r} = \frac{n!}{(n-r)!} \qquad (9)$$

where n and r are whole numbers and $n \geq r$. Another commonly used notation for permutations is nPr.

Combinations

- A combination is a grouping that pays no attention to order.
- Counting Rule for Combinations

Counting rule for combinations

The number of *combinations* of n objects taken r at a time is

$$C_{n,r} = \frac{n!}{r!(n-r)!} \qquad (10)$$

where n and r are whole numbers and $n \geq r$. Other commonly used notations for combinations include nCr and $\binom{n}{r}$.