

Chapters 8 and 9 – Hypothesis testing and confidence intervals for one population

(I) Section 9.2 - Confidence Intervals and Hypothesis testing Regarding the Population Mean μ (σ Known/Unknown)

Assumptions

- We have a simple random sample
- The population is normally distributed or the sample size, n , is large ($n > 30$)

The procedure is robust, which means that minor departures from normality will not adversely affect the results of the test. However, for small samples, if the data have outliers, the procedure should not be used.

Use normal probability plots to assess normality and box plots to check for outliers.

A normal probability plot plots observed data versus normal scores. If the normal probability plot is roughly linear and all the data lie within the bounds provided by the software (our calculator does not show the bounds), then we have reason to believe the data come from a population that is approximately normal.

(II) Using the calculator to test hypothesis or construct confidence intervals for one population mean

For hypothesis testing use items 1 or 2 from the STAT, TESTS menu

For confidence intervals use items 7 or 8 from the STAT, TESTS menu

(III) Section 9.3 - Confidence Intervals and Hypothesis Testing Regarding the Population Proportion

Assumptions

- The sample is a simple random sample. (SRS)
- The conditions for a binomial distribution are satisfied by the sample. That is: there are a fixed number of trials, the trials are independent, there are two categories of outcomes, and the probabilities remain constant for each trial. A “trial” would be the examination of each sample element to see which of the two possibilities it is.
- The normal distribution can be used to approximate the distribution of sample proportions because $np \geq 5$ and $n(1 - p) \geq 5$ are both satisfied.

Technically, many times the trials are not independent, but they can be treated as if they were independent if $n \leq 0.05 N$ (the sample size is no more than 5% of the population size)

Notice that it is possible that in some cases the p-value method may yield a different conclusion than the confidence interval method. This is due to the fact that when constructing confidence intervals, we use an estimated standard deviation based on the sample proportion \hat{p} .

If we are testing claims about proportions, it is recommended to use the p-value method or the traditional method.

(IV) Using the calculator to test hypothesis or construct confidence intervals for one population proportion

For hypothesis testing use item 5 from the STAT, TESTS menu

For confidence intervals use item A from the STAT, TESTS menu

1) In 1990, the mean pH level of the rain in Pierce County, Washington, was 5.03. A biologist claims that the acidity of the rain has increased. (This would mean that the pH level of the rain has decreased.) From a random sample of 19 rain dates in 2000, she obtains the data shown below. Assume that the population is normally distributed and that $\sigma = 0.2$.

5.08, 4.66, 4.7, 4.87, 4.78, 5.00, 4.50, 4.73, 4.79, 4.65,
4.91, 5.07, 5.03, 4.78, 4.77, 4.6, 4.73, 5.05, 4.7

Source: National Atmospheric Deposition Program

Part 1: Construct a 98% confidence interval estimate for the mean pH levels of rain in that area for the year 2000.

Part 2: At the 1% significance level, test the claim of the biologist that the pH level of the rain in that area has decreased, and therefore, the acidity of the rain has increased.

Part 3 - Solve problem 1 in the case when σ is not known

It is not very realistic to know the standard deviation of the population. Assume that in problem (1), page 2, sigma is not given. In that case, we'll need the standard deviation of the sample, and we'll use the t-distribution.

Part 1: Test the claim of the biologist at the 1% significance level.

Part 2 - Use a calculator feature to construct the 98% confidence interval estimate for the mean pH level of rain in Pierce County Washington for the year 2000.

*Note: Because of time constraints, the hypothesis testing problems involving the t-distribution **will be done only with the corresponding calculator feature.** (You explore on your own how the book handles each method, the critical value and the p-value approach showing all steps)*

2) – Side effects of Lipitor

The drug Lipitor is meant to reduce total cholesterol and LDL-cholesterol. In clinical trials, 19 out of 863 patients taking 10 mg of Lipitor daily complained of flu-like symptoms. Suppose that it is known that 1.9% of patients taking competing drugs complain of flu-like symptoms.

Part I - Is there significant evidence to support the claim that more than 1.9% of Lipitor users experience flu-like symptoms as a side effect at the $\alpha = 0.01$ level of significance?

Part II – Construct a 98% confidence interval estimate for the proportion of all patients who experience flu-like symptoms when taking 10 mg Lipitor daily. (Go to page 8 to do this)

Chapters 8 and 9 – Sections 8.5 and 9.5

Hypothesis testing and confidence intervals for two populations – Independent samples

Inferences about Two Means with Unknown Population Standard Deviations – Independent Samples – Population Standard Deviations not Assumed Equal (Non-Pooled t-Test)

Assumptions

- The samples are obtained using simple random sampling
- The samples are independent
- The populations from which the samples are drawn are normally distributed or the sample sizes are large ($n_1 > 30, n_2 > 30$)

The procedure is robust, so minor departures from normality will not adversely affect the results. If the data have outliers, the procedure should not be used.

3) In the Spacelab Life Sciences 2 payload, 14 male rats were sent to space. Upon their return, the red blood cell mass (in milliliters) of the rats was determined. A control group of 14 male rats was held under the same conditions (except for space flight) as the space rats and their red blood cell mass was also determined when the space rats returned. The project, led by Dr. Paul X. Callahan, resulted in the data listed below.

Part 1 - Construct a 95% confidence interval about $\mu_1 - \mu_2$

Part 2 - Test the claim that the flight animals have a different red blood cell mass from the control animals at the 5% level of significance.

Flight

8.59	8.64	7.43	7.21	6.87	7.89	9.79	6.85	7.00	8.80	9.30	8.03	6.39	7.54
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Control

8.65	6.99	8.40	9.66	7.62	7.44	8.55	8.7	7.33	8.58	9.88	9.94	7.14	9.14
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First: Verify assumptions. Because the sample sizes are small, we must verify normality and that the samples does not contain any outliers. Construct a normal probability plot and a boxplot in order to observe if the conditions for testing the hypothesis are satisfied.

4) Neurosurgery Operative Times

Several neurosurgeons wanted to determine whether a dynamic system (Z-plate) reduced the operative time relative to a static system (ALPS plate). R. Jacobowitz, Ph.D., an ASU professor, along with G. Visheth, M.D., and other neurosurgeons, obtained the data displayed below on operative times, in minutes for the two systems.

Dynamic:	370	360	510	445	295	315	490
	345	450	505	335	280	325	500
Static:		430	445	455	455	490	535

Part 1 - At the 1% significance level, do the data provide sufficient evidence to conclude that the mean operative time is less with the dynamic system than with the static system?

Part 2 - Obtain a 98% confidence interval for the difference between the mean operative times of the dynamic and static systems.

Inferences about Two Population Proportions - Sections 8.5 and 9.5

Assumptions

- The samples are independently obtained using simple random sampling.
- For both samples, the conditions $np \geq 5$ and $n(1 - p) \geq 5$ are both satisfied.
For both samples, the sample size, is no more than 5% of the population size

To construct the confidence interval, press STAT, arrow to TESTS, select B:2-PropZInt

To test the hypothesis, press STAT, arrow to TESTS, select 6:2-PropZTest

5) - Nasonex

In clinical trials of Nasonex, 3774 adult adolescent allergy patients (patients 12 years and older) were randomly divided into two groups. The patients in group 1 (experimental group) received 200 mcg of Nasonex, while the patients in Group 2 (control group) received a placebo. Of the 2103 patients in the experimental group, 547 reported headaches as side effect. Of the 1671 patients in the control group, 368 reported headaches as a side effect.

Part 1 – Use a feature of your calculator to construct a 90% confidence interval estimate for the difference between the two population proportions. What is the interval suggesting?

Part 2 - Is there significant evidence to support the claim that the proportion of Nasonex users that experienced headaches as a side effect is greater than the proportion in the control group at the 0.05 significance level?

6) – Vasectomies and Prostate Cancer

Approximately 450,000 vasectomies are performed each year in the U.S. In this surgical procedure for contraception, the tube carrying sperm from the testicles is cut and tied. Several studies have been conducted to analyze the relationship between vasectomies and prostate cancer. The results of one such study by E. Giovannucci et al. appeared in the paper “A Retrospective Cohort Study of Vasectomy and Prostate Cancer in U.S. Men”. Of 21,300 men who had not had a vasectomy, 69 were found to have prostate cancer; of 22,000 men who had had a vasectomy, 113 were found to have prostate cancer.

Part 1 - At the 1% significance level, do the data provide sufficient evidence to conclude that men who have had a vasectomy are at greater risk of having prostate cancer?

Part 2 – Use the calculator to determine a 98% confidence interval for the difference between the prostate cancer rates of men who have had a vasectomy and those who have not.

Section 9.4 – Tests Involving Paired Differences**Inferences About Two Means – Dependent Samples (Matched Pairs – Paired data)**

A sampling method is dependent when the individuals selected to be in one sample are used to determine the individuals to be in the second sample.

Assumptions:

1. The sample is obtained using simple random sampling
2. The sample data are matched pairs
3. The differences are normally distributed with no outliers or the sample size, n , is large ($n \geq 30$)

Procedure

Take the difference d of the data pairs. Find the mean difference \bar{d} . Perform a t-test on \bar{d} as in section 9.2, with $n-1$ degrees of freedom.

7) Professor Andy Neill measured the time (in seconds) required to catch a falling meter stick for 12 randomly selected students' dominant hand and non-dominant hand. Professor Neill claims that the reaction time in an individual's dominant hand is less than the reaction time in their non-dominant hand. Test the claim at the 5% significance level.

Student	1	2	3	4	5	6	7	8	9	10	11	12
Dominant Hand	0.177	0.210	0.186	0.189	0.198	0.194	0.160	0.163	0.166	0.152	0.190	0.172
Non-dominant hand	0.179	0.202	0.208	0.184	0.215	0.193	0.194	0.160	0.209	0.164	0.210	0.197
Differences												

Part 1: Test the claim that the reaction time in an individual's dominant hand is less than the reaction time in their non-dominant hand. (Use a 5% significance level).

- Part 2: Use the calculator to construct a 90% confidence interval estimate of the mean difference**

8) Rat's Hemoglobin

Hemoglobin helps the red blood cells transport oxygen and remove carbon dioxide. Researchers at NASA wanted to determine the effects of space flight on a rat's hemoglobin. The following data represent the hemoglobin (in grams per deciliter) at lift-off minus 3 days (H-L3) and immediately upon the return (H-T0) for 12 randomly selected rats sent to space on the Spacelab Sciences 1 flight. (Source: NASA Life Sciences Data Archive)

Part 1 - Test the claim that the hemoglobin levels at lift-off minus 3 days are less than the hemoglobin levels upon return at the 5% level of significance

Part 2 - Construct a 90% confidence interval about the population mean difference. Interpret your results. Does the interval support the claim from part (b)?

Rat #	1	2	3	4	5	6	7	8	9	10	11	12
H-L3	15.2	16.1	15.3	16.4	15.7	14.7	14.3	14.5	15.2	16.1	15.1	15.8
H-R0	15.8	16.5	16.7	15.7	16.9	13.1	16.4	16.5	16	16.8	17.6	16.9
Differences												

Verify Assumptions: Verify that the differences come from a population that is approximately normally distributed with no outliers because the sample size is small. In order to do this we must construct a normal probability plot and a boxplot.