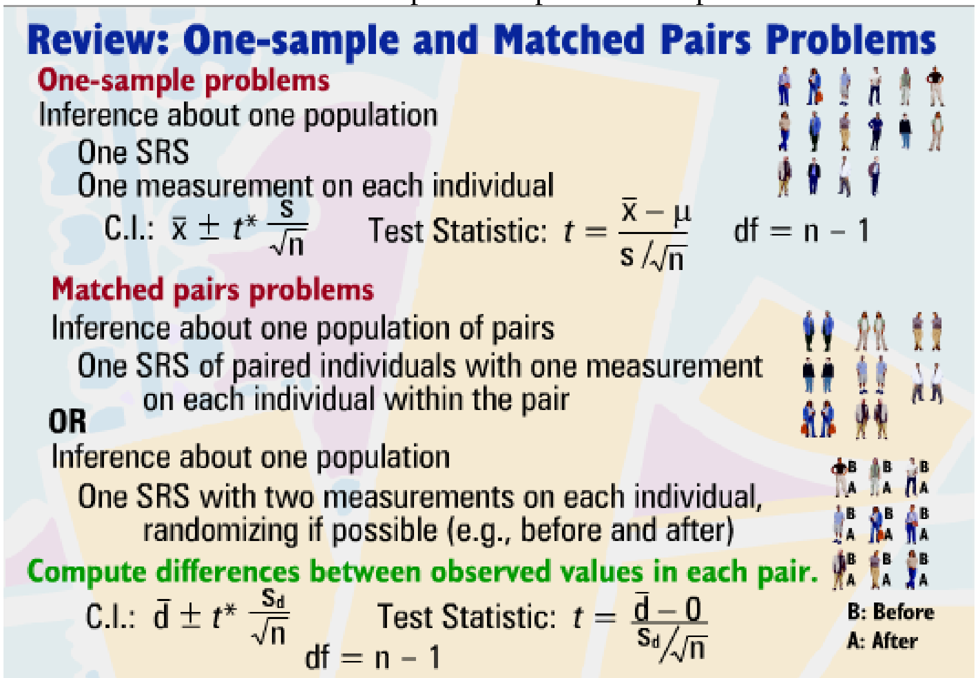
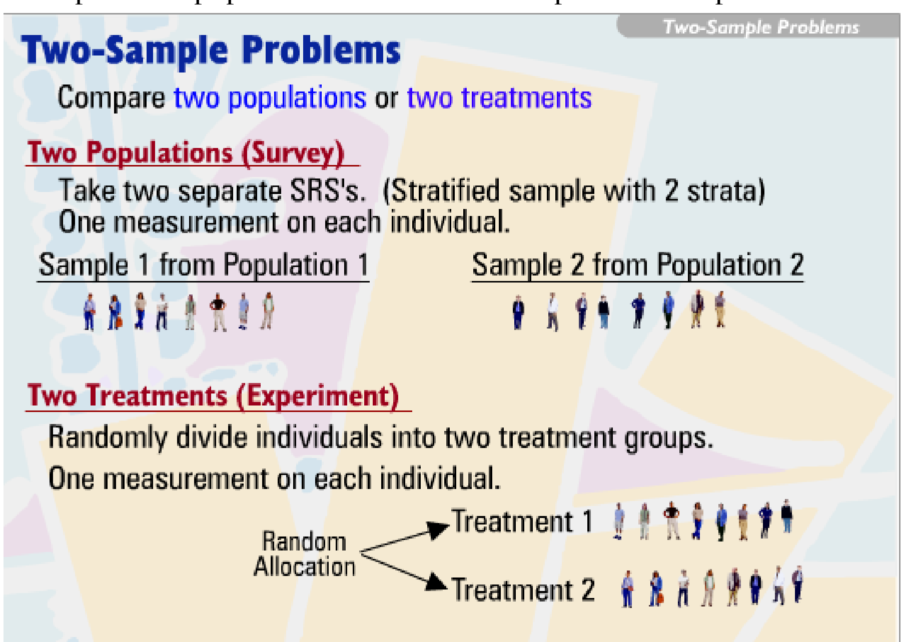
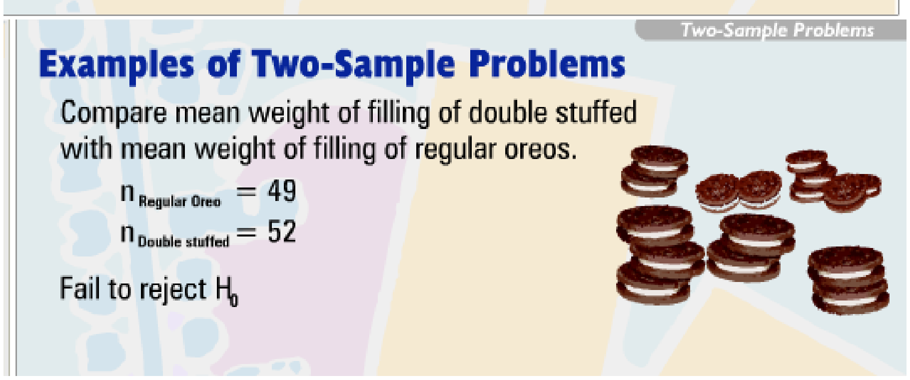
Chapter 18 – Comparing two population means – Independent samples

In chapter 17, we considered the case of Matched pairs – dependent samples



In this chapter 18, we compare two population means from independent samples





Notation for Comparing Two Population Means

	One population	One Sample
Mean	μ	$\bar{x}$
Standard deviation	σ	s

	Two populations		Two Samples	
Means	μ_1	μ_2	$\bar{x}_1$	$\bar{x}_2$
Standard deviations	σ_1	σ_2	s_1	s_2

How should we combine two parameters to get one for comparison?

Sum	$\mu_1 + \mu_2$?	Not informative
Difference	$\mu_1 - \mu_2$?	Informative and mathematically nice
Product	$\mu_1 \bullet \mu_2$?	Not informative
Ratio	μ_1 / μ_2?	Informative but not mathematically nice

What is the corresponding statistic? $\bar{x}_1 - \bar{x}_2$

Two-Sample Problems

Notation for Comparing Two Population Means

	One population	One Sample
Mean	μ	$\bar{x}$
Standard deviation	σ	s

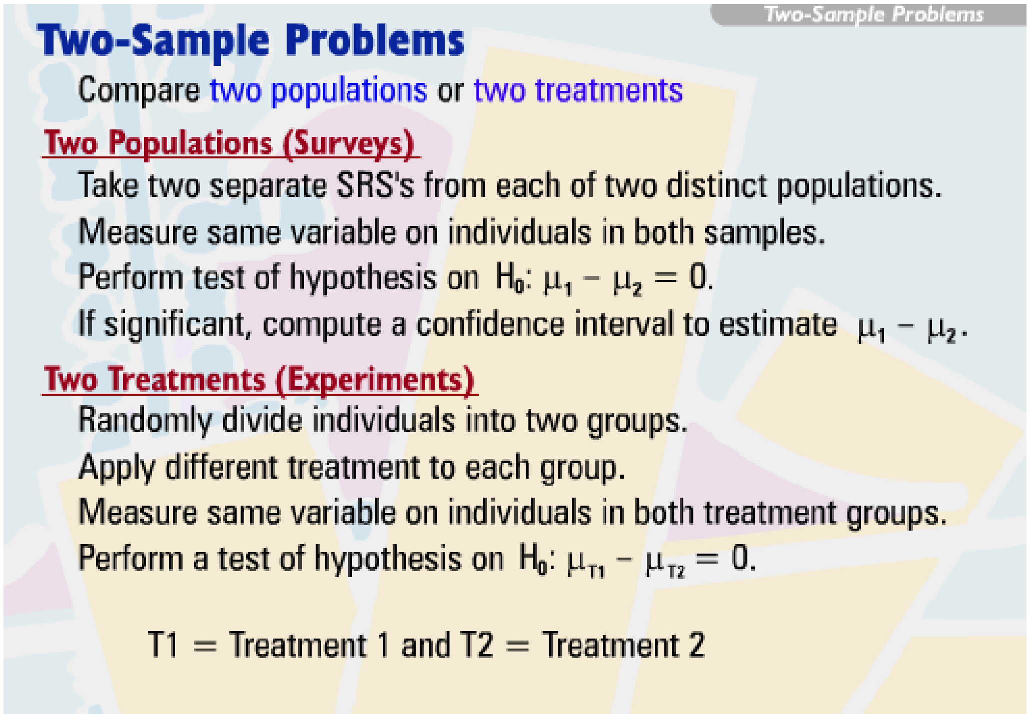
	Two populations		Two Samples	
Means	μ_1	μ_2	$\bar{x}_1$	$\bar{x}_2$
Standard deviations	σ_1	σ_2	s_1	s_2

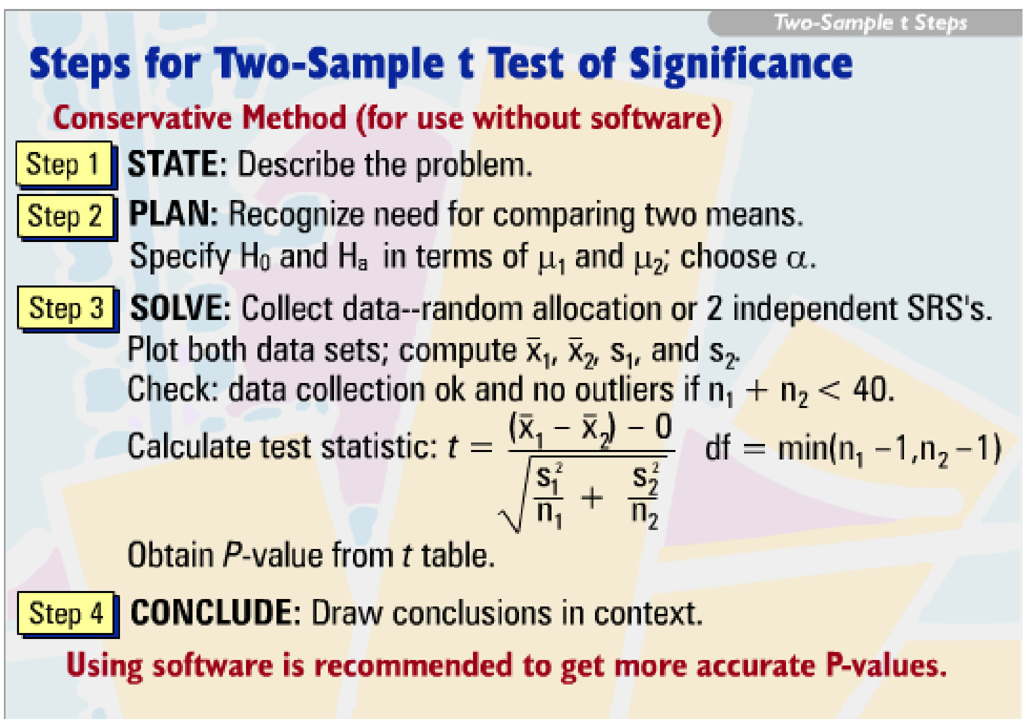
How should we combine two parameters to get one for comparison?

$\mu_1 - \mu_2$ In words: Difference between two population means

$\bar{x}_1 - \bar{x}_2$ In words: Difference between two sample means

What is the corresponding statistic? $\bar{x}_1 - \bar{x}_2$





We will use the calculator to test hypothesis or construct confidence intervals for two population means

Randomized Design Example (Two Treatments)

Conservative Method (for use without software)

Step 1 **STATE** the problem and identify essential information.

A pharmaceutical company is conducting pre-clinical trials of an experimental anti-depressant drug. Since several subjects are complaining of dryness, a technician is assigned to investigate with 20 rats. She plans to randomly allocate 10 rats to receive the drug injection and 10 rats to receive a placebo injection and measure their water intake during the next 24 hour period. Using $\alpha = 0.05$, the research question is:
"***Does the anti-depressant cause an increase in water consumption?***"

Step 2 **PLAN**: Need two-sample *t*-test for means.

Specify H_0 and H_a; choose α.

Type of Alternative:	greater than
Null value ($\mu_D - \mu_P$):	0
n_D:	10
n_P:	10
α:	0.05

H_0: $\mu_D - \mu_P = 0$ versus H_a: $\mu_D - \mu_P > 0$

Randomized Design Example (Two Treatments)

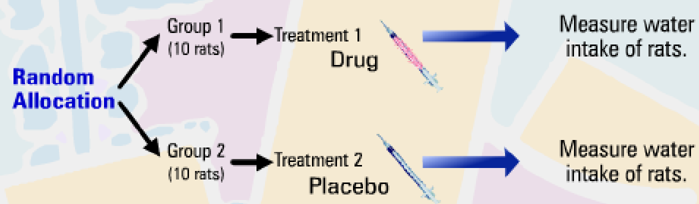
Conservative Method (for use without software)

Step 3 **SOLVE:** Collect data by conducting experiment.

Randomized Design Example (Two Treatments)

Conservative Method (for use without software)

Step 3 **SOLVE:** Collect data by conducting experiment.



Randomized Design Example (Two Treatments)

Conservative Method (for use without software)

Step 3 **SOLVE:** Plot each data set, compute $\bar{x}_D$, $\bar{x}_P$ and s_D and s_P.

Drug	Placebo
8.2	8.5
9.0	7.2
8.8	7.7
8.3	8.0
7.5	8.6
9.2	7.8
9.9	8.7
7.8	7.8
8.4	8.0
7.7	7.0

Side-By-Side Stemplot
Leaf unit = 0.1

Drug		Placebo
	7	0 2
5 7 8	7	7 8 8
2 3 4	8	0 0
8	8	5 6 7
0 2	9	
9	9	

	Drug	Placebo
$\bar{x}$	8.48	7.93
s	0.750	0.564
n	10	10

Randomized Design Example

Conservative Method (for use without s

Step 3 **SOLVE:** (cont.) Check conditions

Rats randomly assigned to treatments; no outliers or strong skewness in either data set; use of *t* procedure is ok.

Randomized Design Example (Two Treatments)

Conservative Method (for use without software)

Step 3 **SOLVE:** (cont.) Calculate test statistic

$$\bar{x}_D - \bar{x}_P = 8.48 - 7.93 = 0.55$$

$$t = \frac{(\bar{x}_D - \bar{x}_P) - 0}{\sqrt{\frac{s_D^2}{n_D} + \frac{s_P^2}{n_P}}} = \frac{(8.48 - 7.93) - 0}{\sqrt{\frac{0.750^2}{10} + \frac{0.564^2}{10}}} = \frac{0.55}{0.297} = 1.85$$

	Drug	Placebo
$\bar{x}$	8.48	7.93
s	0.750	0.564
n	10	10

Randomized Design Example (Two Treatments)

Conservative Method (for use without software)

Step 3 **SOLVE:** (cont.) Find *P*-value for $t = 1.85$.
df = min(10 – 1, 10 – 1) = min(9, 9)

$.05 > P > .025$

9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
One-sided *P*	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005

$1.833 < 1.85 < 2.262$

Randomized Design Example (Two Treatments)

Conservative Method (for use without software)

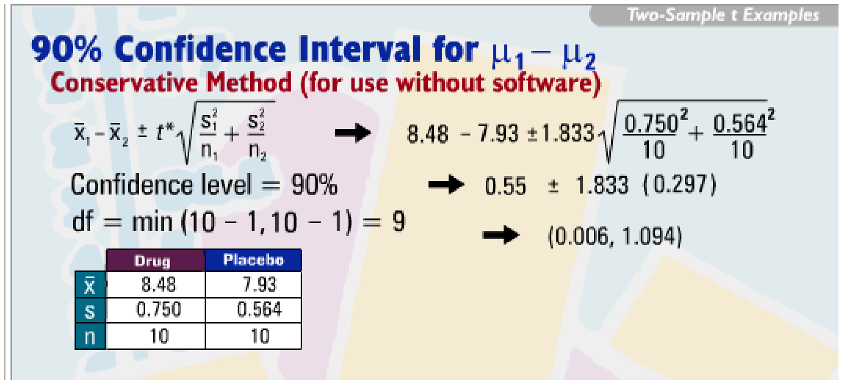
Step 3 **SOLVE:** (cont.) Find *P*-value for $t = 1.85$.
df = min(10 – 1, 10 – 1) = min(9, 9)

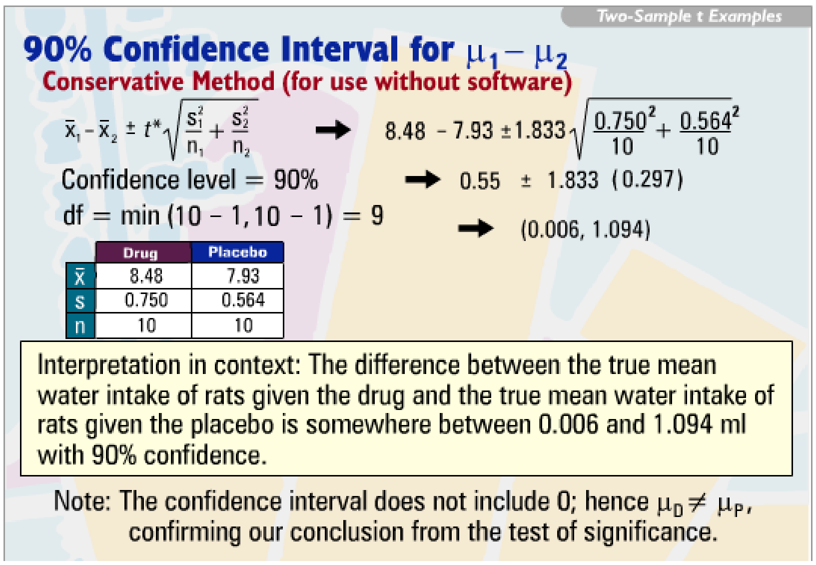
Step 4 **CONCLUDE:** Draw conclusions in context.

P-value: $.025 < P < .05 = \alpha$ (level of significance)

Sufficient evidence to reject the null hypothesis at $\alpha = 0.05$

Conclusion in context: At $\alpha = 0.05$, the average water intake for the rats in the drug group is significantly greater than the average water intake for the rats in the placebo group.
Valid experiment: The anti-depressant drug *causes* an increase in thirst in rats.





Increase in water intake