

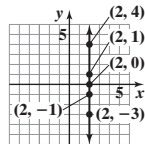
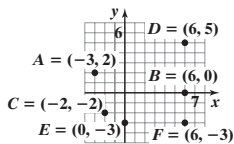
Answers

Chapter 1 Graphs

1.1 Assess Your Understanding (page 13)

7. x -coordinate or abscissa; y -coordinate or ordinate 8. quadrants 9. midpoint 10. F 11. F 12. T 13. d 14. c

15. (a) quadrant II (b) x -axis 17. The points will be on a vertical line that is 2 units to the right of the y -axis.
 (c) quadrant III (d) quadrant I
 (e) y -axis (f) quadrant IV

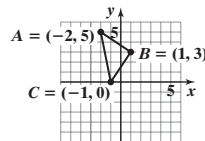


19. $(-1, 4)$; quadrant II
 21. $(3, 1)$; quadrant I
 23. $X_{\min} = -11, X_{\max} = 5, X_{\text{scl}} = 1, Y_{\min} = -3, Y_{\max} = 6, Y_{\text{scl}} = 1$
 25. $X_{\min} = -30, X_{\max} = 50, X_{\text{scl}} = 10, Y_{\min} = -90, Y_{\max} = 50, Y_{\text{scl}} = 10$
 27. $X_{\min} = -10, X_{\max} = 110, X_{\text{scl}} = 10, Y_{\min} = -10, Y_{\max} = 160, Y_{\text{scl}} = 10$
 29. $X_{\min} = -6, X_{\max} = 6, X_{\text{scl}} = 2, Y_{\min} = -4, Y_{\max} = 4, Y_{\text{scl}} = 2$

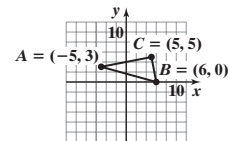
31. $X_{\min} = -6, X_{\max} = 6, X_{\text{scl}} = 2, Y_{\min} = -1, Y_{\max} = 3, Y_{\text{scl}} = 1$ 33. $X_{\min} = 3, X_{\max} = 9, X_{\text{scl}} = 1, Y_{\min} = 2, Y_{\max} = 10, Y_{\text{scl}} = 2$

35. $\sqrt{5}$ 37. $\sqrt{10}$ 39. $2\sqrt{17}$ 41. 20 43. $\sqrt{53}$ 45. $\sqrt{a^2 + b^2}$ 47. $4\sqrt{10}$ 49. $2\sqrt{65}$

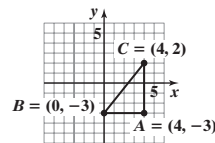
51. $d(A, B) = \sqrt{13}$
 $d(B, C) = \sqrt{13}$
 $d(A, C) = \sqrt{26}$
 $(\sqrt{13})^2 + (\sqrt{13})^2 = (\sqrt{26})^2$
 Area = $\frac{13}{2}$ square units



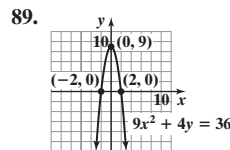
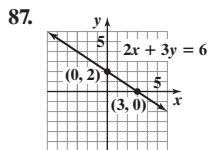
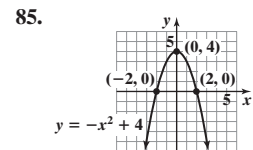
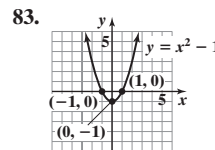
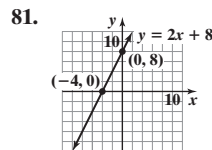
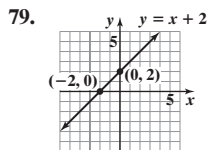
53. $d(A, B) = \sqrt{130}$
 $d(B, C) = \sqrt{26}$
 $d(A, C) = 2\sqrt{26}$
 $(\sqrt{26})^2 + (2\sqrt{26})^2 = (\sqrt{130})^2$
 Area = 26 square units



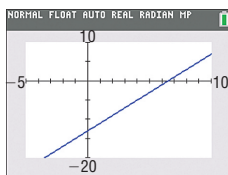
55. $d(A, B) = 4$
 $d(B, C) = \sqrt{41}$
 $d(A, C) = 5$
 $4^2 + 5^2 = (\sqrt{41})^2$
 Area = 10 square units



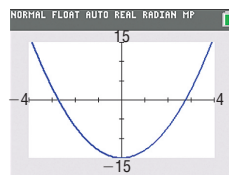
57. $(4, 0)$ 59. $(3, 3)$ 61. $(5, -1)$ 63. $(\frac{a}{2}, \frac{b}{2})$ 65. $(0, 0)$ is on the graph.
 67. $(0, 3)$ is on the graph. 69. $(0, 2)$ and $(\sqrt{2}, \sqrt{2})$ are on the graph. 71. $(-1, 0), (1, 0)$
 73. $(-\frac{\pi}{2}, 0), (0, 1), (\frac{\pi}{2}, 0)$ 75. $(0, 2), (1, 0), (0, -2)$ 77. $(-4, 0), (-1, 0), (0, -3), (4, 0)$



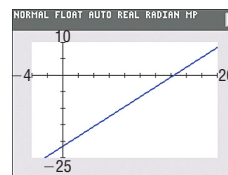
91. x -intercept: 6.5
 y -intercept: -13



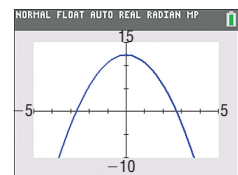
93. x -intercepts: -2.74, 2.74
 y -intercept: -15



95. x -intercept: 14.33
 y -intercept: -21.5



97. x -intercepts: -2.72, 2.72
 y -intercept: 12.33



99. $(5, 3)$ 101. $\sqrt{17}; 2\sqrt{5}; \sqrt{29}$ 103. $d(P_1, P_2) = 6; d(P_2, P_3) = 4; d(P_1, P_3) = 2\sqrt{13}$; right triangle

105. $d(P_1, P_2) = 2\sqrt{17}; d(P_2, P_3) = \sqrt{34}; d(P_1, P_3) = \sqrt{34}$; isosceles right triangle 107. $(5, -2)$ 109. $90\sqrt{2} \approx 127.28$ ft

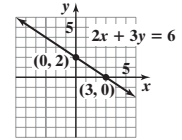
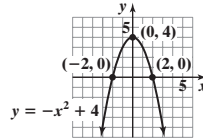
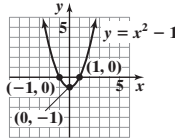
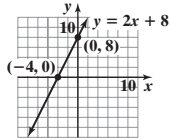
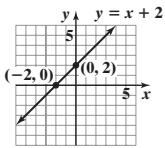
111. (a) $(90, 0), (90, 90), (0, 90)$ (b) $5\sqrt{2161} \approx 232.43$ ft (c) $30\sqrt{149} \approx 366.20$ ft 113. $d = 50t$ mi 115. (a) $(2.65, 1.6)$ (b) ≈ 1.285 units

117. \$21,800

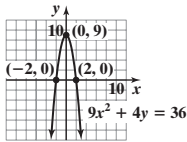
1.2 Assess Your Understanding (page 23)

3. intercepts 4. y-axis 5. 4 6. (-3, 4) 7. T 8. F 9. a 10. c

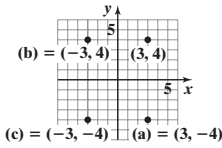
11. (-2, 0), (0, 2) 13. (-4, 0), (0, 8) 15. (-1, 0), (1, 0), (0, -1) 17. (-2, 0), (2, 0), (0, 4) 19. (3, 0), (0, 2)



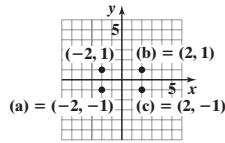
21. (-2, 0), (2, 0), (0, 9)



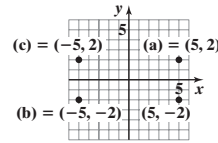
23.



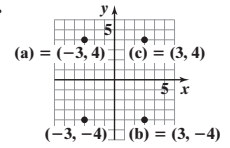
25.



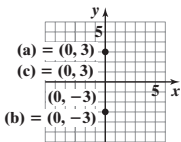
27.



29.



31.



33. (a) (-1, 0), (1, 0)

(b) Symmetric with respect to the x-axis, the y-axis, and the origin

35. (a) $(-\frac{\pi}{2}, 0)$, $(0, 1)$, $(\frac{\pi}{2}, 0)$

(b) Symmetric with respect to the y-axis

37. (a) (0, 0)

(b) Symmetric with respect to the x-axis

39. (a) (-2, 0), (0, 0), (2, 0)

(b) Symmetric with respect to the origin

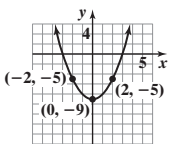
41. (a) $(x, 0)$, $-2 \leq x \leq 1$

(b) No Symmetry

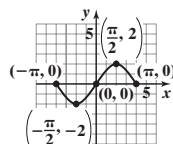
43. (a) No intercepts

(b) Symmetric with respect to the origin

45.



47.



49. (-4, 0), (0, -2), (0, 2); symmetric with respect to the x-axis 51. (0, 0); symmetric with respect to the origin

53. (0, -9), (3, 0), (-3, 0); symmetric with respect to the y-axis 55. (-2, 0), (2, 0), (0, -3), (0, 3); symmetric with respect to the x-axis, y-axis, and origin

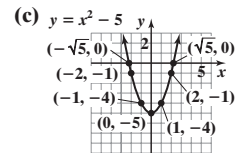
57. (0, -27), (3, 0); no symmetry 59. (0, -4), (4, 0), (-1, 0); no symmetry 61. (0, 0); symmetric with respect to the origin

63. (0, 0); symmetric with respect to the origin

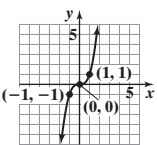
69. $b = 13$ 71. $a = -4$ or $a = 1$

73. (a) $(0, -5)$, $(-\sqrt{5}, 0)$, $(\sqrt{5}, 0)$

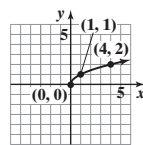
(b) Symmetric with respect to the y-axis



65.



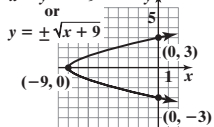
67.



75. (a) (-9, 0), (0, -3), (0, 3)

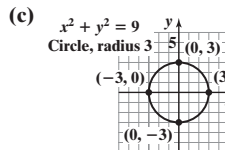
(b) Symmetric with respect to the x-axis

(c) $x - y^2 = -9$



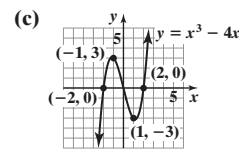
77. (a) (0, 3), (0, -3), (-3, 0), (3, 0)

(b) Symmetric with respect to the x-axis, y-axis, and origin



79. (a) (0, 0), (-2, 0), (2, 0)

(b) Symmetric with respect to the origin



81. (-1, -2) 83. 4 85. (a) (0, 0), (2, 0), (0, 1), (0, -1) (b) x-axis symmetry

1.3 Assess Your Understanding (page 29)

3. ZERO (or ROOT) 4. F 5. {-2.21, 0.54, 1.68} 7. {-1.55, 1.15} 9. {-1.12, 0.36} 11. {-2.69, -0.49, 1.51} 13. {-2.86, -1.34, 0.20, 1.00}

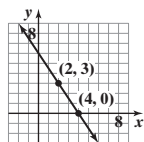
15. No real solutions 17. {-18} 19. {-4} 21. $\{\frac{46}{5}\}$ 23. {3} 25. {2} 27. {-4, 7} 29. $\{-\frac{2}{3}, 2\}$

31. {-2, -1, 2} 33. {15} 35. $\{-4, -\frac{1}{8}\}$

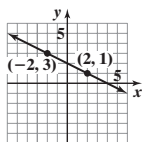
1.4 Assess Your Understanding (page 40)

1. undefined; 0 2. 3; 2 3. T 4. F 5. T 6. $m_1 = m_2$; y-intercepts; $m_1 m_2 = -1$ 7. 2 8. $-\frac{1}{2}$ 9. F 10. d 11. c 12. b
 13. (a) Slope = $\frac{1}{2}$ (b) If x increases by 2 units, y will increase by 1 unit. 15. (a) Slope = $-\frac{1}{3}$ (b) If x increases by 3 units, y will decrease by 1 unit.

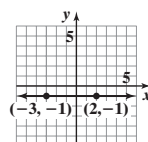
17. Slope = $-\frac{3}{2}$



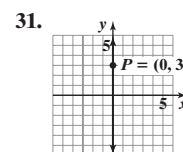
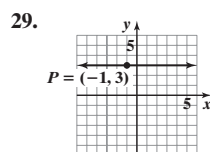
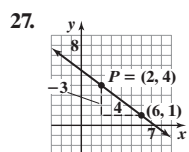
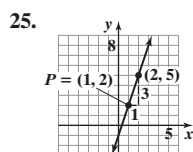
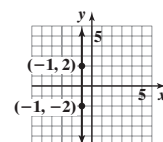
19. Slope = $-\frac{1}{2}$



21. Slope = 0

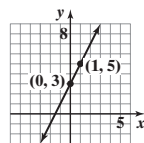


23. Slope undefined

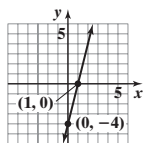


33. (2, 6); (3, 10); (4, 14) 35. (4, -7); (6, -10); (8, -13) 37. (-1, -5); (0, -7); (1, -9) 39. $x - 2y = 0$ or $y = \frac{1}{2}x$
 41. $x + y = 2$ or $y = -x + 2$ 43. $2x - y = 3$ or $y = 2x - 3$ 45. $x + 2y = 5$ or $y = -\frac{1}{2}x + \frac{5}{2}$ 47. $3x - y = -9$ or $y = 3x + 9$
 49. $2x + 3y = -1$ or $y = -\frac{2}{3}x - \frac{1}{3}$ 51. $3x + y = 3$ or $y = -3x + 3$ 53. $x - 2y = -5$ or $y = \frac{1}{2}x + \frac{5}{2}$ 55. $x - 2y = 2$ or $y = \frac{1}{2}x - 1$
 57. $x = 2$; no slope-intercept form 59. $y = 2$ 61. $4x - y = -6$ or $y = 4x + 6$ 63. $5x - y = 0$ or $y = 5x$ 65. $x = 4$; no slope-intercept form
 67. $6x + y = 0$ or $y = -6x$ 69. $5x - 2y = -3$ or $y = \frac{5}{2}x + \frac{3}{2}$ 71. $y = 4$

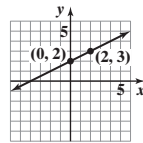
73. Slope = 2; y-intercept = 3



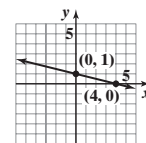
75. Slope = 4; y-intercept = -4



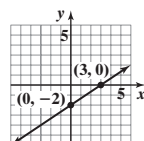
77. Slope = $\frac{1}{2}$; y-intercept = 2



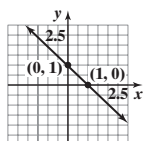
79. Slope = $-\frac{1}{4}$; y-intercept = 1



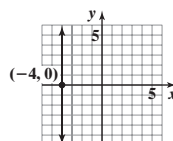
81. Slope = $\frac{2}{3}$; y-intercept = -2



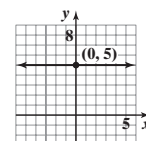
83. Slope = -1; y-intercept = 1



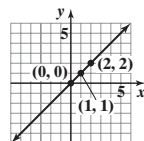
85. Slope undefined; no y-intercept



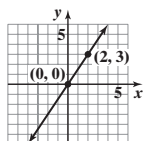
87. Slope = 0; y-intercept = 5



89. Slope = 1; y-intercept = 0



91. Slope = $\frac{3}{2}$; y-intercept = 0



93. (a) x-intercept: 3; y-intercept: 2
(b)

95. (a) x-intercept: -10; y-intercept: 8
(b)

97. (a) x-intercept: 3; y-intercept: $\frac{21}{2}$
(b)

99. (a) x-intercept: 2; y-intercept: 3
(b)

101. (a) x-intercept: 5; y-intercept: -2
(b)

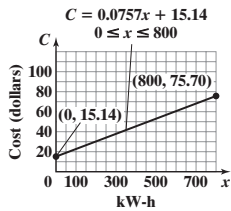
103. $y = 0$
 105. Parallel
 107. Neither
 109. $x - y = -2$ or $y = x + 2$
 111. $x + 3y = 3$ or $y = -\frac{1}{3}x + 1$

113. $P_1 = (-2, 5)$, $P_2 = (1, 3)$, $m_1 = -\frac{2}{3}$; $P_2 = (1, 3)$, $P_3 = (-1, 0)$, $m_2 = \frac{3}{2}$, because $m_1 m_2 = -1$, the lines are perpendicular and the points $(-2, 5)$, $(1, 3)$, and $(-1, 0)$ are the vertices of a right triangle.

AN-4 ANSWERS Section 1.4

115. $P_1 = (-1, 0)$, $P_2 = (2, 3)$, $m = 1$; $P_3 = (1, -2)$, $P_4 = (4, 1)$, $m = 1$; $P_1 = (-1, 0)$, $P_3 = (1, -2)$, $m = -1$; $P_2 = (2, 3)$, $P_4 = (4, 1)$, $m = -1$; opposite sides are parallel, and adjacent sides are perpendicular; the points are the vertices of a rectangle.
 117. $C = 0.60x + 39$; \$105; \$177 119. $C = 0.16x + 1461$

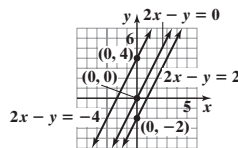
121. (a) $C = 0.0757x + 15.14$, $0 \leq x \leq 800$



- (b) $C = 0.0757x + 15.14$, $0 \leq x \leq 800$
 (c) \$30.28 (d) \$52.99
 (e) Each additional kW-h used adds \$0.0757 to the bill.

123. $^{\circ}\text{C} = \frac{5}{9} (^{\circ}\text{F} - 32)$; approximately 21.1°C 125. (a) $y = -\frac{2}{25}x + 30$ (b) x-intercept: 375; The ramp meets the floor 375 in. (31.25 ft) from the base of the platform. (c) The ramp does not meet design requirements. It has a run of 31.25 ft. (d) The only slope possible for the ramp to comply with the requirement is for it to drop 1 in. for every 12-in. run.

127. (a) $A = \frac{1}{5}x + 20,000$ (b) \$80,000 (c) Each additional box sold requires an additional \$0.20 in advertising. 129. All have the same slope, 2; the lines are parallel.



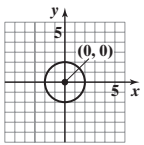
131. (b), (c), (e), (g) 133. (c) 139. No; no 141. They are the same line. 143. Yes, if the y-intercept is 0.

1.5 Assess Your Understanding (page 49)

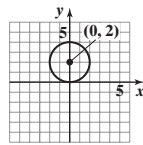
3. F 4. radius 5. T 6. F 7. d 8. a 9. Center $(2, 1)$; radius = 2; $(x - 2)^2 + (y - 1)^2 = 4$

11. Center $(\frac{5}{2}, 2)$; radius = $\frac{3}{2}$; $(x - \frac{5}{2})^2 + (y - 2)^2 = \frac{9}{4}$

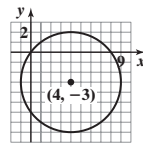
13. $x^2 + y^2 = 4$;
 $x^2 + y^2 - 4 = 0$



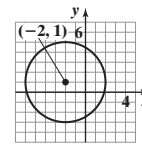
15. $x^2 + (y - 2)^2 = 4$;
 $x^2 + y^2 - 4y = 0$



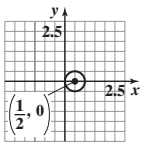
17. $(x - 4)^2 + (y + 3)^2 = 25$;
 $x^2 + y^2 - 8x + 6y = 0$



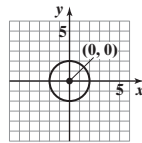
19. $(x + 2)^2 + (y - 1)^2 = 16$;
 $x^2 + y^2 + 4x - 2y - 11 = 0$



21. $(x - \frac{1}{2})^2 + y^2 = \frac{1}{4}$;
 $x^2 + y^2 - x = 0$

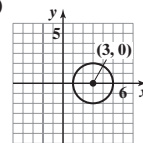


23. (a) $(h, k) = (0, 0)$; $r = 2$
 (b)



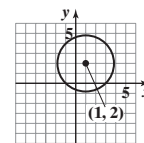
- (c) $(\pm 2, 0)$; $(0, \pm 2)$

25. (a) $(h, k) = (3, 0)$; $r = 2$
 (b)



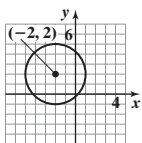
- (c) $(1, 0)$; $(5, 0)$

27. (a) $(h, k) = (1, 2)$; $r = 3$
 (b)



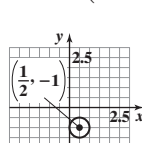
- (c) $(1 \pm \sqrt{5}, 0)$; $(0, 2 \pm 2\sqrt{2})$

29. (a) $(h, k) = (-2, 2)$; $r = 3$
 (b)



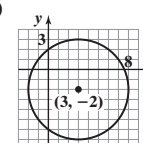
- (c) $(-2 \pm \sqrt{5}, 0)$;
 $(0, 2 \pm \sqrt{5})$

31. (a) $(h, k) = (\frac{1}{2}, -1)$; $r = \frac{1}{2}$
 (b)



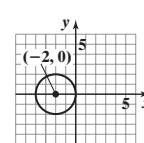
- (c) $(0, -1)$

33. (a) $(h, k) = (3, -2)$; $r = 5$
 (b)



- (c) $(3 \pm \sqrt{21}, 0)$;
 $(0, -6)$, $(0, 2)$

35. (a) $(h, k) = (-2, 0)$; $r = 2$
 (b)

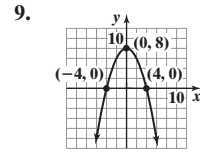
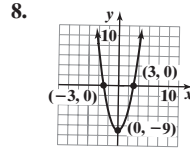
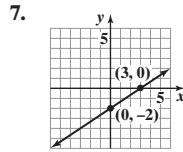
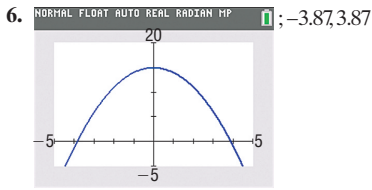


- (c) $(0, 0)$, $(-4, 0)$

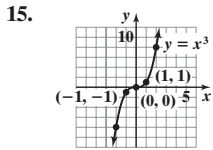
37. $x^2 + y^2 = 13$ 39. $(x - 2)^2 + (y - 3)^2 = 9$ 41. $(x + 1)^2 + (y - 3)^2 = 5$ 43. $(x + 1)^2 + (y - 3)^2 = 1$ 45. (c) 47. (b) 49. 18 units²
 51. $x^2 + (y - 139)^2 = 15,625$ 53. $x^2 + y^2 + 2x + 4y - 4168.16 = 0$ 55. $\sqrt{2}x + 4y - 9\sqrt{2} = 0$ 57. $(1, 0)$ 59. $y = 2$ 61. (b), (c), (e), (g)

Review Exercises (page 53)

1. (a) $2\sqrt{5}$ (b) $(2, 1)$ (c) $\frac{1}{2}$ (d) For each run of 2, there is a rise of 1. 2. (a) 5 (b) $(-\frac{1}{2}, 1)$ (c) $-\frac{4}{3}$
 (d) For each run of 3, there is a rise of -4 . 3. (a) 12 (b) $(4, 2)$ (c) Undefined (d) No change in x 4. (a) 5 (b) $(\frac{1}{2}, -1)$ (c) 0
 (d) No change in y 5. $(-4, 0)$, $(0, 2)$, $(0, 0)$, $(0, -2)$, $(2, 0)$



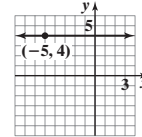
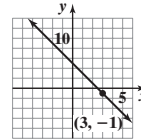
10. x-axis 11. x-axis, y-axis, origin 12. y-axis 13. Origin 14. No symmetry



16. $\{-2.49, 0.66, 1.83\}$
17. $\{-1.14, 1.64\}$

18. $2x + y = 5$ or $y = -2x + 5$

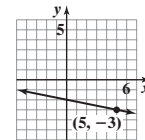
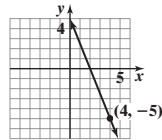
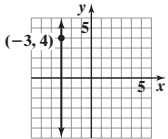
19. $y = 4$



20. $x = -3$; no slope-intercept form

21. $5x + 2y = 10$ or $y = -\frac{5}{2}x + 5$

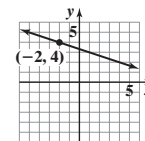
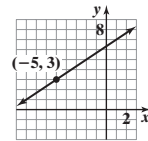
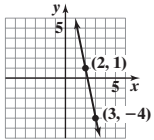
22. $x + 5y = -10$ or $y = -\frac{1}{5}x - 2$



23. $5x + y = 11$ or $y = -5x + 11$

24. $2x - 3y = -19$ or $y = \frac{2}{3}x + \frac{19}{3}$

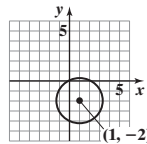
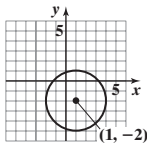
25. $x + 3y = 10$ or $y = -\frac{1}{3}x + \frac{10}{3}$



26. $(x + 2)^2 + (y - 3)^2 = 16$ 27. $(x + 1)^2 + (y + 2)^2 = 1$

28. Center $(1, -2)$; radius = 3

29. Center $(1, -2)$; radius = $\sqrt{5}$



$(0, -2 \pm 2\sqrt{2}), (1 \pm \sqrt{5}, 0)$

$(0, 0), (0, -4), (2, 0)$

30. (a) $d(A, B) = 2\sqrt{5}$; $d(B, C) = \sqrt{145}$; $d(A, C) = 5\sqrt{5}$; Since $d(A, B)^2 + d(A, C)^2 = d(B, C)^2$, by the converse of the Pythagorean Theorem, the points $A, B,$ and C are vertices of a right triangle.

- (b) Slope of $AB = -2$; Slope of $BC = \frac{1}{12}$; Slope of $AC = \frac{1}{2}$; Since $(-2)\left(\frac{1}{2}\right) = -1$, the lines AB and AC are perpendicular and hence form a right angle.

31. $m_{AB} = -1$; $m_{BC} = -1$

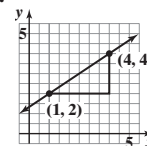
32. Let $D = (-1, 2)$. Then $d(A, D) = d(B, D) = d(C, D) = \sqrt{13}$.

$A, B,$ and C lie on the circle $(x + 1)^2 + (y - 2)^2 = 13$ which has radius $\sqrt{13}$.

33. Center $(1, -2)$; radius = $4\sqrt{2}$; $x^2 + y^2 - 2x + 4y - 27 = 0$

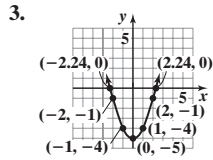
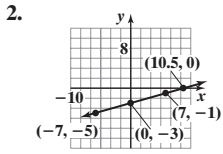
34. -4 and 8

- 35.



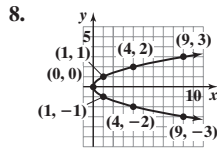
Chapter Test (page 54)

1. (a) 10 (b) (1, 1)



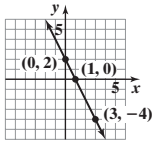
- 4. $\{-1, 0.5, 1\}$
- 5. $\{-2.50, 2.50\}$
- 6. $\{-2.46, -0.24, 1.70\}$

7. (a) $m = -\frac{2}{3}$
 (b) For every 3-unit change in x , y will change by -2 units.



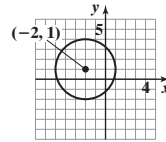
9. Intercepts: $(-3, 0)$, $(3, 0)$, $(0, 9)$; symmetric with respect to the y -axis

10. $y = -2x + 2$



11. $x^2 + y^2 - 8x + 6y = 0$

12. Center: $(-2, 1)$; radius: 3



13. Parallel line: $y = -\frac{2}{3}x - \frac{1}{3}$;
 perpendicular line: $y = \frac{3}{2}x + 3$

Chapter 2 Functions and Their Graphs

2.1 Assess Your Understanding (page 68)

7. independent; dependent 8. $[0, 5]$ 9. $\neq; f; g$ 10. $(g - f)(x)$ 11. F 12. T 13. F 14. F 15. a 16. c 17. d 18. a
 19. Function; Domain: {Elvis, Colleen, Kaleigh, Marissa}; Range: {January 8, March 15, September 17} 21. Not a function; Domain: {20, 30, 40}; Range: {200, 300, 350, 425} 23. Not a function; Domain: $\{-3, 2, 4\}$; Range: {6, 9, 10} 25. Function; Domain: {1, 2, 3, 4}; Range: {3}
 27. Not a function; Domain: $\{-2, 0, 3\}$; Range: {3, 4, 6, 7} 29. Function; Domain: $\{-2, -1, 0, 1\}$; Range: {0, 1, 4} 31. Function 33. Function
 35. Not a function 37. Not a function 39. Function 41. Not a function 43. (a) -4 (b) 1 (c) -3 (d) $3x^2 - 2x - 4$ (e) $-3x^2 - 2x + 4$
 (f) $3x^2 + 8x + 1$ (g) $12x^2 + 4x - 4$ (h) $3x^2 + 6xh + 3h^2 + 2x + 2h - 4$ 45. (a) 0 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) $\frac{-x}{x^2 + 1}$ (e) $\frac{-x}{x^2 + 1}$ (f) $\frac{x + 1}{x^2 + 2x + 2}$
 (g) $\frac{2x}{4x^2 + 1}$ (h) $\frac{x + h}{x^2 + 2xh + h^2 + 1}$ 47. (a) 4 (b) 5 (c) 5 (d) $|x| + 4$ (e) $-|x| - 4$ (f) $|x + 1| + 4$ (g) $2|x| + 4$ (h) $|x + h| + 4$
 49. (a) $-\frac{1}{5}$ (b) $-\frac{3}{2}$ (c) $\frac{1}{8}$ (d) $\frac{2x - 1}{3x + 5}$ (e) $\frac{-2x - 1}{3x - 5}$ (f) $\frac{2x + 3}{3x - 2}$ (g) $\frac{4x + 1}{6x - 5}$ (h) $\frac{2x + 2h + 1}{3x + 3h - 5}$ 51. All real numbers 53. All real numbers
 55. $\{x|x \neq -4, x \neq 4\}$ 57. $\{x|x \neq 0\}$ 59. $\{x|x \geq 4\}$ 61. $\{x|x > 1\}$ 63. $\{x|x > 4\}$ 65. $\{t|t \geq 4, t \neq 7\}$
 67. (a) $(f + g)(x) = 5x + 1$; All real numbers (b) $(f - g)(x) = x + 7$; All real numbers (c) $(f \cdot g)(x) = 6x^2 - x - 12$; All real numbers
 (d) $\left(\frac{f}{g}\right)(x) = \frac{3x + 4}{2x - 3}$; $\left\{x \mid x \neq \frac{3}{2}\right\}$ (e) 16 (f) 11 (g) 10 (h) -7 69. (a) $(f + g)(x) = 2x^2 + x - 1$; All real numbers
 (b) $(f - g)(x) = -2x^2 + x - 1$; All real numbers (c) $(f \cdot g)(x) = 2x^3 - 2x^2$; All real numbers (d) $\left(\frac{f}{g}\right)(x) = \frac{x - 1}{2x^2}$; $\{x|x \neq 0\}$ (e) 20
 (f) -29 (g) 8 (h) 0
 71. (a) $(f + g)(x) = \sqrt{x} + 3x - 5$; $\{x|x \geq 0\}$ (b) $(f - g)(x) = \sqrt{x} - 3x + 5$; $\{x|x \geq 0\}$ (c) $(f \cdot g)(x) = 3x\sqrt{x} - 5\sqrt{x}$; $\{x|x \geq 0\}$
 (d) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{3x - 5}$; $\left\{x \mid x \geq 0, x \neq \frac{5}{3}\right\}$ (e) $\sqrt{3} + 4$ (f) -5 (g) $\sqrt{2}$ (h) $-\frac{1}{2}$ 73. (a) $(f + g)(x) = 1 + \frac{2}{x}$; $\{x|x \neq 0\}$
 (b) $(f - g)(x) = 1$; $\{x|x \neq 0\}$ (c) $(f \cdot g)(x) = \frac{1}{x} + \frac{1}{x^2}$; $\{x|x \neq 0\}$ (d) $\left(\frac{f}{g}\right)(x) = x + 1$; $\{x|x \neq 0\}$ (e) $\frac{5}{3}$ (f) 1 (g) $\frac{3}{4}$ (h) 2
 75. (a) $(f + g)(x) = \frac{6x + 3}{3x - 2}$; $\left\{x \mid x \neq \frac{2}{3}\right\}$ (b) $(f - g)(x) = \frac{-2x + 3}{3x - 2}$; $\left\{x \mid x \neq \frac{2}{3}\right\}$ (c) $(f \cdot g)(x) = \frac{8x^2 + 12x}{(3x - 2)^2}$; $\left\{x \mid x \neq \frac{2}{3}\right\}$
 (d) $\left(\frac{f}{g}\right)(x) = \frac{2x + 3}{4x}$; $\left\{x \mid x \neq 0, x \neq \frac{2}{3}\right\}$ (e) 3 (f) $-\frac{1}{2}$ (g) $\frac{7}{2}$ (h) $\frac{5}{4}$ 77. $g(x) = 5 - \frac{7}{2}x$ 79. 4 81. $2x + h$ 83. $2x + h - 1$
 85. $\frac{-(2x + h)}{x^2(x + h)^2}$ 87. $\frac{6}{(x + 3)(x + h + 3)}$ 89. $\frac{1}{\sqrt{x + h - 2} + \sqrt{x - 2}}$ 91. $\{-2, 4\}$ 93. $A = -\frac{7}{2}$ 95. $A = -4$ 97. $A(x) = \frac{1}{2}x^2$ 99. $G(x) = 14x$

101. (a) P is the dependent variable; a is the independent variable. (b) $P(20) = 231.427$ million; In 2012, there were 231.427 million people 20 years of age or older. (c) $P(0) = 327.287$ million; In 2012, there were 327.287 million people.

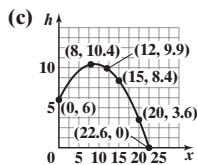
103. (a) 15.1 m, 14.071 m, 12.944 m, 11.719 m (b) 1.01 sec, 1.43 sec, 1.75 sec (c) 2.02 sec 105. (a) \$222 (b) \$225 (c) \$220 (d) \$230

107. $R(x) = \frac{L(x)}{P(x)}$ 109. $H(x) = P(x) \cdot I(x)$ 111. (a) $P(x) = -0.05x^3 + 0.8x^2 + 155x - 500$ (b) $P(15) = \$1836.25$
 (c) When 15 hundred cellphones are sold, the profit is \$1836.25. 113. (a) $D(v) = 0.05v^2 + 2.6v - 15$ (b) 321 feet (c) The car will need 321 feet to stop once the impediment is observed. 115. No; domain of f is all real numbers; domain of g is $\{x|x \neq -1\}$
 117. $H(x) = \frac{3x - x^3}{\text{age}}$ 118. Intercepts: $(-16, 0), (-8, 0)$; x -axis symmetry 119. $(4, 32)$ 120. $(-\frac{3}{2}, -2)$ 121. $(x - 4)^2 + (y + 1)^2 = 9$

2.2 Assess Your Understanding (page 76)

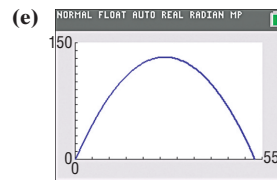
3. vertical 4. 5; -3 5. $a = -2$ 6. F 7. F 8. T 9. c 10. a 11. (a) $f(0) = 3; f(-6) = -3$ (b) $f(6) = 0; f(11) = 1$ (c) Positive
 (d) Negative (e) -3, 6, and 10 (f) $-3 < x < 6; 10 < x \leq 11$ (g) $\{x|-6 \leq x \leq 11\}$ (h) $\{y|-3 \leq y \leq 4\}$ (i) -3, 6, 10 (j) 3 (k) 3 times
 (l) Once (m) 0, 4 (n) -5, 8 13. Not a function 15. Function (a) Domain: $\{x|-\pi \leq x \leq \pi\}$; Range: $\{y|-1 \leq y \leq 1\}$
 (b) $(-\frac{\pi}{2}, 0), (\frac{\pi}{2}, 0), (0, 1)$ (c) y -axis 17. Not a function 19. Function (a) Domain: $\{x|0 < x < 3\}$; Range: $\{y|y < 2\}$ (b) $(1, 0)$ (c) None
 21. Function (a) Domain: all real numbers; Range: $\{y|y \leq 2\}$ (b) $(-3, 0), (3, 0), (0, 2)$ (c) y -axis 23. Function
 (a) Domain: all real numbers; Range: $\{y|y \geq -3\}$ (b) $(1, 0), (3, 0), (0, 9)$ (c) None 25. (a) Yes (b) $f(-2) = 9; (-2, 9)$
 (c) $0, \frac{1}{2}; (0, -1), (\frac{1}{2}, -1)$ (d) All real numbers (e) $-\frac{1}{2}, 1$ (f) -1 27. (a) No (b) $f(4) = -3; (4, -3)$ (c) 14; $(14, 2)$ (d) $\{x|x \neq 6\}$
 (e) -2 (f) $-\frac{1}{3}$ 29. (a) Yes (b) $f(2) = \frac{8}{17}; (2, \frac{8}{17})$ (c) -1, 1; $(-1, 1), (1, 1)$ (d) All real numbers (e) 0 (f) 0
 31. (a) 3 (b) -2 (c) -1 (d) 1 (e) 2 (f) $-\frac{1}{3}$

33. (a) Approximately 10.4 ft high
 (b) Approximately 9.9 ft high



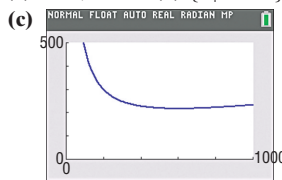
- (d) The ball will not go through the hoop; $h(15) \approx 8.4$ ft. If $v = 30$ ft/sec, $h(15) = 10$ ft.

35. (a) About 81.07 ft (b) About 129.59 ft (c) About 26.63 ft
 (d) About 528.13 ft



- (f) About 115.07 ft and 413.05 ft
 (g) 275 ft; maximum height shown in the table is 131.8 ft (h) 264 ft

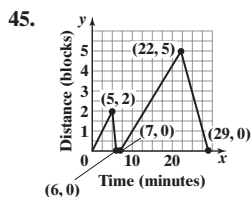
37. (a) \$223; \$220 (b) $\{x|x > 0\}$



X	Y	ERROR
50	825	
100	470	
150	355	
200	300	
250	269	
300	250	
350	237.86	
400	229	
450	225	
500	222	

39. (a) \$30; It costs \$30 if you use 0 gigabytes. (b) \$30; It costs \$30 if you use 5 gigabytes. (c) \$90; It costs \$90 if you use 15 gigabytes.
 (d) $\{g|0 \leq g \leq 60\}$. There are at most 60 gigabytes in a month.
 41. The x -intercepts can number anywhere from 0 to infinitely many. There is at most one y -intercept.

43. (a) III (b) IV (c) I (d) V (e) II



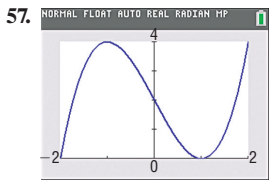
- (e) 600 mi/h

47. (a) 2 hr elapsed during which Kevin was between 0 and 3 mi from home (b) 0.5 hr elapsed during which Kevin was 3 mi from home (c) 0.3 hr elapsed during which Kevin was between 0 and 3 mi from home
 (d) 0.2 hr elapsed during which Kevin was 0 mi from home (e) 0.9 hr elapsed during which Kevin was between 0 and 2.8 mi from home (f) 0.3 hr elapsed during which Kevin was 2.8 mi from home (g) 1.1 hr elapsed during which Kevin was between 0 and 2.8 mi from home (h) 3 mi (i) Twice
 49. No points whose x -coordinate is 5 or whose y -coordinate is 0 can be on the graph.

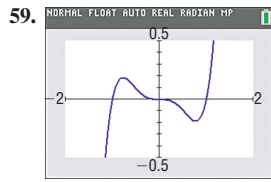
52. $-x^2 + 5x - 9$ 53. $2\sqrt{10}$ 54. $y = \frac{2}{3}x + 8$ 55. $(-\infty, \infty)$

2.3 Assess Your Understanding (page 90)

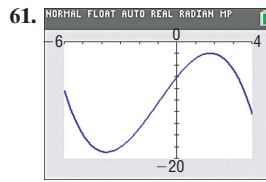
6. increasing 7. even; odd 8. T 9. T 10. F 11. c 12. d 13. Yes 15. No 17. $[-8, -2]; [0, 2]; [5, 7]$ 19. Yes; 10 21. -2, 2; 6, 10
 23. $f(-8) = -4$ 25. (a) $(-2, 0), (0, 3), (2, 0)$ (b) Domain: $\{x|-4 \leq x \leq 4\}$ or $[-4, 4]$; Range: $\{y|0 \leq y \leq 3\}$ or $[0, 3]$
 (c) Increasing on $[-2, 0]$ and $[2, 4]$; Decreasing on $[-4, -2]$ and $[0, 2]$ (d) Even 27. (a) $(0, 1)$
 (b) Domain: all real numbers; Range: $\{y|y > 0\}$ or $(0, \infty)$ (c) Increasing on $(-\infty, \infty)$ (d) Neither
 29. (a) $(-\pi, 0), (0, 0), (\pi, 0)$ (b) Domain: $\{x|-\pi \leq x \leq \pi\}$ or $[-\pi, \pi]$; Range: $\{y|-1 \leq y \leq 1\}$ or $[-1, 1]$
 (c) Increasing on $[-\frac{\pi}{2}, \frac{\pi}{2}]$; Decreasing on $[-\pi, -\frac{\pi}{2}]$ and $[\frac{\pi}{2}, \pi]$ (d) Odd 31. (a) $(0, \frac{1}{2}), (\frac{1}{3}, 0), (\frac{5}{2}, 0)$
 (b) Domain: $\{x|-3 \leq x \leq 3\}$ or $[-3, 3]$; Range: $\{y|-1 \leq y \leq 2\}$ or $[-1, 2]$ (c) Increasing on $[2, 3]$; Decreasing on $[-1, 1]$; Constant on $[-3, -1]$ and $[1, 2]$ (d) Neither 33. (a) 0; 3 (b) -2, 2; 0, 0
 35. (a) $\frac{\pi}{2}; 1$ (b) $-\frac{\pi}{2}; -1$ 37. Odd 39. Even 41. Odd 43. Neither 45. Even 47. Odd
 49. Absolute maximum: $f(1) = 4$; absolute minimum: $f(5) = 1$; local maximum: $f(3) = 3$; local minimum: $f(2) = 2$
 51. Absolute maximum: $f(3) = 4$; absolute minimum: $f(1) = 1$; local maximum: $f(3) = 4$; local minimum: $f(1) = 1$
 53. Absolute maximum: none; absolute minimum: $f(0) = 0$; local maximum: $f(2) = 3$; local minimum: $f(0) = 0$ and $f(3) = 2$
 55. Absolute maximum: none; absolute minimum: none; local maximum: none; local minimum: none



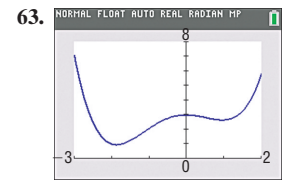
Increasing: $[-2, -1]$, $[1, 2]$
 Decreasing: $[-1, 1]$
 Local maximum: $f(-1) = 4$
 Local minimum: $f(1) = 0$



Increasing:
 $[-2, -0.77]$, $[0.77, 2]$
 Decreasing: $[-0.77, 0.77]$
 Local maximum: $f(-0.77) = 0.19$
 Local minimum: $f(0.77) = -0.19$

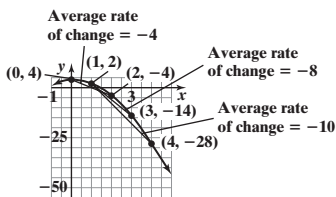


Increasing: $[-3.77, 1.77]$
 Decreasing:
 $[-6, -3.77]$, $[1.77, 4]$
 Local maximum:
 $f(1.77) = -1.91$
 Local minimum:
 $f(-3.77) = -18.89$

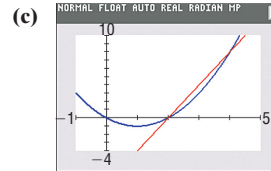
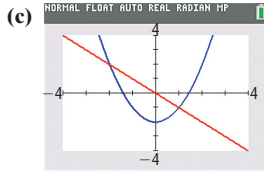


Increasing:
 $[-1.87, 0]$, $[0.97, 2]$
 Decreasing:
 $[-3, -1.87]$, $[0, 0.97]$
 Local maximum: $f(0) = 3$
 Local minima: $f(-1.87) = 0.95$,
 $f(0.97) = 2.65$

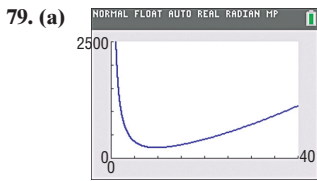
65. (a) -4 (b) -8 (c) -10
 (d)



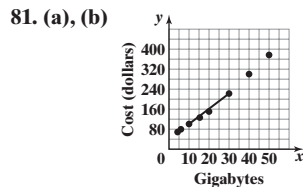
67. (a) 17 (b) -1 (c) 11 69. (a) 5 (b) $y = 5x - 2$
 71. (a) -1 (b) $y = -x$ 73. (a) 4 (b) $y = 4x - 8$



75. (a) Odd (b) Local maximum value: 54 at $x = -3$ 77. (a) Even (b) Local maximum value: 25 at $x = -2$ (c) 50.4 sq. units



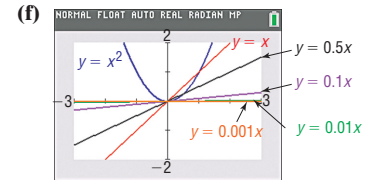
(b) 10 riding lawn mowers
 (c) \$239/mower



(c) \$5/gigabyte
 (d) \$6.25/gigabyte
 (e) \$750/gigabyte
 (f) The average rate of change is increasing as the number of gigabytes increases.

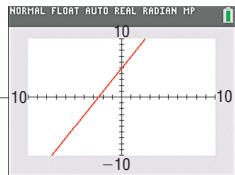
83. (a) On average, the population is increasing at a rate of 0.036 g/h from 0 to 2.5 h. (b) On average, from 4.5 to 6 h, the population is increasing at a rate of 0.1 g/h. (c) The average rate of change is increasing over time.

85. (a) 1 (b) 0.5 (c) 0.1 (d) 0.01
 (e) 0.001

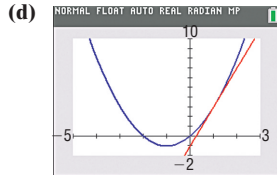


(g) They are getting closer to the tangent line at $(0, 0)$.
 (h) They are getting closer to 0.

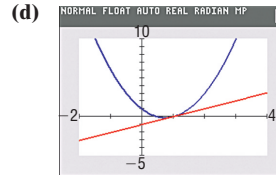
87. (a) 2
 (b) 2; 2; 2
 (c) $y = 2x + 5$
 (d)



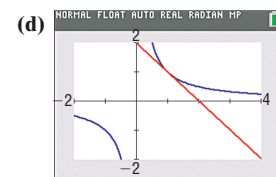
89. (a) $2x + h + 2$
 (b) 4.5; 4.1; 4.01; 4
 (c) $y = 4.01x - 1.01$



91. (a) $4x + 2h - 3$
 (b) 2; 1.2; 1.02; 1
 (c) $y = 1.02x - 1.02$



93. (a) $-\frac{1}{(x+h)x}$
 (b) $-\frac{2}{3}, -\frac{10}{11}, -\frac{100}{101}, -1$
 (c) $y = -\frac{100}{101}x + \frac{201}{101}$

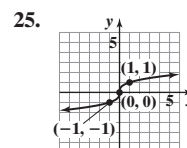
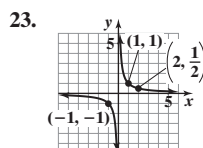
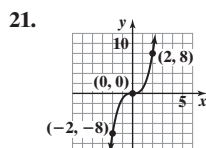
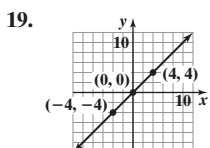


97. At most one 99. Yes; the function $f(x) = 0$ is both even and odd. 101. Not necessarily. It just means $f(5) > f(2)$.

103. $f(-3) = -9$; $(-3, -9)$ 104. $C(x) = 0.80x + 40$ 105. $y = -\frac{5}{3}x + 4$ 106. $6x - 5 + 3h$

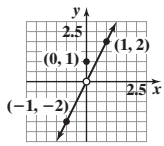
2.4 Assess Your Understanding (page 102)

4. $(-\infty, 0]$ 5. piecewise-defined 6. T 7. F 8. F 9. b 10. a 11. C 13. E 15. B 17. F



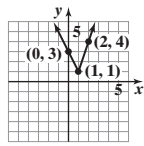
27. (a) 4 (b) 2 (c) 5 29. (a) -4 (b) -2 (c) 0 (d) 25

31. (a) All real numbers
(b) (0, 1)
(c)



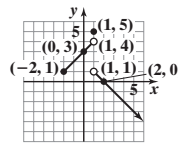
(d) $\{y | y \neq 0\}; (-\infty, 0) \cup (0, \infty)$

33. (a) All real numbers
(b) (0, 3)
(c)



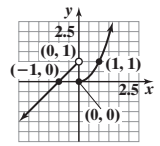
(d) $\{y | y \geq 1\}; [1, \infty)$

35. (a) $\{x | x \geq -2\}; [-2, \infty)$
(b) (0, 3), (2, 0)
(c)



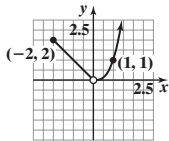
(d) $\{y | y < 4, y = 5\}; (-\infty, 4) \cup \{5\}$

37. (a) All real numbers
(b) (-1, 0), (0, 0)
(c)



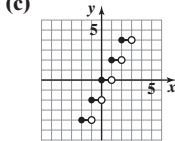
(d) All real numbers

39. (a) $\{x | x \geq -2, x \neq 0\}; [-2, 0) \cup (0, \infty)$
(b) No intercepts
(c)



(d) $\{y | y > 0\}; (0, \infty)$

41. (a) All real numbers
(b) (x, 0) for $0 \leq x < 1$
(c)

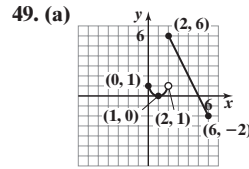


(d) Set of even integers

43. $f(x) = \begin{cases} -x & \text{if } -1 \leq x \leq 0 \\ \frac{1}{2}x & \text{if } 0 < x \leq 2 \end{cases}$ (Other answers are possible.)

45. $f(x) = \begin{cases} -x & \text{if } x \leq 0 \\ -x + 2 & \text{if } 0 < x \leq 2 \end{cases}$ (Other answers are possible.)

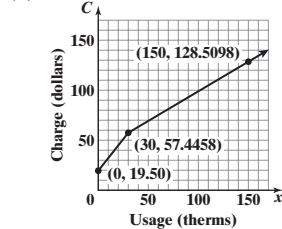
47. (a) 2 (b) 3 (c) -4



(b) $[0, 6]$
(c) Absolute maximum: $f(2) = 6$; absolute minimum: $f(6) = -2$
(d) Local maximum: $f(2) = 6$; local minimum: $f(1) = 0$

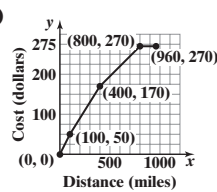
51. (a) \$34.99 (b) \$64.99 (c) \$184.99
53. (a) \$44.80 (b) \$128.51

(c) $C(x) = \begin{cases} 1.26486x + 19.50 & \text{if } 0 \leq x \leq 30 \\ 0.5922x + 39.6798 & \text{if } x > 30 \end{cases}$



(d) $55. f(x) = \begin{cases} 0.10x & \text{if } 0 < x \leq 9,225 \\ 922.50 + 0.15(x - 9,225) & \text{if } 9,225 < x \leq 37,450 \\ 5,156.25 + 0.25(x - 37,450) & \text{if } 37,450 < x \leq 90,750 \\ 18,481.25 + 0.28(x - 90,750) & \text{if } 90,750 < x \leq 189,300 \\ 46,075.25 + 0.33(x - 189,300) & \text{if } 189,300 < x \leq 411,500 \\ 119,401.25 + 0.35(x - 411,500) & \text{if } 411,500 < x \leq 413,200 \\ 119,996.25 + 0.396(x - 413,200) & \text{if } x > 413,200 \end{cases}$

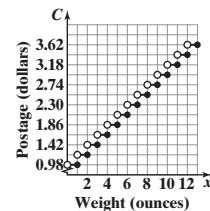
57. (a) (b) $C(x) = 10 + 0.4x$ (c) $C(x) = 70 + 0.25x$



59. (a) $C(s) = \begin{cases} 9000 & \text{if } s \leq 659 \\ 7500 & \text{if } 660 \leq s \leq 679 \\ 5250 & \text{if } 680 \leq s \leq 699 \\ 3000 & \text{if } 700 \leq s \leq 719 \\ 1500 & \text{if } 720 \leq s \leq 739 \\ 750 & \text{if } s \geq 740 \end{cases}$ (b) \$1500 (c) \$7500

61. (a) 10°C (b) 4°C (c) -3°C (d) -4°C
(e) The wind chill is equal to the air temperature.
(f) At wind speed greater than 20 m/sec, the wind chill factor depends only on the air temperature.

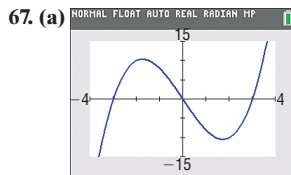
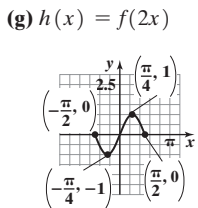
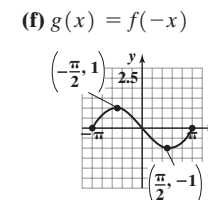
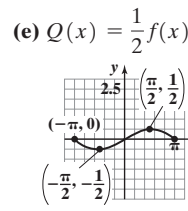
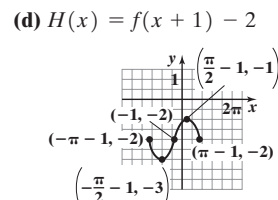
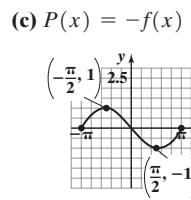
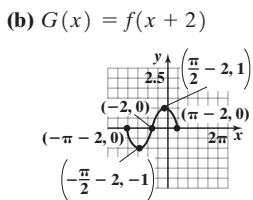
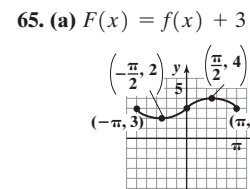
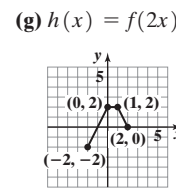
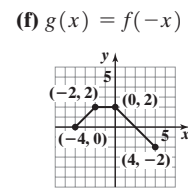
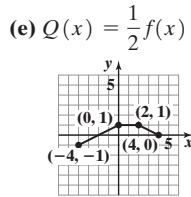
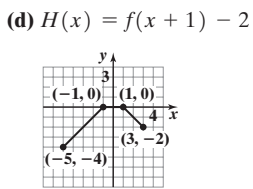
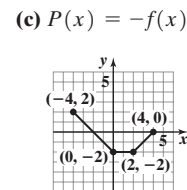
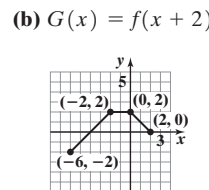
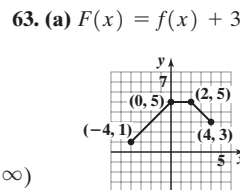
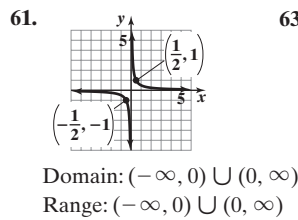
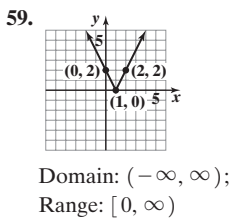
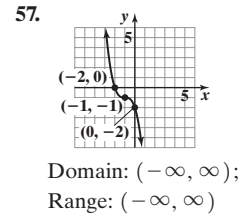
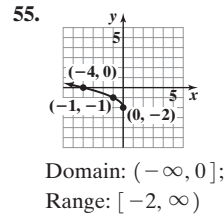
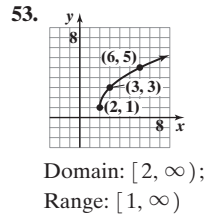
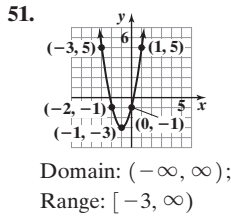
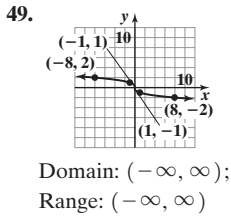
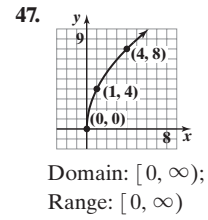
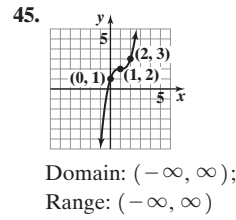
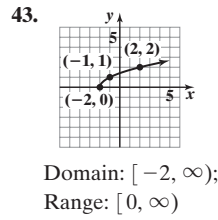
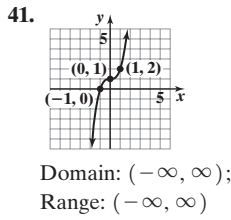
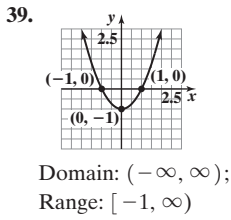
63. $C(x) = \begin{cases} 0.98 & \text{if } 0 < x \leq 1 \\ 1.20 & \text{if } 1 < x \leq 2 \\ 1.42 & \text{if } 2 < x \leq 3 \\ 1.64 & \text{if } 3 < x \leq 4 \\ 1.86 & \text{if } 4 < x \leq 5 \\ 2.08 & \text{if } 5 < x \leq 6 \\ 2.30 & \text{if } 6 < x \leq 7 \\ 2.52 & \text{if } 7 < x \leq 8 \\ 2.74 & \text{if } 8 < x \leq 9 \\ 2.96 & \text{if } 9 < x \leq 10 \\ 3.18 & \text{if } 10 < x \leq 11 \\ 3.40 & \text{if } 11 < x \leq 12 \\ 3.62 & \text{if } 12 < x \leq 13 \end{cases}$



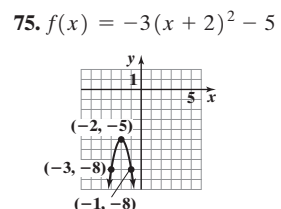
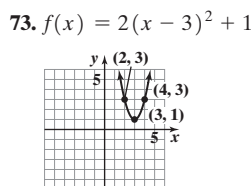
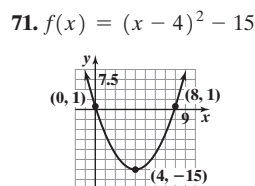
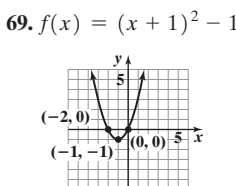
65. Each graph is that of $y = x^2$, but shifted horizontally. If $y = (x - k)^2, k > 0$, the shift is right k units; if $y = (x + k)^2, k > 0$, the shift is left k units. 67. The graph of $y = -f(x)$ is the reflection about the x -axis of the graph of $y = f(x)$. 69. Yes. The graph of $y = (x - 1)^3 + 2$ is the graph of $y = x^3$ shifted right 1 unit and up 2 units. 71. They all have the same general shape. All three go through the points $(-1, -1), (0, 0),$ and $(1, 1)$. As the exponent increases, the steepness of the curve increases (except near $x = 0$). 74. $\pm \sqrt{13}$ 75. $(h, k) = (0, 3); r = 5$
76. $\frac{3}{4}$ 77. local max: $f(-0.26) \approx 0.41$; local min: $f(-1.93) \approx -10.04$

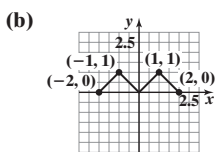
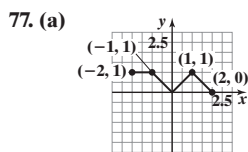
2.5 Assess Your Understanding (page 114)

1. horizontal; right 2. y 3. F 4. T 5. d 6. a 7. B 9. H 11. I 13. L 15. F 17. G 19. $y = (x - 4)^3$ 21. $y = x^3 + 4$ 23. $y = -x^3$
 25. $y = 4x^3$ 27. $y = -(\sqrt{-x} + 2)$ 29. $y = -\sqrt{x + 3} + 2$ 31. c 33. c 35. (a) -7 and 1 (b) -3 and 5 (c) -5 and 3 (d) -3 and 5
 37. (a) $[-3, 3]$ (b) $[4, 10]$ (c) Decreasing on $[-1, 5]$ (d) Decreasing on $[-5, 1]$

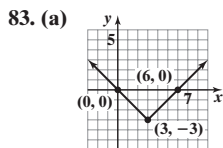
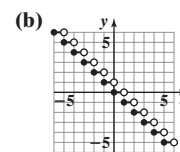
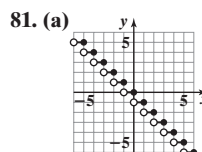


- (b) -3, 0, 3 (c) Local maximum: 10.39 at $x = -1.73$; local minimum: -10.39 at $x = 1.73$ (d) Increasing: $[-4, -1.73]$, $[1.73, 4]$; decreasing: $[-1.73, 1.73]$ (e) Intercepts: -5, -2, 1; local maximum: 10.39 at $x = -3.73$; local minimum: -10.39 at $x = -0.27$; increasing: $[-6, -3.73]$, $[-0.27, 2]$; decreasing: $[-3.73, -0.27]$
 (f) Intercepts: -3, 0, 3; Local maximum: 20.78 at $x = -1.73$; local minimum: -20.78 at $x = 1.73$; increasing: $[-4, -1.73]$, $[1.73, 4]$; decreasing: $[-1.73, 1.73]$ (g) Intercepts: -3, 0, 3; local maximum: 10.39 at $x = 1.73$; local minimum: -10.39 at $x = -1.73$; increasing: $[-1.73, 1.73]$; decreasing: $[-4, -1.73]$, $[1.73, 4]$



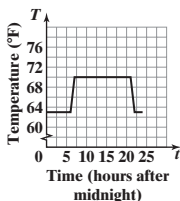


79. (a) $(-2, 2)$
 (b) $(3, -5)$ (c) $(-1, 3)$

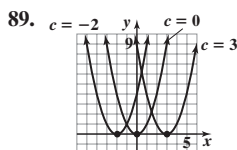
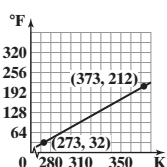
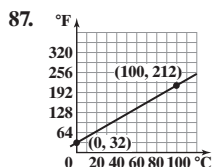
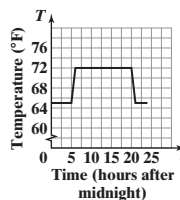


(b) 9 square units

85. (a) $72^\circ\text{F}; 65^\circ\text{F}$
 (b) The temperature decreases by 2° to 70°F during the day and 63°F overnight.



(c) The time at which the temperature adjusts between the daytime and overnight settings is moved to 1 hr sooner. It begins warming up at 5:00 AM instead of 6:00 AM, and it begins cooling down at 8:00 PM instead of 9:00 PM.



91. The graph of $y = 4f(x)$ is a vertical stretch by a factor of 4. The graph of $y = f(4x)$ is a horizontal compression by a factor of $\frac{1}{4}$.

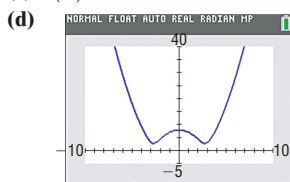
93. $\frac{16}{3}$ sq. units 95. The domain of $g(x) = \sqrt{x}$ is $[0, \infty)$. The graph of $g(x - k)$ is the graph of g shifted k units to the right, so the domain of $g(x - k)$ is $[k, \infty)$. 96. $m = \frac{3}{5}$; $b = -6$ 97. odd 98. 3 99. Intercepts: $(0, -2), (0, 2), (-4, 0)$; x -axis symmetry

2.6 Assess Your Understanding (page 120)

1. (a) $d(x) = \sqrt{x^4 - 15x^2 + 64}$

(b) $d(0) = 8$

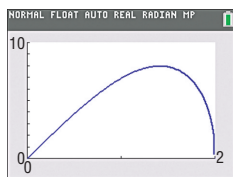
(c) $d(1) = \sqrt{50} \approx 7.07$



(e) d is smallest when $x \approx -2.74$ or $x \approx 2.74$

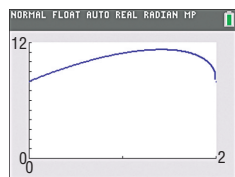
9. (a) $A(x) = 4x\sqrt{4 - x^2}$

(c) A is largest when $x \approx 1.41$.

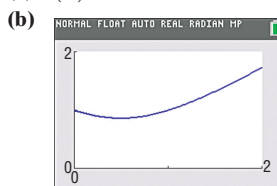


(b) $p(x) = 4x + 4\sqrt{4 - x^2}$

(d) p is largest when $x \approx 1.41$.



3. (a) $d(x) = \sqrt{x^2 - x + 1}$



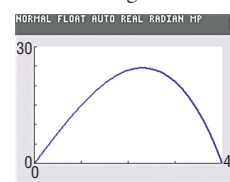
(c) d is smallest when $x = 0.5$.

5. $A(x) = \frac{1}{2}x^4$

7. (a) $A(x) = x(16 - x^2)$

(b) Domain: $\{x \mid 0 < x < 4\}$

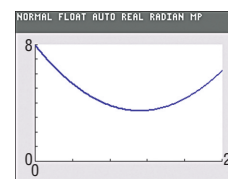
(c) The area is largest when $x \approx 2.31$.



11. (a) $A(x) = x^2 + \frac{25 - 20x + 4x^2}{\pi}$

(b) Domain: $\{x \mid 0 < x < 2.5\}$

(c) A is smallest when $x \approx 1.40$ m.



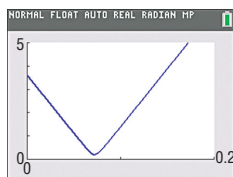
13. (a) $C(x) = x$ (b) $A(x) = \frac{x^2}{4\pi}$

15. (a) $A(r) = 2r^2$ (b) $p(r) = 6r$

17. $A(x) = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)x^2$

19. (a) $d(t) = \sqrt{2500t^2 - 360t + 13}$

(b) d is smallest when $t \approx 0.07$ hr.



21. $V(r) = \frac{\pi H(R - r)r^2}{R}$

23. (a) $T(x) = \frac{12 - x}{5} + \frac{\sqrt{x^2 + 4}}{3}$

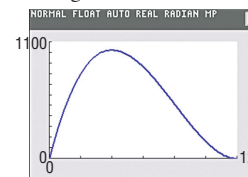
(b) $\{x \mid 0 \leq x \leq 12\}$

(c) 3.09 hr (d) 3.55 hr

25. (a) $V(x) = x(24 - 2x)^2$

(b) 972 in.^3 (c) 160 in.^3

(d) V is largest when $x = 4$.



27. $(h, k) = (-1, -1); r = \sqrt{41}$

28. $y = (x + 4)^2 - 2$

29. $m = -4$

30. $(-\infty, -2) \cup (-2, 5]$

Review Exercises (page 125)

1. Function; domain $\{-1, 2, 4\}$, range $\{0, 3\}$ 2. Not a function 3. (a) 2 (b) -2 (c) $-\frac{3x}{x^2-1}$ (d) $-\frac{3x}{x^2-1}$ (e) $\frac{3(x-2)}{x^2-4x+3}$ (f) $\frac{6x}{4x^2-1}$

4. (a) 0 (b) 0 (c) $\sqrt{x^2-4}$ (d) $-\sqrt{x^2-4}$ (e) $\sqrt{x^2-4x}$ (f) $2\sqrt{x^2-1}$ 5. (a) 0 (b) 0 (c) $\frac{x^2-4}{x^2}$ (d) $-\frac{x^2-4}{x^2}$ (e) $\frac{x(x-4)}{(x-2)^2}$ (f) $\frac{x^2-1}{x^2}$

6. $\{x|x \neq -3, x \neq 3\}$ 7. $\{x|x \leq 2\}$ 8. $\{x|x \neq 0\}$ 9. $\{x|x \neq -3, x \neq 1\}$ 10. $[-1, 2) \cup (2, \infty)$ 11. $\{x|x > -8\}$

12. $(f+g)(x) = 2x+3$; Domain: all real numbers
 $(f-g)(x) = -4x+1$; Domain: all real numbers
 $(f \cdot g)(x) = -3x^2+5x+2$; Domain: all real numbers
 $\left(\frac{f}{g}\right)(x) = \frac{2-x}{3x+1}$; Domain: $\left\{x \mid x \neq -\frac{1}{3}\right\}$

13. $(f+g)(x) = 3x^2+4x+1$; Domain: all real numbers
 $(f-g)(x) = 3x^2-2x+1$; Domain: all real numbers
 $(f \cdot g)(x) = 9x^3+3x^2+3x$; Domain: all real numbers
 $\left(\frac{f}{g}\right)(x) = \frac{3x^2+x+1}{3x}$; Domain: $\{x|x \neq 0\}$

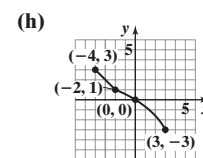
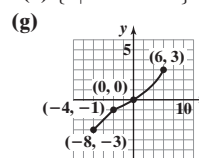
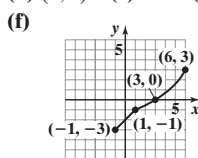
14. $(f+g)(x) = \frac{x^2+2x-1}{x(x-1)}$; Domain: $\{x|x \neq 0, x \neq 1\}$

15. $-4x+1-2h$ 16. (a) Domain: $\{x|-4 \leq x \leq 3\}$; Range: $\{y|-3 \leq y \leq 3\}$
 (b) (0, 0) (c) -1 (d) -4 (e) $\{x|0 < x \leq 3\}$

$(f-g)(x) = \frac{x^2+1}{x(x-1)}$; Domain: $\{x|x \neq 0, x \neq 1\}$

$(f \cdot g)(x) = \frac{x+1}{x(x-1)}$; Domain: $\{x|x \neq 0, x \neq 1\}$

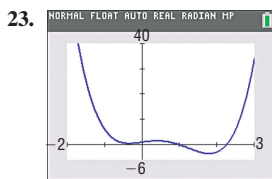
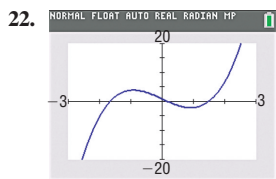
$\left(\frac{f}{g}\right)(x) = \frac{x(x+1)}{x-1}$; Domain: $\{x|x \neq 0, x \neq 1\}$



17. (a) Domain: $\{x|x \leq 4\}$ or $(-\infty, 4]$
 Range: $\{y|y \leq 3\}$ or $(-\infty, 3]$
 (b) Increasing on $(-\infty, -2]$ and $[2, 4]$; Decreasing on $[-2, 2]$
 (c) Local maximum value is 1 and occurs at $x = -2$.
 Local minimum value is -1 and occurs at $x = 2$.

(d) Absolute maximum: $f(4) = 3$
 Absolute minimum: none
 (e) No symmetry
 (f) Neither
 (g) x -intercepts: -3, 0, 3 y -intercept: 0

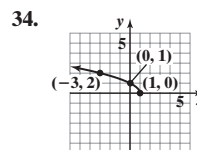
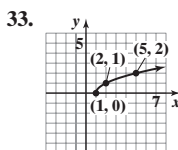
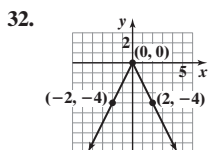
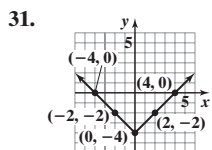
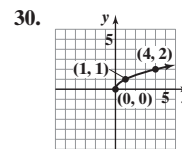
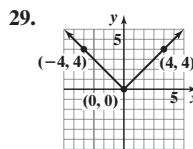
18. Odd 19. Even 20. Neither 21. Odd



Local maximum value: 4.04 at $x = -0.91$
 Local minimum value: -2.04 at $x = 0.91$
 Increasing: $[-3, -0.91]$; $[0.91, 3]$
 Decreasing: $[-0.91, 0.91]$

Local maximum value: 1.53 at $x = 0.41$
 Local minima values: 0.54 at $x = -0.34$ and -3.56 at $x = 1.80$
 Increasing: $[-0.34, 0.41]$; $[1.80, 3]$
 Decreasing: $[-2, -0.34]$; $[0.41, 1.80]$

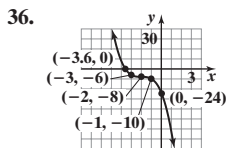
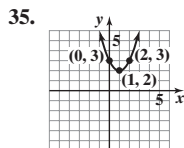
24. (a) 23 (b) 7 (c) 47 25. -5 26. -17 27. No 28. Yes



Intercepts: $(-4, 0)$, $(4, 0)$, $(0, -4)$ Intercept: $(0, 0)$
 Domain: all real numbers Domain: all real numbers
 Range: $\{y|y \geq -4\}$ or $[-4, \infty)$ Range: $\{y|y \leq 0\}$ or $(-\infty, 0]$

Intercept: $(1, 0)$ Domain: $\{x|x \geq 1\}$ or $[1, \infty)$
 Range: $\{y|y \geq 0\}$ or $[0, \infty)$

Intercepts: $(0, 1)$, $(1, 0)$ Domain: $\{x|x \leq 1\}$ or $(-\infty, 1]$
 Range: $\{y|y \geq 0\}$ or $[0, \infty)$

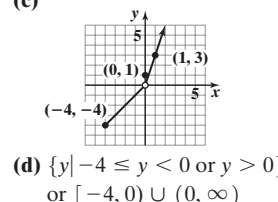
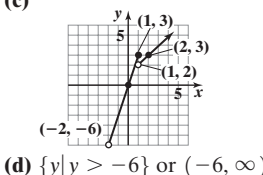


37. (a) $\{x|x > -2\}$ or $(-2, \infty)$
 (b) $(0, 0)$
 (c)

38. (a) $\{x|x \geq -4\}$ or $[-4, \infty)$
 (b) $(0, 1)$
 (c)

Intercept: $(0, 3)$
 Domain: all real numbers
 Range: $\{y|y \geq 2\}$ or $[2, \infty)$

Intercepts: $(0, -24)$, $(-2 - \sqrt[3]{4}, 0)$ or about $(-3.6, 0)$
 Domain: all real numbers
 Range: all real numbers

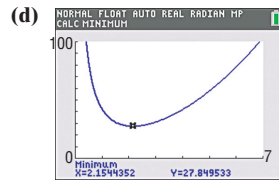


39. $A = 11$

40. (a) $A(x) = 2x^2 + \frac{40}{x}$

(b) 42 ft^2

(c) 28 ft^2



A is smallest when $x \approx 2.15 \text{ ft}$.

41. (a) $A(x) = 10x - x^3$

(b) The largest area that can be enclosed by the rectangle is approximately 12.17 square units.

Chapter Test (page 127)

1. (a) Function; Domain: $\{2, 4, 6, 8\}$; Range: $\{5, 6, 7, 8\}$ (b) Not a function (c) Not a function (d) Function; Domain: all real numbers; Range: $\{y|y \geq 2\}$

2. Domain: $\{x|x \leq \frac{4}{5}\}$; $f(-1) = 3$ 3. Domain: $\{x|x \neq -2\}$; $g(-1) = 1$ 4. Domain: $\{x|x \neq -9, x \neq 4\}$; $h(-1) = \frac{1}{8}$

5. (a) Domain: $\{x|-5 \leq x \leq 5\}$; Range: $\{y|-3 \leq y \leq 3\}$ (b) $(0, 2)$, $(-2, 0)$, and $(2, 0)$ (c) $f(1) = 3$ (d) $x = -5$ and $x = 3$
 (e) $\{x|-5 \leq x < -2 \text{ or } 2 < x \leq 5\}$ or $[-5, -2) \cup (2, 5]$ 6. Local maxima values: $f(-0.85) \approx -0.86$; $f(2.35) \approx 15.55$; local minimum value: $f(0) = -2$; the function is increasing on the intervals $[-5, -0.85]$ and $[0, 2.35]$ and decreasing on the intervals $[-0.85, 0]$ and $[2.35, 5]$.

7. (a) (b) $(0, -4)$, $(4, 0)$ 8. 19

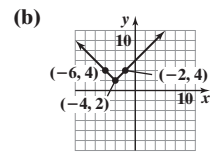
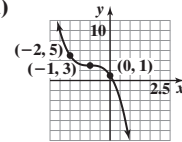
9. (a) $(f - g)(x) = 2x^2 - 3x + 3$ 10. (a)

(c) $g(-5) = -9$

(b) $(f \cdot g)(x) = 6x^3 - 4x^2 + 3x - 2$

(d) $g(2) = -2$

(c) $f(x + h) - f(x) = 4xh + 2h^2$



11. (a) 8.67% occurring in 1997 ($x \approx 5$) (b) The model predicts that the interest rate will be -10.343% . This is not reasonable.

12. (a) $V(x) = \frac{x^2}{8} - \frac{5x}{4} + \frac{\pi x^2}{64}$ (b) 1297.61 ft^3

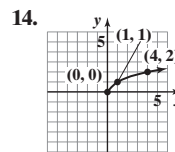
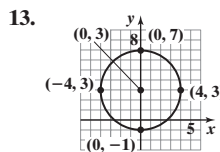
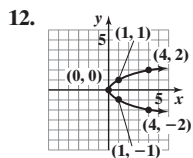
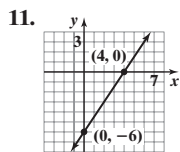
Cumulative Review (page 128)

1. $\{6\}$ 2. $\{0, \frac{1}{3}\}$ 3. $\{-1, 9\}$ 4. $\{\frac{1}{3}, \frac{1}{2}\}$ 5. $\{-\frac{7}{2}, \frac{1}{2}\}$ 6. $\{\frac{1}{2}\}$

7. $\{x|x < -\frac{4}{3}\}$; $(-\infty, -\frac{4}{3})$ 8. $\{x|1 < x < 4\}$; $(1, 4)$ 9. $\{x|x \leq -2 \text{ or } x \geq \frac{3}{2}\}$; $(-\infty, -2] \cup [\frac{3}{2}, \infty)$

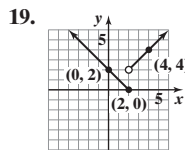
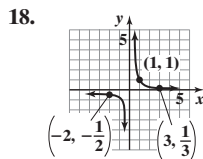
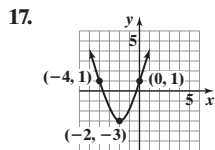


10. (a) distance: $\sqrt{29}$ (b) midpoint: $(\frac{1}{2}, -4)$ (c) slope: $-\frac{2}{5}$



15. Intercepts: $(0, -3)$, $(-2, 0)$, $(2, 0)$; symmetry with respect to the y-axis

16. $y = \frac{1}{2}x + 5$

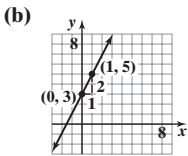


Chapter 3 Linear and Quadratic Functions

3.1 Assess Your Understanding (page 137)

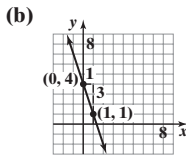
7. slope; y-intercept 8. positive 9. T 10. F 11. a 12. d

13. (a) $m = 2; b = 3$



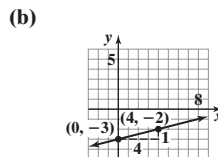
(c) 2 (d) Increasing

15. (a) $m = -3; b = 4$



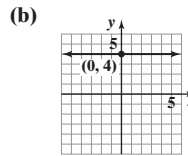
(c) -3 (d) Decreasing

17. (a) $m = \frac{1}{4}; b = -3$



(c) $\frac{1}{4}$ (d) Increasing

19. (a) $m = 0; b = 4$



(c) 0 (d) Constant

21. Linear; $f(x) = -3x - 2$

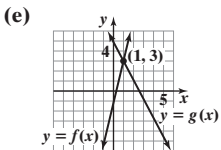
23. Nonlinear

25. Nonlinear

27. Linear; $f(x) = 8$

29. (a) $\frac{1}{4}$ (b) $\{x \mid x > \frac{1}{4}\}$ or $(\frac{1}{4}, \infty)$

(c) 1 (d) $\{x \mid x \leq 1\}$ or $(-\infty, 1]$



31. (a) 40 (b) 88 (c) -40 (d) $\{x \mid x > 40\}$ or $(40, \infty)$ (e) $\{x \mid x \leq 88\}$ or $(-\infty, 88]$

(f) $\{x \mid -40 < x < 88\}$ or $(-40, 88)$ 33. (a) -4 (b) $\{x \mid x < -4\}$ or $(-\infty, -4)$ 35. (a) -6

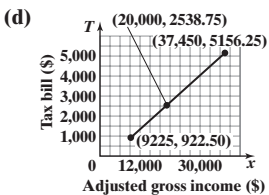
(b) $\{x \mid -6 \leq x < 5\}$ or $[-6, 5)$ 37. (a) \$59 (b) 180 mi (c) 300 mi (d) $\{x \mid x \geq 0\}$ or $[0, \infty)$

(e) The cost of renting the car for a day increases \$0.35 for each mile driven, or there is a charge of \$0.35 per mile to rent the car in addition to a fixed charge of \$45. (f) It costs \$45 to rent the car if 0 miles are driven, or there is a fixed charge of \$45 to rent the car in addition to a charge that depends on mileage. 39. (a) \$24; 600 T-shirts (b) $0 \leq p < \$24$ (c) The price will increase.

41. (a) $\{x \mid 9225 \leq x \leq 37,450\}$ or $[9225, 37,450]$

(b) \$2538.75

(c) The independent variable is adjusted gross income, x . The dependent variable is the tax bill, T .



(e) \$27,500

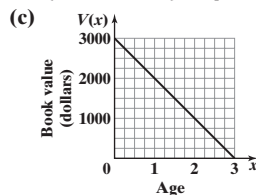
(f) For each additional dollar of taxable income between \$9225 and \$37,450, the tax bill of a single person in 2015 increased by \$0.15.

43. (a) $x = 5000$

(b) $x > 5000$

45. (a) $V(x) = -1000x + 3000$

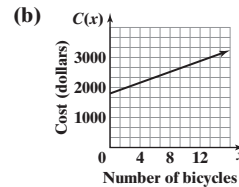
(b) $\{x \mid 0 \leq x \leq 3\}$ or $[0, 3]$



(d) \$1000

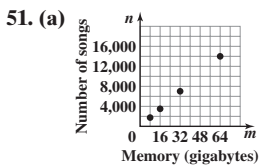
(e) After 1 year

47. (a) $C(x) = 90x + 1800$



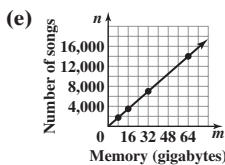
(c) \$3060 (d) 22 bicycles

49. (a) $C(x) = 0.89x + 39.95$ (b) \$13785; \$244.65



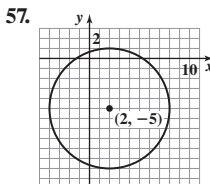
(b) Since each input (memory) corresponds to a single output (number of songs), we know that number of songs is a function of memory. Also, because the average rate of change is a constant 218.75 songs per gigabyte, the function is linear.

(c) $n(m) = 218.75m$ (d) $\{m \mid m \geq 0\}$ or $[0, \infty)$



(f) If memory increases by 1 GB, then the number of songs increases by 218.75.

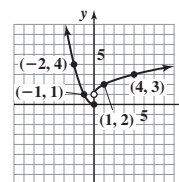
53. (d), (e) 55. $b = 0$; yes, $f(x) = b$



58. 6

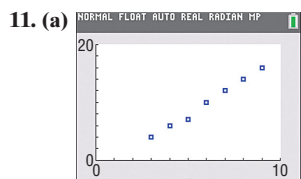
59. 7

60.

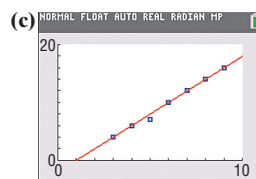


3.2 Assess Your Understanding (page 144)

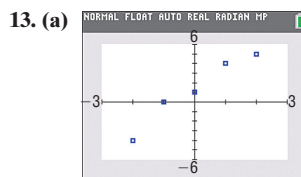
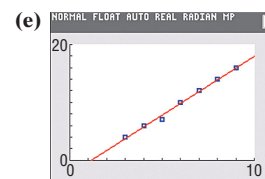
3. scatter diagram 4. decrease; 0.008 5. Linear relation, $m > 0$ 7. Linear relation, $m < 0$ 9. Nonlinear relation



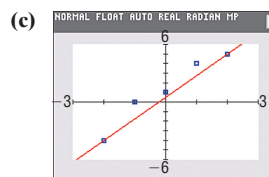
(b) Answers will vary. Using (4, 6) and (8, 14), $y = 2x - 2$.



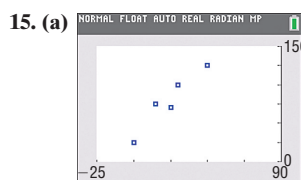
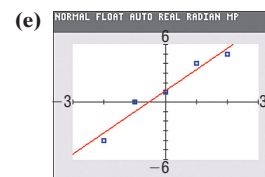
(d) $y = 2.0357x - 2.3571$



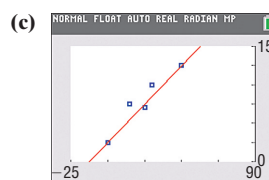
(b) Answers will vary. Using (-2, -4) and (2, 5), $y = \frac{9}{4}x + \frac{1}{2}$.



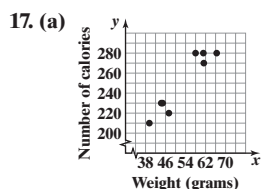
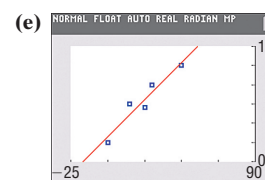
(d) $y = 2.2x + 1.2$



(b) Answers will vary. Using (-20, 100) and (-10, 140), $y = 4x + 180$.

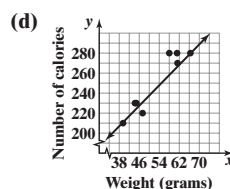


(d) $y = 3.8613x + 180.2920$



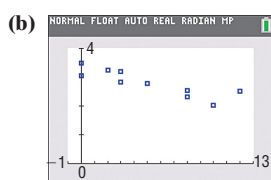
(b) Linear with positive slope

(c) Answers will vary. Using the points (39.52, 210) and (66.45, 280), $y = 2.599x + 107.288$.

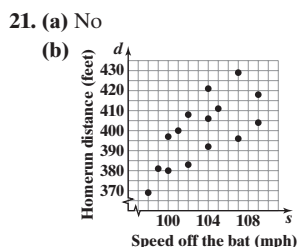


(e) 269 calories
(f) If the weight of a candy bar is increased by 1 gram, the number of calories will increase by 2.599, on average.

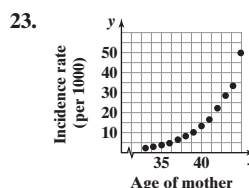
19. (a) The independent variable is the number of hours spent playing video games, and cumulative grade-point average is the dependent variable, because we are using number of hours playing video games to predict (or explain) cumulative grade-point average.



(c) $G(h) = -0.0942h + 3.2763$
(d) If the number of hours playing video games in a week increases by 1 hour, the cumulative grade-point average decreases 0.09, on average.
(e) 2.52
(f) Approximately 9.3 hours



(c) $d = 3.3641s + 51.8233$
(d) If the speed off the bat increases by 1 mile per hour, the homerun distance increases by 3.3641 feet, on average.
(e) $d(s) = 3.3641s + 51.8233$
(f) $\{s | s > 0\}$ or $(0, \infty)$
(g) Approximately 398 feet

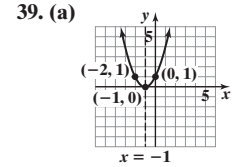
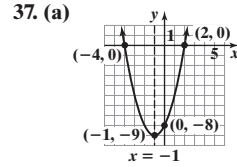
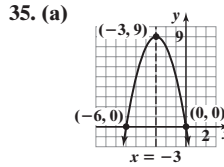
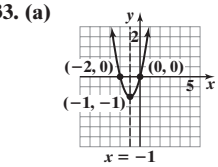
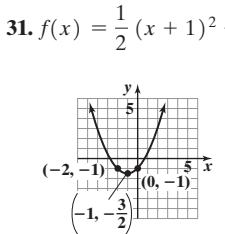
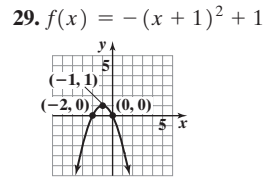
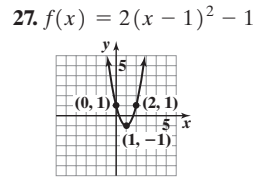
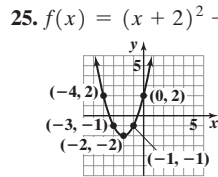
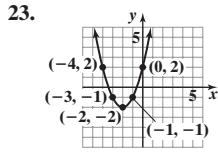
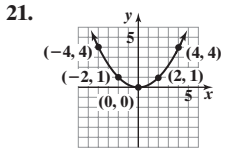


No, the data do not follow a linear pattern.

25. No linear relation 27. 34.8 hours; A student whose GPA is 0 spends 34.8 hours each week playing video games. $G(0) = 3.28$. The average GPA of a student who does not play video games is 3.28. 28. $2x + y = 3$ or $y = -2x + 3$ 29. $\{x | x \neq -5, x \neq 5\}$ 30. $(g - f)(x) = x^2 - 8x + 12$
31. $y = (x + 3)^2 - 4$

3.3 Assess Your Understanding (page 156)

5. parabola 6. axis or axis of symmetry 7. $-\frac{b}{2a}$ 8. T 9. T 10. T 11. a 12. d 13. C 15. F 17. G 19. H

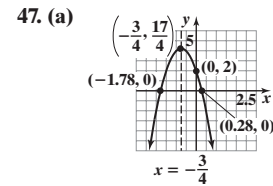
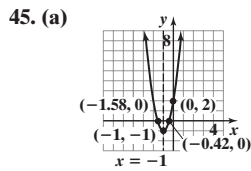
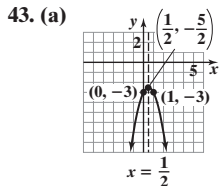
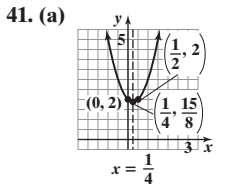


(b) Domain: $(-\infty, \infty)$
Range: $[-1, \infty)$
(c) Decreasing: $(-\infty, -1]$
Increasing: $[-1, \infty)$

(b) Domain: $(-\infty, \infty)$
Range: $(-\infty, 9]$
(c) Increasing: $(-\infty, -3]$
Decreasing: $[-3, \infty)$

(b) Domain: $(-\infty, \infty)$
Range: $[-9, \infty)$
(c) Decreasing: $(-\infty, -1]$
Increasing: $[-1, \infty)$

(b) Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
(c) Decreasing: $(-\infty, -1]$
Increasing: $[-1, \infty)$



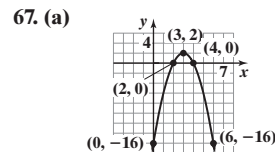
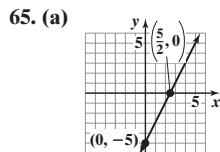
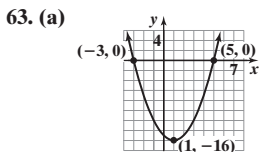
(b) Domain: $(-\infty, \infty)$
Range: $[\frac{15}{8}, \infty)$
(c) Decreasing: $(-\infty, \frac{1}{4}]$
Increasing: $[\frac{1}{4}, \infty)$

(b) Domain: $(-\infty, \infty)$
Range: $(-\infty, -\frac{5}{2}]$
(c) Increasing: $(-\infty, \frac{1}{2}]$
Decreasing: $[\frac{1}{2}, \infty)$

(b) Domain: $(-\infty, \infty)$
Range: $[-1, \infty)$
(c) Decreasing: $(-\infty, -1]$
Increasing: $[-1, \infty)$

(b) Domain: $(-\infty, \infty)$
Range: $(-\infty, \frac{17}{4}]$
(c) Increasing: $(-\infty, -\frac{3}{4}]$
Decreasing: $[-\frac{3}{4}, \infty)$

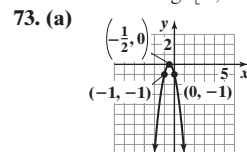
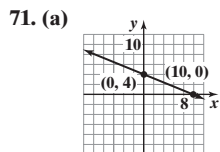
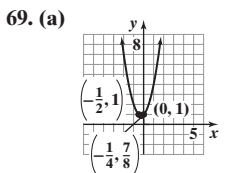
49. $f(x) = (x + 1)^2 - 2 = x^2 + 2x - 1$ 51. $f(x) = -(x + 3)^2 + 5 = -x^2 - 6x - 4$ 53. $f(x) = 2(x - 1)^2 - 3 = 2x^2 - 4x - 1$
55. Minimum value; -18 57. Minimum value; -21 59. Maximum value; 21 61. Maximum value; 13



(b) Domain: $(-\infty, \infty)$
Range: $[-16, \infty)$
(c) Decreasing: $(-\infty, 1]$
Increasing: $[1, \infty)$

(b) Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
(c) Increasing: $(-\infty, \infty)$

(b) Domain: $(-\infty, \infty)$
Range: $(-\infty, 2]$
(c) Increasing: $(-\infty, 3]$
Decreasing: $[3, \infty)$



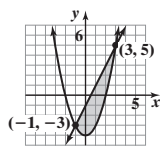
(b) Domain: $(-\infty, \infty)$
Range: $[\frac{7}{8}, \infty)$
(c) Decreasing: $(-\infty, -\frac{1}{4}]$
Increasing: $[-\frac{1}{4}, \infty)$

(b) Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
(c) Decreasing: $(-\infty, \infty)$

(b) Domain: $(-\infty, \infty)$
Range: $(-\infty, 0]$
(c) Increasing: $(-\infty, -\frac{1}{2}]$
Decreasing: $[-\frac{1}{2}, \infty)$

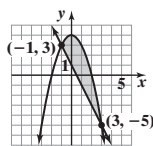
75. $a = 6, b = 0, c = 2$

77. (a), (c), (d)



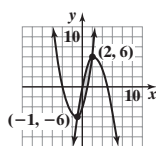
(b) $\{-1, 3\}$

79. (a), (c), (d)



(b) $\{-1, 3\}$

81. (a), (c), (d)



(b) $\{-1, 2\}$

83. (a) $a = 1: f(x) = (x + 3)(x - 1) = x^2 + 2x - 3$

$a = 2: f(x) = 2(x + 3)(x - 1) = 2x^2 + 4x - 6$

$a = -2: f(x) = -2(x + 3)(x - 1) = -2x^2 - 4x + 6$

$a = 5: f(x) = 5(x + 3)(x - 1) = 5x^2 + 10x - 15$

(b) The value of a does not affect the x -intercepts, but it changes the y -intercept by a factor of a .

(c) The value of a does not affect the axis of symmetry. It is $x = -1$ for all values of a .

(d) The value of a does not affect the x -coordinate of the vertex. However, the y -coordinate of the vertex is multiplied by a .

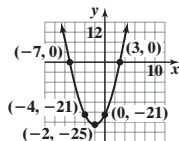
(e) The mean of the x -intercepts is the x -coordinate of the vertex.

85. (a) $(-2, -25)$

(b) $-7, 3$

(c) $-4, 0; (-4, -21), (0, -21)$

(d)



87. $(2, 2)$ 89. \$500; \$1,000,000 91. (a) 70,000 digital music players (b) \$2500

93. (a) 187 or 188 watches; \$7031.20 (b) $P(x) = -0.2x^2 + 43x - 1750$

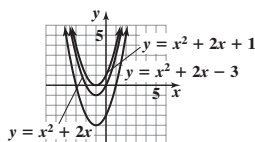
(c) 107 or 108 watches; \$561.20 95. (a) 171 ft (b) 49 mph (c) Reaction time

97. If x is even, then ax^2 and bx are even and $ax^2 + bx$ is even, which means that $ax^2 + bx + c$ is odd.

If x is odd, then ax^2 and bx are odd and $ax^2 + bx$ is even, which means that $ax^2 + bx + c$ is odd.

In either case, $f(x)$ is odd.

99.



101. $b^2 - 4ac < 0$ 103. No

105. Symmetric with respect to the x -axis, the y -axis, and the origin.

106. $\{x | x \leq 4\}$ or $(-\infty, 4]$ 107. Center $(5, -2)$; radius = 3

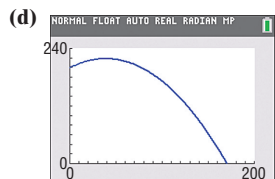
108. $y = \sqrt{-x}$

3.4 Assess Your Understanding (page 165)

3. (a) $R(x) = -\frac{1}{6}x^2 + 100x$ (b) $\{x | 0 \leq x \leq 600\}$ (c) \$13,333.33 (d) 300; \$15,000 (e) \$50 5. (a) $R(x) = -\frac{1}{5}x^2 + 20x$ (b) \$255

(c) 50; \$500 (d) \$10 (e) Between \$8 and \$12 7. (a) $A(w) = -w^2 + 200w$ (b) A is largest when $w = 100$ yd. (c) 10,000 yd² 9. 2,000,000 m²

11. (a) $\frac{625}{16} \approx 39$ ft (b) $\frac{7025}{32} \approx 219.5$ ft (c) About 170 ft

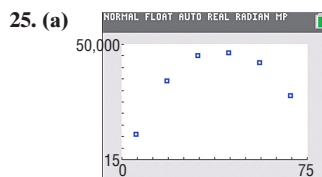


(d) When the height is 100 ft, the projectile is about 135.7 ft from the cliff.

13. 18.75 m 15. (a) 3 in. (b) Between 2 in. and 4 in.

17. $\frac{750}{\pi} \approx 238.73$ m by 375 m

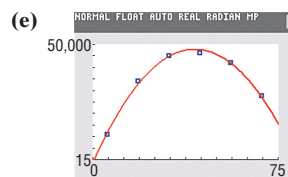
19. $x = \frac{a}{2}$ 21. $\frac{38}{3}$ 23. $\frac{248}{3}$



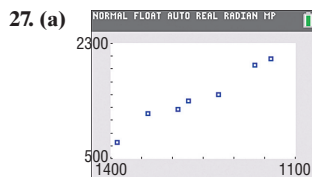
(b) $I(x) = -44.759x^2 + 4295.356x - 55,045.418$

(c) About 48.0 years of age

(d) Approximately \$48,007

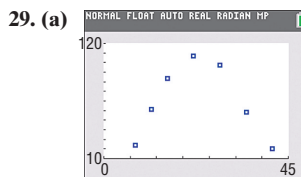


The data appear to follow a quadratic relation with $a < 0$.



The data appear to be linearly related with positive slope.

(b) $R(x) = 1.229x + 917.385$ (c) \$1993



The data appear to follow a quadratic relation with $a < 0$.

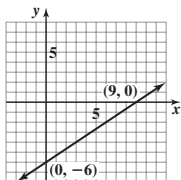
(b) $B(a) = -0.547a^2 + 31.190a - 342.218$ (c) 79.357

32. $2x + 3y = -4$ or $y = -\frac{2}{3}x - \frac{4}{3}$ 33. 13 34. $(x + 6)^2 + y^2 = 7$ 35. $(0, -12), (-4, 0), (4, 0)$

3.5 Assess Your Understanding (page 172)

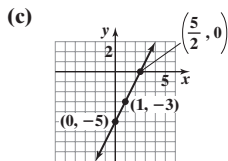
3. (a) $\{x|x < -2 \text{ or } x > 2\}; (-\infty, -2) \cup (2, \infty)$ (b) $\{x|-2 \leq x \leq 2\}; [-2, 2]$
 5. (a) $\{x|-2 \leq x \leq 1\}; [-2, 1]$ (b) $\{x|x < -2 \text{ or } x > 1\}; (-\infty, -2) \cup (1, \infty)$ 7. $\{x|-2 < x < 5\}; (-2, 5)$
 9. $\{x|x < 0 \text{ or } x > 4\}; (-\infty, 0) \cup (4, \infty)$ 11. $\{x|-3 < x < 3\}; (-3, 3)$ 13. $\{x|x < -4 \text{ or } x > 3\}; (-\infty, -4) \cup (3, \infty)$
 15. $\left\{x \mid -\frac{1}{2} < x < 3\right\}; \left(-\frac{1}{2}, 3\right)$ 17. No real solution 19. No real solution 21. $\left\{x \mid x < -\frac{2}{3} \text{ or } x > \frac{3}{2}\right\}; \left(-\infty, -\frac{2}{3}\right) \cup \left(\frac{3}{2}, \infty\right)$
 23. $\{x|x \leq -4 \text{ or } x \geq 4\}; (-\infty, -4] \cup [4, \infty)$
 25. (a) $\{-1, 1\}$ (b) $\{-1\}$ (c) $\{-1, 4\}$ (d) $\{x|x < -1 \text{ or } x > 1\}; (-\infty, -1) \cup (1, \infty)$ (e) $\{x|x \leq -1\}; (-\infty, -1]$
 (f) $\{x|x < -1 \text{ or } x > 4\}; (-\infty, -1) \cup (4, \infty)$ (g) $\{x|x \leq -\sqrt{2} \text{ or } x \geq \sqrt{2}\}; (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$
 27. (a) $\{-1, 1\}$ (b) $\left\{-\frac{1}{4}\right\}$ (c) $\{-4, 0\}$ (d) $\{x|-1 < x < 1\}; (-1, 1)$ (e) $\left\{x \mid x \leq -\frac{1}{4}\right\}; \left(-\infty, -\frac{1}{4}\right]$ (f) $\{x|-4 < x < 0\}; (-4, 0)$ (g) $\{0\}$
 29. (a) $\{-2, 2\}$ (b) $\{-2, 2\}$ (c) $\{-2, 2\}$ (d) $\{x|x < -2 \text{ or } x > 2\}; (-\infty, -2) \cup (2, \infty)$ (e) $\{x|x \leq -2 \text{ or } x \geq 2\}; (-\infty, -2] \cup [2, \infty)$
 (f) $\{x|x < -2 \text{ or } x > 2\}; (-\infty, -2) \cup (2, \infty)$ (g) $\{x|x \leq -\sqrt{5} \text{ or } x \geq \sqrt{5}\}; (-\infty, -\sqrt{5}] \cup [\sqrt{5}, \infty)$
 31. (a) $\{-1, 2\}$ (b) $\{-2, 1\}$ (c) $\{0\}$ (d) $\{x|x < -1 \text{ or } x > 2\}; (-\infty, -1) \cup (2, \infty)$ (e) $\{x|-2 \leq x \leq 1\}; [-2, 1]$ (f) $\{x|x < 0\}; (-\infty, 0)$
 (g) $\left\{x \mid x \leq \frac{1 - \sqrt{13}}{2} \text{ or } x \geq \frac{1 + \sqrt{13}}{2}\right\}; \left(-\infty, \frac{1 - \sqrt{13}}{2}\right] \cup \left[\frac{1 + \sqrt{13}}{2}, \infty\right)$

33. (a) 5 sec (b) The ball is more than 96 ft above the ground for time t between 2 and 3 sec, $2 < t < 3$.
 35. (a) \$0, \$1000 (b) The revenue is more than \$800,000 for prices between \$276.39 and \$723.61, $\$276.39 < p < \723.61 .
 37. (a) $\{c \mid 0.112 < c < 81.907\}; (0.112, 81.907)$ (b) It is possible to hit a target 75 km away if $c = 0.651$ or $c = 1.536$. 44. $\{x|x \leq 5\}$
 45. (a) $(9, 0), (0, -6)$ 46. Odd 47. Neither

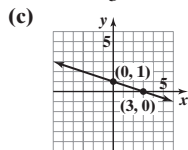


Review Exercises (page 175)

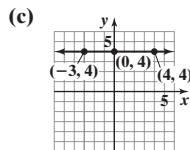
1. (a) $m = 2; b = -5$ (b) 2 2. (a) $m = -\frac{1}{3}; b = 1$ (b) $-\frac{1}{3}$ 3. (a) $m = 0; b = 4$ (b) 0



(d) Increasing



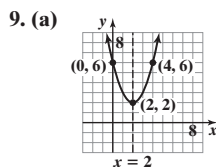
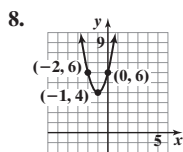
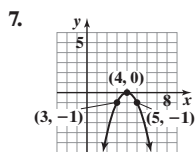
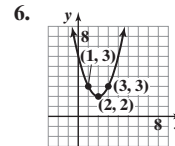
(d) Decreasing



(d) Constant

4. Linear; $f(x) = 5x + 3$

5. Nonlinear

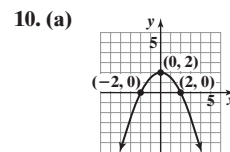


(b) Domain: $(-\infty, \infty)$

Range: $[2, \infty)$

(c) Decreasing: $(-\infty, 2]$

Increasing: $[2, \infty)$

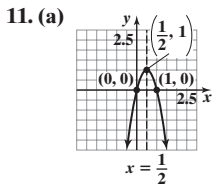


(b) Domain: $(-\infty, \infty)$

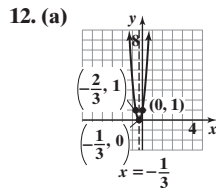
Range: $(-\infty, 2]$

(c) Increasing: $(-\infty, 0]$

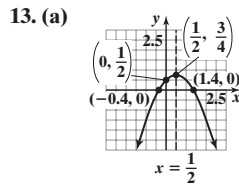
Decreasing: $[0, \infty)$



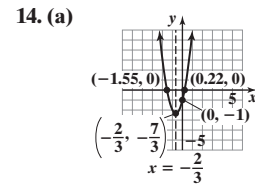
- (b) Domain: $(-\infty, \infty)$
Range: $(-\infty, 1]$
- (c) Increasing: $(-\infty, \frac{1}{2}]$
Decreasing: $[\frac{1}{2}, \infty)$



- (b) Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
- (c) Decreasing: $(-\infty, -\frac{1}{3}]$
Increasing: $[-\frac{1}{3}, \infty)$



- (b) Domain: $(-\infty, \infty)$
Range: $(-\infty, \frac{3}{4}]$
- (c) Increasing: $(-\infty, \frac{1}{2}]$
Decreasing: $[\frac{1}{2}, \infty)$



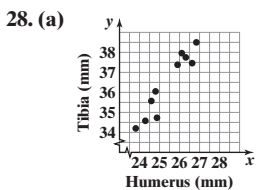
- (b) Domain: $(-\infty, \infty)$
Range: $[-\frac{7}{3}, \infty)$
- (c) Decreasing: $(-\infty, -\frac{2}{3}]$
Increasing: $[-\frac{2}{3}, \infty)$

15. Minimum value; 1 16. Maximum value; 12 17. Maximum value; 4 18. $\{x | -8 < x < 2\}$; $(-8, -2)$

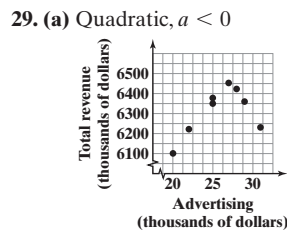
19. $\{x | x \leq -\frac{1}{3} \text{ or } x \geq 5\}$; $(-\infty, -\frac{1}{3}] \cup [5, \infty)$ 20. $y = x^2 + 2x + 3$ 21. $y = x^2 - 4x + 5$

22. (a) $S(x) = 0.01x + 25,000$ (b) \$35,000 (c) \$7,500,000 (d) $x > \$12,500,000$ 23. (a) $R(x) = -\frac{1}{10}x^2 + 150x$ (b) \$14,000 (c) 750; \$56,250 (d) \$75

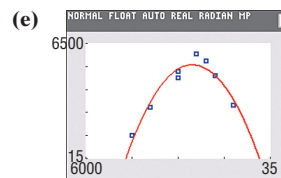
24. 4,166,666.7 m² 25. (a) 63 clubs (b) \$151.90 26. (a) $A(x) = -x^2 + 10x$; 25 units² 27. 3.6 ft



- (b) Yes
(c) $y = 1.3092x + 1.1140$
(d) 3795 mm

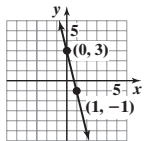


- (b) About \$26.5 thousand
(c) \$6408 thousand

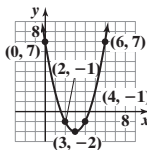


Chapter Test (page 177)

1. (a) Slope: -4 ; y-intercept: 3
(b) Decreasing
(c)



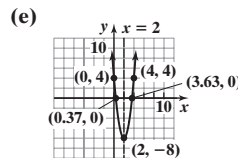
2. Linear; $y = -5x + 2$
3.



4. (a) Opens up (b) $(2, -8)$ (c) $x = 2$

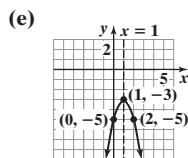
(d) x-intercepts: $\frac{6 - 2\sqrt{6}}{3}, \frac{6 + 2\sqrt{6}}{3}$

y-intercept: 4



- (f) Domain: All real numbers; $(-\infty, \infty)$
Range: $\{y | y \geq -8\}$; $[-8, \infty)$
(g) Decreasing: $(-\infty, 2]$; Increasing: $[2, \infty)$

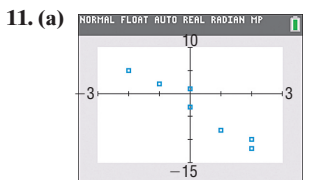
5. (a) Opens down (b) $(1, -3)$ (c) $x = 1$
(d) No x-intercepts; y-intercept: -5



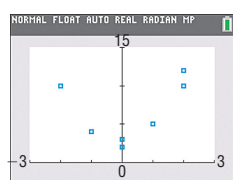
- (f) Domain: All real numbers; $(-\infty, \infty)$
Range: $\{y | y \leq -3\}$; $(-\infty, -3]$
(g) Increasing: $(-\infty, 1]$; Decreasing: $[1, \infty)$

6. $f(x) = 2x^2 - 4x - 30$ 7. Maximum value; 21 8. $\{x | x \leq 4 \text{ or } x \geq 6\}$; $(-\infty, 4] \cup [6, \infty)$ 9. (a) $C(m) = 0.15m + 129.50$ (b) \$258.50

(c) 562 miles 10. (a) $R(x) = -\frac{1}{10}x^2 + 1000x$ (b) \$384,000 (c) 5000 units; \$2,500,000 (d) \$500



Linear with negative slope



Quadratic that opens up

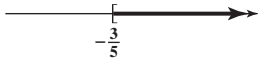
(b) $y = -4.234x - 2.362$

(c) $y = 1.993x^2 + 0.289x + 2.503$

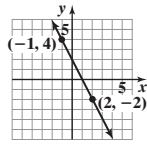
Cumulative Review (page 178)

1. $5\sqrt{2}$; $(\frac{3}{2}, \frac{1}{2})$ 2. $(-2, -1)$ and $(2, 3)$ are on the graph.

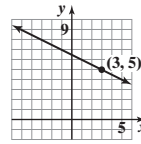
3. $\{x \mid x \geq -\frac{3}{5}\}$ or $[-\frac{3}{5}, \infty)$



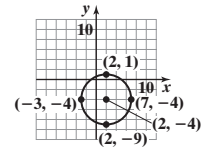
4. $y = -2x + 2$



5. $y = -\frac{1}{2}x + \frac{13}{2}$



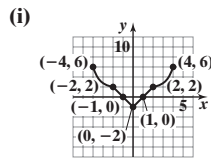
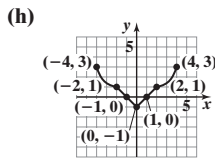
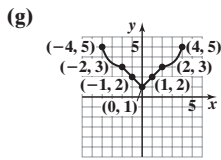
6. $(x - 2)^2 + (y + 4)^2 = 25$



7. Yes 8. (a) -3 (b) $x^2 - 4x - 2$ (c) $x^2 + 4x + 1$ (d) $-x^2 + 4x - 1$ (e) $x^2 - 3$ (f) $2x + h - 4$ 9. $\{z \mid z \neq \frac{7}{6}\}$ 10. Yes

11. (a) No (b) -1 ; $(-2, -1)$ is on the graph. (c) -8 ; $(-8, 2)$ is on the graph. 12. Neither 13. Local maximum value is 5.30 and occurs at $x = -1.29$. Local minimum value is -3.30 and occurs at $x = 1.29$. Increasing: $[-4, -1.29]$ and $[1.29, 4]$; Decreasing: $[-1.29, 1.29]$

14. (a) -4 (b) $\{x \mid x > -4\}$ or $(-4, \infty)$ 15. (a) Domain: $\{x \mid -4 \leq x \leq 4\}$; Range: $\{y \mid -1 \leq y \leq 3\}$ (b) $(-1, 0)$, $(0, -1)$, $(1, 0)$ (c) y -axis (d) 1 (e) -4 and 4 (f) $\{x \mid -1 < x < 1\}$



(j) Even (k) $[0, 4]$

Chapter 4 Polynomial and Rational Functions

4.1 Assess Your Understanding (page 196)

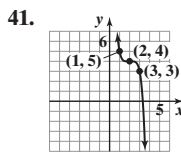
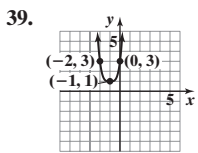
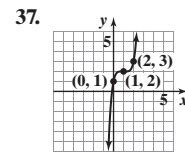
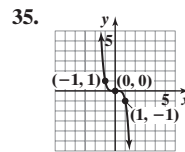
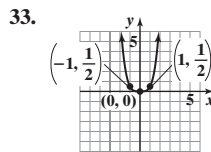
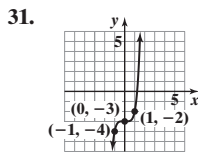
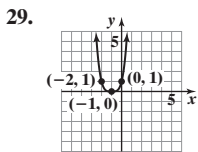
7. smooth; continuous 8. touches 9. $(-1, 1)$; $(0, 0)$; $(1, 1)$ 10. r is a real zero of f ; r is an x -intercept of the graph of f ; $x - r$ is a factor of f .

11. turning points 12. ∞ ; ∞ 13. ∞ ; $-\infty$ 14. As x increases in the positive direction, $f(x)$ decreases without bound. 15. b 16. d

17. Yes; degree 3; $f(x) = x^3 + 4x$; leading term: x^3 ; constant term: 0 19. Yes; degree 2; $g(x) = -\frac{1}{2}x^2 + \frac{1}{2}$; leading term: $-\frac{1}{2}x^2$; constant term: $\frac{1}{2}$

21. No; x is raised to the -1 power 23. No; x is raised to the $\frac{3}{2}$ power 25. Yes; degree 4; $F(x) = 5x^4 - \pi x^3 + \frac{1}{2}$; leading term: $5x^4$; constant term: $\frac{1}{2}$

27. Yes; degree 4; $G(x) = 2x^4 - 4x^3 + 4x^2 - 4x + 2$; leading term: $2x^4$; constant term: 2



43. $f(x) = (x + 1)(x - 1)(x - 3)$
 $= x^3 - 3x^2 - x + 3$ for $a = 1$

45. $f(x) = x(x + 3)(x - 4)$
 $= x^3 - x^2 - 12x$ for $a = 1$

49. $f(x) = (x + 1)(x - 3)^2$
 $= x^3 - 5x^2 + 3x + 9$ for $a = 1$

47. $f(x) = (x + 4)(x + 1)(x - 2)(x - 3)$
 $= x^4 - 15x^2 + 10x + 24$ for $a = 1$

51. $f(x) = 2(x + 3)(x - 1)(x - 4)$
 $= 2x^3 - 4x^2 - 22x + 24$

53. $f(x) = 16x(x + 1)(x - 2)(x - 4)$
 $= 16x^4 - 80x^3 + 32x^2 + 128x$

55. $f(x) = 5(x + 1)^2(x - 1)^2$
 $= 5x^4 - 10x^2 + 5$

57. (a) 7, multiplicity 1; -3 , multiplicity 2 (b) Graph touches the x -axis at -3 and crosses it at 7. (c) 2 (d) $y = 3x^3$

59. (a) 2, multiplicity 3 (b) Graph crosses the x -axis at 2. (c) 4 (d) $y = 4x^5$ 61. (a) $-\frac{1}{2}$, multiplicity 2; -4 , multiplicity 3

(b) Graph touches the x -axis at $-\frac{1}{2}$ and crosses at -4 . (c) 4 (d) $y = -2x^5$ 63. (a) 5, multiplicity 3; -4 , multiplicity 2

(b) Graph touches the x -axis at -4 and crosses it at 5. (c) 4 (d) $y = x^5$ 65. (a) No real zeros (b) Graph neither crosses nor touches the x -axis.

(c) 5 (d) $y = 3x^6$ 67. (a) 0, multiplicity 2; $-\sqrt{2}$, $\sqrt{2}$, multiplicity 1 (b) Graph touches the x -axis at 0 and crosses at $-\sqrt{2}$ and $\sqrt{2}$. (c) 3

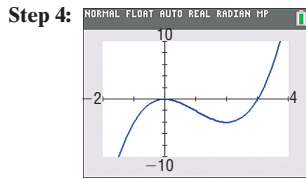
(d) $y = -2x^4$ 69. Could be; zeros: $-1, 1, 2$; Least degree is 3. 71. Cannot be the graph of a polynomial; gap at $x = -1$ 73. $f(x) = x(x - 1)(x - 2)$

75. $f(x) = -\frac{1}{2}(x + 1)(x - 1)^2(x - 2)$ 77. $f(x) = 0.2(x + 4)(x + 1)^2(x - 3)$ 79. $f(x) = -x(x + 3)^2(x - 3)^2$

81. Step 1: $y = x^3$

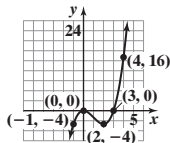
Step 2: x -intercepts: 0, 3;
 y -intercept: 0

Step 3: 0: multiplicity 2; touches;
3: multiplicity 1; crosses



Step 5: (2, -4); (0, 0)

Step 6:



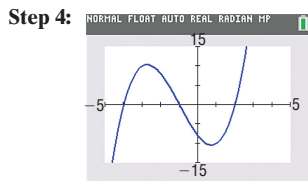
Step 7: Domain: $(-\infty, \infty)$;
Range: $(-\infty, \infty)$

Step 8: Increasing on $(-\infty, 0]$ and $[2, \infty)$
Decreasing on $[0, 2]$

87. Step 1: $y = x^3$

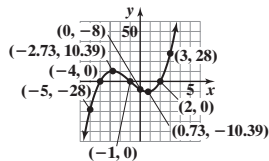
Step 2: x -intercepts: -4, -1, 2;
 y -intercept: -8

Step 3: -4, -1, 2: multiplicity 1, crosses



Step 5: (-2.73, 10.39); (0.73, -10.39)

Step 6:



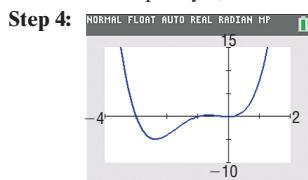
Step 7: Domain: $(-\infty, \infty)$;
Range: $(-\infty, \infty)$

Step 8: Increasing on $(-\infty, -2.73]$
and $[0.73, \infty)$
Decreasing on $[-2.73, 0.73]$

93. Step 1: $y = x^4$

Step 2: x -intercepts: -1, 0, -3; y -intercept: 0

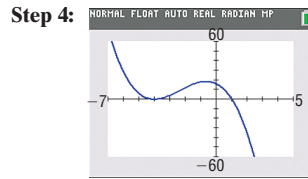
Step 3: -1, -3: multiplicity 1, crosses;
0: multiplicity 2, touches



83. Step 1: $y = -x^3$

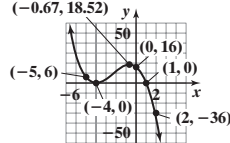
Step 2: x -intercepts: -4, 1;
 y -intercept: 16

Step 3: -4: multiplicity 2, touches;
1: multiplicity 1, crosses



Step 5: (-4, 0); (-0.67, 18.52)

Step 6:



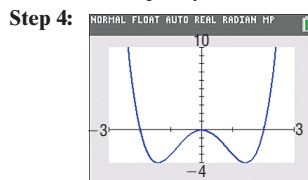
Step 7: Domain: $(-\infty, \infty)$;
Range: $(-\infty, \infty)$

Step 8: Increasing on $[-4, -0.67]$
Decreasing on $(-\infty, -4]$ and
 $[-0.67, \infty)$

89. Step 1: $y = x^4$

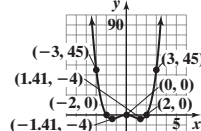
Step 2: x -intercepts: -2, 0, 2;
 y -intercept: 0

Step 3: -2, 2: multiplicity 1, crosses;
0: multiplicity 2, touches



Step 5: (-1.41, -4); (1.41, -4); (0, 0)

Step 6:

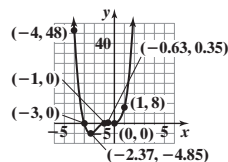


Step 7: Domain: $(-\infty, \infty)$;
Range: $[-4, \infty)$

Step 8: Increasing on $[-1.41, 0]$ and $[1.41, \infty)$
Decreasing on $(-\infty, -1.41]$, and $[0, 1.41]$

Step 5: (-2.37, -4.85); (-0.63, 0.35); (0, 0)

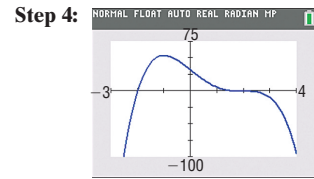
Step 6:



85. Step 1: $y = -2x^4$

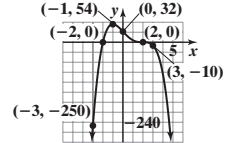
Step 2: x -intercepts: -2, 2;
 y -intercept: 32

Step 3: -2: multiplicity 1, crosses;
2: multiplicity 3, crosses



Step 5: (-1, 54)

Step 6:



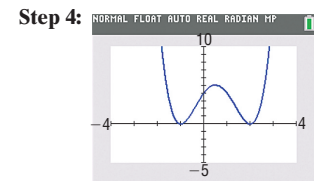
Step 7: Domain: $(-\infty, \infty)$;
Range: $(-\infty, 54]$

Step 8: Increasing on $(-\infty, -1]$
Decreasing on $[-1, \infty)$

91. Step 1: $y = x^4$

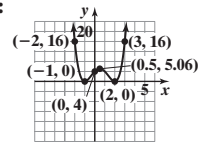
Step 2: x -intercepts: -1, 2;
 y -intercept: 4

Step 3: -1, 2: multiplicity 2, touches



Step 5: (-1, 0); (2, 0); (0.5, 5.06)

Step 6:



Step 7: Domain: $(-\infty, \infty)$;
Range: $[0, \infty)$

Step 8: Increasing on $[-1, 0.5]$ and $[2, \infty)$
Decreasing on $(-\infty, -1]$ and $[0.5, 2]$

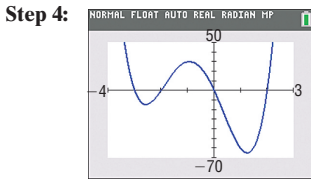
Step 7: Domain: $(-\infty, \infty)$;
Range: $[-4.85, \infty)$

Step 8: Increasing on $[-2.37, -0.63]$ and
 $[0, \infty)$
Decreasing on $(-\infty, -2.37]$ and
 $[-0.63, 0]$

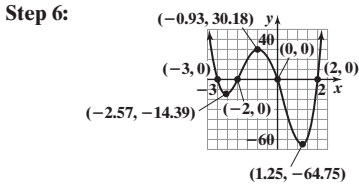
95. Step 1: $y = 5x^4$

Step 2: x -intercepts: $-3, -2, 0, 2$;
 y -intercept: 0

Step 3: $-3, -2, 0, 2$: multiplicity 1, crosses



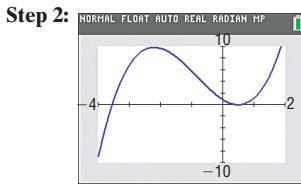
Step 5: $(-2.57, -14.39)$; $(-0.93, 30.18)$;
 $(1.25, -64.75)$



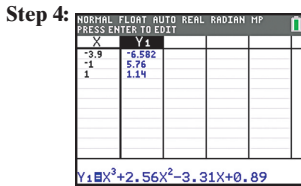
Step 7: Domain: $(-\infty, \infty)$; Range: $[-64.75, \infty)$

Step 8: Increasing on $[-2.57, -0.93]$
 and $[1.25, \infty)$
 Decreasing on $(-\infty, -2.57]$ and
 $[-0.93, 1.25]$

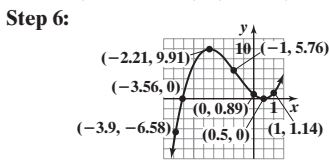
101. Step 1: $y = x^3$



Step 3: x -intercepts: $-3.56, 0.50$;
 y -intercept: 0.89



Step 5: $(-2.21, 9.91)$; $(0.50, 0)$



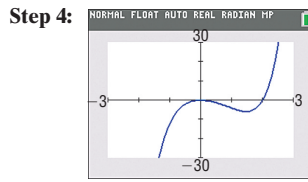
Step 7: Domain: $(-\infty, \infty)$;
 Range: $(-\infty, \infty)$

Step 8: Increasing on $(-\infty, -2.21]$
 and $[0.50, \infty)$
 Decreasing on $[-2.21, 0.50]$

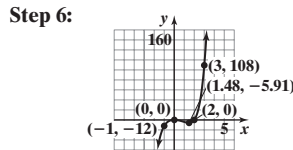
97. Step 1: $y = x^5$

Step 2: x -intercepts: $0, 2$; y -intercept: 0

Step 3: 0 : multiplicity 2, touches;
 2 : multiplicity 1, crosses



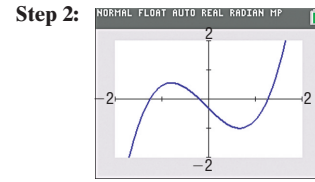
Step 5: $(0, 0)$; $(1.48, -5.91)$



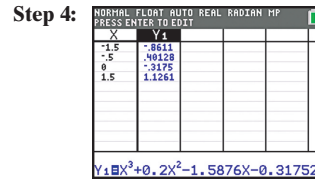
Step 7: Domain: $(-\infty, \infty)$;
 Range: $(-\infty, \infty)$

Step 8: Increasing on $(-\infty, 0]$ and $[1.48, \infty)$
 Decreasing on $[0, 1.48]$

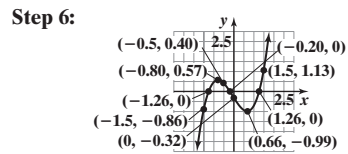
99. Step 1: $y = x^3$



Step 3: x -intercepts: $-1.26, -0.20, 1.26$;
 y -intercept: -0.31752



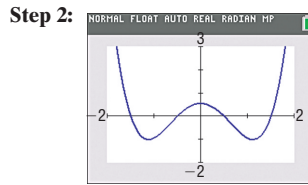
Step 5: $(-0.80, 0.57)$; $(0.66, -0.99)$



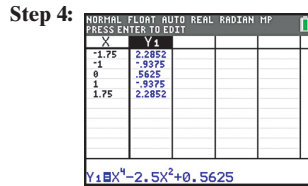
Step 7: Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$

Step 8: Increasing on $(-\infty, -0.80]$ and $[0.66, \infty)$
 Decreasing on $[-0.80, 0.66]$

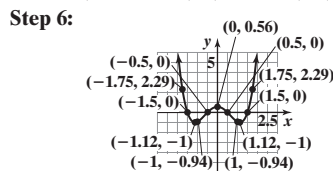
103. Step 1: $y = x^4$



Step 3: x -intercepts: $-1.5, -0.5, 0.5, 1.5$;
 y -intercept: 0.5625



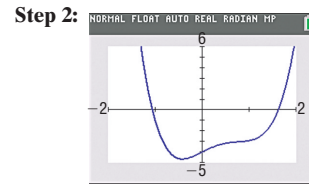
Step 5: $(-1.12, -1)$; $(1.12, -1)$, $(0, 0.56)$



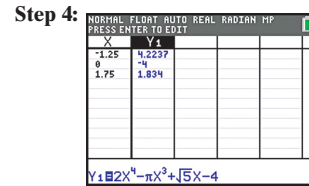
Step 7: Domain: $(-\infty, \infty)$;
 Range: $[-1, \infty)$

Step 8: Increasing on $[-1.12, 0]$ and $[1.12, \infty)$
 Decreasing on $(-\infty, -1.12]$
 and $[0, 1.12]$

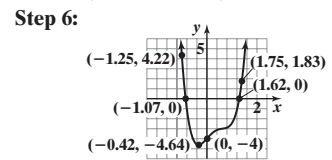
105. Step 1: $y = 2x^4$



Step 3: x -intercepts: $-1.07, 1.62$;
 y -intercept: -4



Step 5: $(-0.42, -4.64)$



Step 7: Domain: $(-\infty, \infty)$;
 Range: $[-4.64, \infty)$

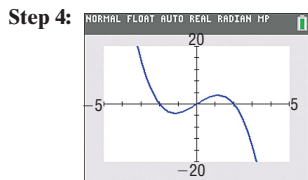
Step 8: Increasing on $[-0.42, \infty)$
 Decreasing on $(-\infty, -0.42]$

107. $f(x) = -x(x + 2)(x - 2)$

Step 1: $y = -x^3$

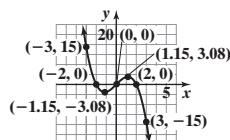
Step 2: x-intercepts: -2, 0, 2;
y-intercept: 0

Step 3: -2, 0, 2: multiplicity 1, crosses



Step 5: (-1.15, -3.08); (1.15, 3.08)

Step 6:



Step 7: Domain: $(-\infty, \infty)$;
Range: $(-\infty, \infty)$

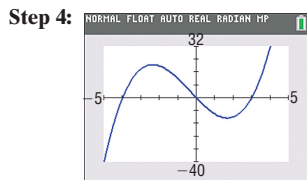
Step 8: Increasing on $[-1.15, 1.15]$
Decreasing on $(-\infty, -1.15]$ and $[1.15, \infty)$

109. $f(x) = x(x + 4)(x - 3)$

Step 1: $y = x^3$

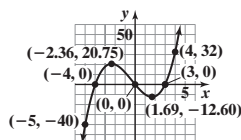
Step 2: x-intercepts: -4, 0, 3;
y-intercept: 0

Step 3: -4, 0, 3: multiplicity 1, crosses



Step 5: (-2.36, 20.75); (1.69, -12.60)

Step 6:



Step 7: Domain: $(-\infty, \infty)$;
Range: $(-\infty, \infty)$

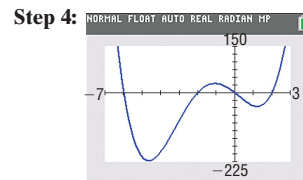
Step 8: Increasing on $(-\infty, -2.36]$ and $[1.69, \infty)$
Decreasing on $[-2.36, 1.69]$

111. $f(x) = 2x(x + 6)(x - 2)(x + 2)$

Step 1: $y = 2x^4$

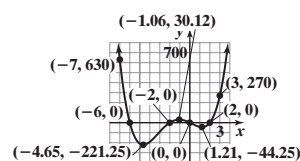
Step 2: x-intercepts: -6, -2, 0, 2;
y-intercept: 0

Step 3: -6, -2, 0, 2: multiplicity 1, crosses



Step 5: (-4.65, -221.25); (-1.06, 30.12);
(1.21, -44.25)

Step 6:



Step 7: Domain: $(-\infty, \infty)$;
Range: $[-221.25, \infty)$

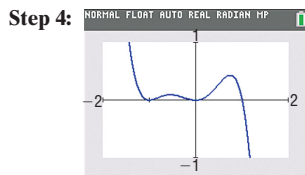
Step 8: Decreasing on $(-\infty, -4.65]$ and $[-1.06, 1.21]$
Increasing on $[-4.65, -1.06]$ and $[1.21, \infty)$

113. $f(x) = -x^2(x + 1)^2(x - 1)$

Step 1: $y = -x^5$

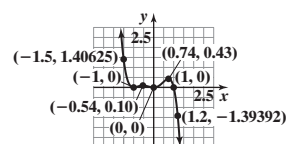
Step 2: x-intercepts: -1, 0, 1;
y-intercept: 0

Step 3: 1: multiplicity 1, crosses; -1,
0: multiplicity 2, touches



Step 5: (-1, 0); (-0.54, 0.10); (0, 0); (0.74, 0.43)

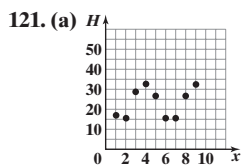
Step 6:



Step 7: Domain: $(-\infty, \infty)$;
Range: $(-\infty, \infty)$

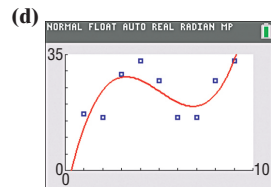
Step 8: Increasing on $[-1, -0.54]$ and $[0, 0.74]$
Decreasing on $(-\infty, -1]$, $[-0.54, 0]$,
and $[0.74, \infty)$

115. $f(x) = 3(x + 3)(x - 1)(x - 4)$ 117. $f(x) = -2(x + 5)^2(x - 2)(x - 4)$ 119. (a) -3, 2 (b) -6, -1

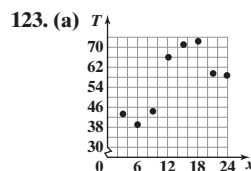


The relation appears to be cubic.

(b) $H(x) = 0.3948x^3 - 5.9563x^2 + 26.1965x - 7.4127$ (c) ≈ 24



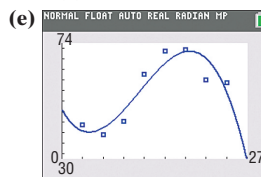
(e) ≈ 54 ; no. The end behavior of the x^3 model indicates that as time goes on, the number of major hurricanes will continue to increase each decade without limit. This is unrealistic. End behavior should not be used to make predictions too far outside the data used to create the model.



The relation appears to be cubic.

(b) $6^\circ/\text{h}$ (c) $0.17^\circ/\text{h}$

(d) $T(x) = -0.01992x^2 + 0.6745x^2 - 4.4360x + 48.4643$; 70.1°F



(f) The predicted temperature at midnight is 48.5°F .

125. (a)

X	Y ₁	Y ₂
-1	5	0
-1	52632	181
-1	55556	328
-1	58824	447
-1	625	544
-1	66667	625
-1	71429	696
-1	76923	763
-1	83333	832
-1	90909	903
0	1	1

$Y_1 \square 1 / (1 - X)$

X	Y ₁	Y ₂
0	1	1.111
1	1.1111	1.111
2	1.25	1.248
3	1.4286	1.417
4	1.6667	1.624
5	2	1.875
6	2.5	2.176
7	3.3333	2.533
8	5	2.952
9	10	3.439
0	ERROR	4

$Y_2 \square 1 + X + X^2 + X^3$

(b)

X	Y ₁	Y ₂
-1	5	8371
-1	52632	8371
-1	55556	7376
-1	58824	6871
-1	625	6736
-1	66667	6875
-1	71429	7216
-1	76923	7711
-1	83333	8336
-1	90909	9091
0	1	1

$Y_1 \square 1 / (1 - X)$

X	Y ₁	Y ₂
0	1	1.1111
1	1.1111	1.2496
2	1.25	1.4251
3	1.4286	1.6496
4	1.6667	1.9375
5	2	2.2856
6	2.5	2.7231
7	3.3333	3.2616
8	5	3.9051
9	10	4.6551
0	ERROR	5

$Y_2 \square 1 + X + X^2 + X^3 + X^4$

NORMAL FLOAT AUTO REAL RADIAN MP		
PRESS ENTER TO EDIT		
X	Y1	Y2
-1	.5	.24661
-.9	.52632	.40992
-.8	.55556	.51903
-.7	.58824	.625
-.6	.625	.65904
-.5	.66667	.65625
-.4	.71429	.7125
-.3	.76923	.76867
-.2	.83333	.8325
-.1	.90909	.90909
0	1	1

Y1: $1/(1-X)$

NORMAL FLOAT AUTO REAL RADIAN MP		
PRESS ENTER TO EDIT		
X	Y1	Y2
0	1.1111	1.1111
.1	1.25	1.2499
.2	1.5238	1.5225
.3	1.9231	1.6598
.4	2.5	1.9688
.5	3.3333	2.3834
.6	5	2.9412
.7	7.6923	3.6893
.8	10	4.6856
.9	ERROR	6

Y2: $1+X+X^2+X^3+X^4+X^5$

(d) As more terms are added, the values of the polynomial function get closer to the values of f . The approximations near 0 are better than those near -1 or 1 .

131. (a)–(d) 135. $y = -\frac{2}{5}x - \frac{11}{5}$ 136. $\{x|x \neq -5\}$ 137. $\frac{-2-\sqrt{7}}{2}, \frac{-2+\sqrt{7}}{2}$ 138. $(-7, 3)$

Historical Problems (page 213)

1. $(x - \frac{b}{3})^3 + b(x - \frac{b}{3})^2 + c(x - \frac{b}{3}) + d = 0$
 $x^3 - bx^2 + \frac{b^2x}{3} - \frac{b^3}{27} + bx^2 - \frac{2b^2x}{3} + \frac{b^3}{9} + cx - \frac{bc}{3} + d = 0$
 $x^3 + (c - \frac{b^2}{3})x + (\frac{2b^3}{27} - \frac{bc}{3} + d) = 0$
 Let $p = c - \frac{b^2}{3}$ and $q = \frac{2b^3}{27} - \frac{bc}{3} + d$. Then $x^3 + px + q = 0$.

2. $(H + K)^3 + p(H + K) + q = 0$
 $H^3 + 3H^2K + 3HK^2 + K^3 + pH + pK + q = 0$
 Let $3HK = -p$.
 $H^3 - pH - pK + K^3 + pH + pK + q = 0, H^3 + K^3 = -q$

4. $H^3 + K^3 = -q$
 $K^3 = -q - H^3$
 $K^3 = -q - \left[\frac{-q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}} \right]$
 $K^3 = \frac{-q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}$
 $K = \sqrt[3]{\frac{-q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$

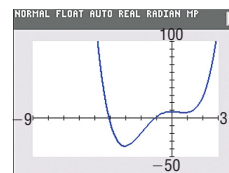
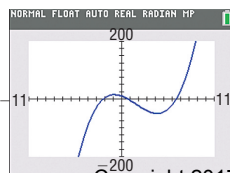
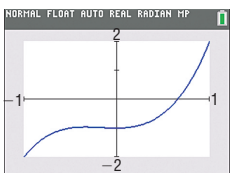
3. $3HK = -p$
 $K = -\frac{p}{3H}$
 $H^3 + \left(-\frac{p}{3H}\right)^3 = -q$
 $H^3 - \frac{p^3}{27H^3} = -q$
 $27H^6 - p^3 = -27qH^3$
 $27H^6 + 27qH^3 - p^3 = 0$
 $H^3 = \frac{-27q \pm \sqrt{(27q)^2 - 4(27)(-p^3)}}{2 \cdot 27}$
 $H^3 = \frac{-q}{2} \pm \sqrt{\frac{27^2q^2}{2^2(27^2)} + \frac{4(27)p^3}{2^2(27^2)}}$
 $H^3 = \frac{-q}{2} \pm \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}$
 $H = \sqrt[3]{\frac{-q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$

Choose the positive root for now.

5. $x = H + K$
 $x = \sqrt[3]{\frac{-q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{\frac{-q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$
 (Note that had we used the negative root in 3 the result would be the same.)
 6. $x = 3$ 7. $x = 2$ 8. $x = 2$

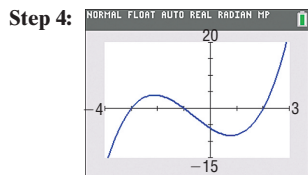
4.2 Assess Your Understanding (page 213)

5. a 6. $f(c)$ 7. b 8. F 9. 0 10. T 11. $R = f(2) = 8$; no 13. $R = f(2) = 0$; yes 15. $R = f(-3) = 0$; yes 17. $R = f(-4) = 1$; no
 19. $R = f\left(\frac{1}{2}\right) = 0$; yes 21. 7; 3 or 1 positive; 2 or 0 negative 23. 6; 2 or 0 positive; 2 or 0 negative 25. 3; 2 or 0 positive; 1 negative
 27. 4; 2 or 0 positive; 2 or 0 negative 29. 5; 0 positive; 3 or 1 negative 31. 6; 1 positive; 1 negative 33. 4; $\pm 1, \pm \frac{1}{3}$ 35. 5; $\pm 1, \pm 3$
 37. 3; $\pm 1, \pm 2, \pm \frac{1}{4}, \pm \frac{1}{2}$ 39. 4; $\pm 1, \pm 3, \pm 9, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm \frac{3}{2}, \pm \frac{9}{2}$ 41. 5; $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm \frac{1}{2}, \pm \frac{3}{2}$
 43. 4; $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{5}{3}, \pm \frac{10}{3}, \pm \frac{20}{3}, \pm \frac{1}{6}, \pm \frac{5}{6}$
 45. -1 and 1 47. -11 and 11 49. -9 and 3

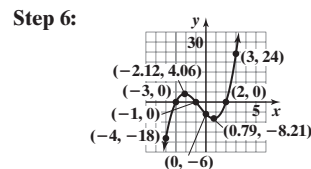


51. $-3, -1, 2; f(x) = (x + 3)(x + 1)(x - 2)$ 53. $\frac{1}{2}, 3, 3; f(x) = (2x - 1)(x - 3)^2$ 55. $-\frac{1}{3}; f(x) = (3x + 1)(x^2 + x + 1)$
 57. $4, 3 - \sqrt{5}, 3 + \sqrt{5}; f(x) = (x - 4)(x - 3 + \sqrt{5})(x - 3 - \sqrt{5})$ 59. $-2, -1, 1, 1; f(x) = (x + 2)(x + 1)(x - 1)^2$
 61. $-\frac{7}{3}, -1, \frac{2}{7}, 2; f(x) = (x - 2)(x + 1)(3x + 7)(7x - 2)$ 63. $3, \frac{5 + \sqrt{17}}{2}, \frac{5 - \sqrt{17}}{2}; f(x) = (x - 3)\left(x - \frac{5 + \sqrt{17}}{2}\right)\left(x - \frac{5 - \sqrt{17}}{2}\right)$
 65. $-\frac{1}{2}, \frac{1}{2}; f(x) = (2x + 1)(2x - 1)(x^2 + 2)$ 67. $\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 2; f(x) = (x - 2)(\sqrt{2}x - 1)(\sqrt{2}x + 1)(2x^2 + 1)$ 69. $-5.9, -0.3, 3$
 71. $-3.8, 4.5$ 73. $-43.5, 1, 23$ 75. $\{-1, 2\}$ 77. $\left\{\frac{2}{3}, -1 + \sqrt{2}, -1 - \sqrt{2}\right\}$ 79. $\left\{\frac{1}{3}, \sqrt{5}, -\sqrt{5}\right\}$ 81. $\{-3, -2\}$ 83. $\left\{-\frac{1}{3}\right\}$
 85. $f(0) = -1; f(1) = 10; \text{Zero: } 0.21$ 87. $f(-5) = -58; f(-4) = 2; \text{Zero: } -4.04$ 89. $f(1.4) = -0.17536; f(1.5) = 1.40625; \text{Zero: } 1.41$

91. **Step 1:** $y = x^3$
Step 2: x-intercepts: $-3, -1, 2$;
 y-intercept: -6
Step 3: $-3, -1, 2$: multiplicity 1, crosses

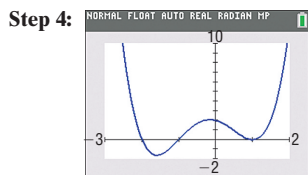


Step 5: $(-2.12, 4.06); (0.79, -8.21)$

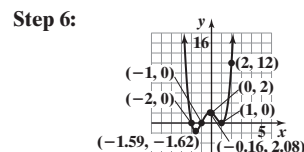


Step 7: Domain and range: $(-\infty, \infty)$
Step 8: Increasing: $(-\infty, -2.12], [0.79, \infty)$
 Decreasing: $[-2.12, 0.79]$

93. **Step 1:** $y = x^4$
Step 2: x-intercepts: $-2, -1, 1$;
 y-intercept: 2
Step 3: $-2, -1$: multiplicity 1, crosses;
 1 : multiplicity 2, touches

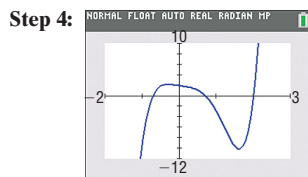


Step 5: $(-1.59, -1.62), (-0.16, 2.08), (1, 0)$

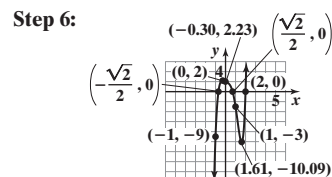


Step 7: Domain: $(-\infty, \infty)$;
 Range: $[-1.62, \infty)$
Step 8: Increasing: $[-1.59, -0.16], [1, \infty)$
 Decreasing: $(-\infty, -1.59], [-0.16, 1]$

95. **Step 1:** $y = 4x^5$
Step 2: x-intercepts: $-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 2$;
 y-intercept: 2
Step 3: $-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 2$: multiplicity 1, crosses

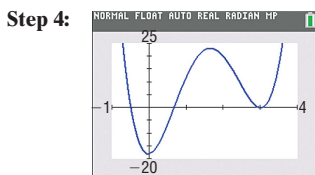


Step 5: $(-0.30, 2.23); (1.61, -10.09)$

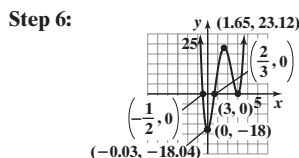


Step 7: Domain and range: $(-\infty, \infty)$
Step 8: Increasing: $(-\infty, -0.30], [1.61, \infty)$
 Decreasing: $[-0.30, 1.61]$

97. **Step 1:** $y = 6x^4$
Step 2: x-intercepts: $-\frac{1}{2}, \frac{2}{3}, 3$;
 y-intercept: -18
Step 3: $-\frac{1}{2}, \frac{2}{3}$: multiplicity 1, crosses;
 3 : multiplicity 2, touches



Step 5: $(-0.03, -18.04), (1.65, 23.12), (3, 0)$



Step 7: Domain: $(-\infty, \infty)$;
 Range: $[-18.04, \infty)$
Step 8: Increasing: $[-0.03, 1.65], [3, \infty)$;
 Decreasing: $(-\infty, -0.03], [1.65, 3]$

99. $k = 5$ 101. -7 103. If $f(x) = x^n - c^n$, then $f(c) = c^n - c^n = 0$; so $x - c$ is a factor of f . 105. 5 107. 7 in.
 109. All the potential rational zeros are integers, so r is either an integer or is not a rational zero (and is, therefore, irrational). 111. 0.215
 113. No; by the Rational Zeros Theorem, $\frac{1}{3}$ is not a potential rational zero. 115. No; by the Rational Zeros Theorem, $\frac{2}{3}$ is not a potential rational zero.
 116. $(-1, 13)$ 117. $f(x) = -3(x - 5)^2 + 71$ 118. $(0, -2\sqrt{3}), (0, 2\sqrt{3}), (4, 0)$ 119. $[-3, 2]$ and $[5, \infty)$

4.3 Assess Your Understanding (page 220)

3. one 4.3 $-4i$ 5. T 6. F 7. $4 + i$ 9. $-i, 1 - i$ 11. $-i, -2i$ 13. $-i$ 15. $2 - i, -3 + i$ 17. $f(x) = x^4 - 14x^3 + 77x^2 - 200x + 208; a = 1$
 19. $f(x) = x^5 - 4x^4 + 7x^3 - 8x^2 + 6x - 4; a = 1$ 21. $f(x) = x^4 - 6x^3 + 10x^2 - 6x + 9; a = 1$ 23. $-2i, 4$ 25. $2i, -3, \frac{1}{2}$ 27. $3 + 2i, -2, 5$
 29. $4i, -\sqrt{11}, \sqrt{11}, -\frac{2}{3}$ 31. $1, -\frac{1}{2} - \frac{\sqrt{3}}{2}i, -\frac{1}{2} + \frac{\sqrt{3}}{2}i; f(x) = (x - 1)\left(x + \frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\left(x + \frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$
 33. $2, 3 - 2i, 3 + 2i; f(x) = (x - 2)(x - 3 + 2i)(x - 3 - 2i)$ 35. $-i, i, -2i, 2i; f(x) = (x + i)(x - i)(x + 2i)(x - 2i)$

37. $-5i, 5i, -3, 1; f(x) = (x + 5i)(x - 5i)(x + 3)(x - 1)$ 39. $-4, \frac{1}{3}, 2 - 3i, 2 + 3i; f(x) = 3(x + 4)\left(x - \frac{1}{3}\right)(x - 2 + 3i)(x - 2 - 3i)$

41. 130 43. (a) $f(x) = (x^2 - \sqrt{2}x + 1)(x^2 + \sqrt{2}x + 1)$ (b) $-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i, -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i, \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

45. Zeros that are complex numbers must occur in conjugate pairs; or a polynomial with real coefficients of odd degree must have at least one real zero.

47. If the remaining zero were a complex number, its conjugate would also be a zero, creating a polynomial of degree 5.

49. 50. -22 51. $6x^3 - 13x^2 - 13x + 20$ 52. perpendicular

4.4 Assess Your Understanding (page 229)

5. F 6. horizontal asymptote 7. vertical asymptote 8. proper 9. T 10. F 11. $y = 0$ 12. T 13. d 14. a

15. All real numbers except 3; $\{x|x \neq 3\}$ 17. All real numbers except 2 and -4 ; $\{x|x \neq 2, x \neq -4\}$

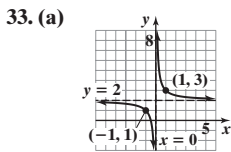
19. All real numbers except $-\frac{1}{2}$ and 3; $\{x|x \neq -\frac{1}{2}, x \neq 3\}$ 21. All real numbers except 2; $\{x|x \neq 2\}$ 23. All real numbers

25. All real numbers except -3 and 3; $\{x|x \neq -3, x \neq 3\}$

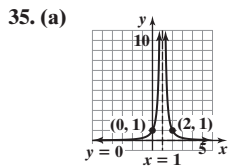
27. (a) Domain: $\{x|x \neq 2\}$; range: $\{y|y \neq 1\}$ (b) $(0, 0)$ (c) $y = 1$ (d) $x = 2$ (e) None

29. (a) Domain: $\{x|x \neq 0\}$; range: all real numbers (b) $(-1, 0), (1, 0)$ (c) None (d) $x = 0$ (e) $y = 2x$

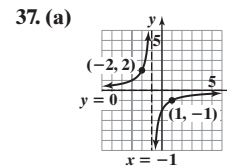
31. (a) Domain: $\{x|x \neq -2, x \neq 2\}$; range: $\{y|y \leq 0, y > 1\}$ (b) $(0, 0)$ (c) $y = 1$ (d) $x = -2, x = 2$ (e) None



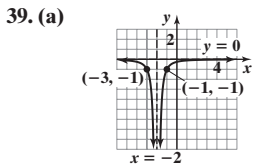
(b) Domain: $\{x|x \neq 0\}$; range: $\{y|y \neq 2\}$
(c) Vertical asymptote: $x = 0$;
horizontal asymptote: $y = 2$



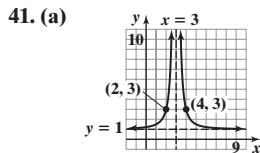
(b) Domain: $\{x|x \neq 1\}$; range: $\{y|y > 0\}$
(c) Vertical asymptote: $x = 1$;
horizontal asymptote: $y = 0$



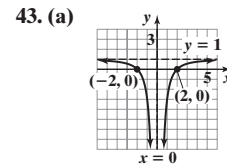
(b) Domain: $\{x|x \neq -1\}$; range: $\{y|y \neq 0\}$
(c) Vertical asymptote: $x = -1$;
horizontal asymptote: $y = 0$



(b) Domain: $\{x|x \neq -2\}$; range: $\{y|y < 0\}$
(c) Vertical asymptote: $x = -2$;
horizontal asymptote: $y = 0$



(b) Domain: $\{x|x \neq 3\}$; range: $\{y|y > 1\}$
(c) Vertical asymptote: $x = 3$;
horizontal asymptote: $y = 1$



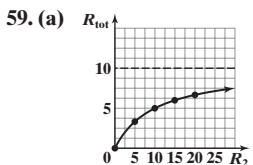
(b) Domain: $\{x|x \neq 0\}$; range: $\{y|y < 1\}$
(c) Vertical asymptote: $x = 0$;
horizontal asymptote: $y = 1$

45. Vertical asymptote: $x = -4$; horizontal asymptote: $y = 3$ 47. Vertical asymptote: $x = 3$; oblique asymptote: $y = x + 5$

49. Vertical asymptotes: $x = 1, x = -1$; horizontal asymptote: $y = 0$ 51. Vertical asymptote: $x = -\frac{1}{3}$; horizontal asymptote: $y = \frac{2}{3}$

53. No asymptotes 55. Vertical asymptote: $x = 0$; no horizontal or oblique asymptote

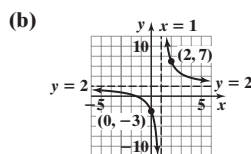
57. (a) 9.8208 m/sec^2 (b) 9.8195 m/sec^2 (c) 9.7936 m/sec^2 (d) $y = 0$ (e) \emptyset



(b) Horizontal: $R_{\text{tot}} = 10$; as the resistance of R_2 increases without bound, the total resistance approaches 10 ohms, the resistance R_1 .

(c) $R_1 \approx 103.5$ ohms

61. (a) $R(x) = 2 + \frac{5}{x-1} = 5\left(\frac{1}{x-1}\right) + 2$



(c) Vertical asymptote: $x = 1$;
horizontal asymptote: $y = 2$

67. $x = 5$ 68. -30

69. x -axis symmetry

70. $(-3, 11), (2, -4)$

4.5 Assess Your Understanding (page 240)

2. False 3. c 4. True 5. (a) $\{x|x \neq 2\}$ (b) 0 6. a

7. **Step 1:** Domain: $\{x|x \neq 0, x \neq -4\}$

Step 2: R is in lowest terms

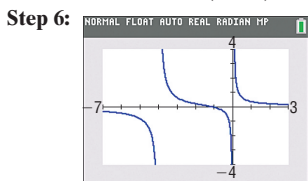
Step 3: no y -intercept; x -intercept: -1

Step 4: R is in lowest terms;

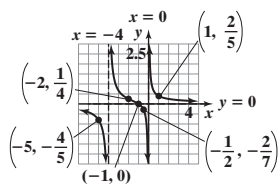
vertical asymptotes: $x = 0, x = -4$

Step 5: Horizontal asymptote: $y = 0$,

intersected at $(-1, 0)$



Step 7:



13. **Step 1:** $P(x) = \frac{(x^2 + x + 1)(x^2 - x + 1)}{(x + 1)(x - 1)}$;

domain: $\{x|x \neq -1, x \neq 1\}$

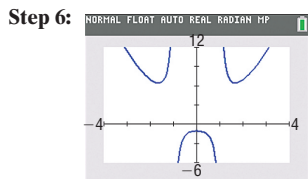
Step 2: P is in lowest terms

Step 3: y -intercept: -1 ; no x -intercept

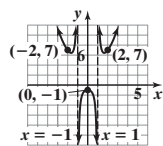
Step 4: P is in lowest terms;

vertical asymptotes: $x = -1, x = 1$

Step 5: No horizontal or oblique asymptote



Step 7:



19. **Step 1:** $G(x) = \frac{x}{(x + 2)(x - 2)}$;

domain: $\{x|x \neq -2, x \neq 2\}$

Step 2: G is in lowest terms

Step 3: y -intercept: 0 ; x -intercept: 0

Step 4: G is in lowest terms;

vertical asymptotes: $x = -2, x = 2$

Step 5: Horizontal asymptote: $y = 0$,

intersected at $(0, 0)$

9. **Step 1:** $R(x) = \frac{3(x + 1)}{2(x + 2)}$;

domain: $\{x|x \neq -2\}$

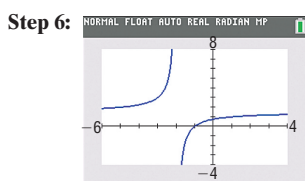
Step 2: R is in lowest terms

Step 3: y -intercept: $\frac{3}{4}$; x -intercept: -1

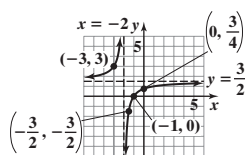
Step 4: R is in lowest terms;

vertical asymptote: $x = -2$

Step 5: Horizontal asymptote: $y = \frac{3}{2}$,
not intersected



Step 7:



15. **Step 1:** $H(x) = \frac{(x - 1)(x^2 + x + 1)}{(x + 3)(x - 3)}$;

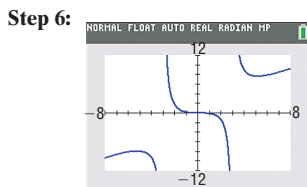
domain: $\{x|x \neq -3, x \neq 3\}$

Step 2: H is in lowest terms

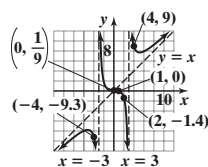
Step 3: y -intercept: $\frac{1}{9}$; x -intercept: 1

Step 4: H is in lowest terms; vertical asymptotes: $x = 3, x = -3$

Step 5: Oblique asymptote: $y = x$,
intersected at $(\frac{1}{9}, \frac{1}{9})$



Step 7:



11. **Step 1:** $R(x) = \frac{3}{(x + 2)(x - 2)}$;

domain: $\{x|x \neq -2, x \neq 2\}$

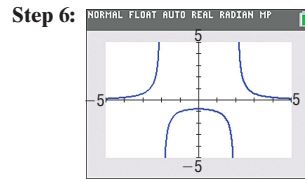
Step 2: R is in lowest terms

Step 3: y -intercept: $-\frac{3}{4}$; no x -intercept

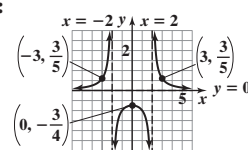
Step 4: R is in lowest terms;

vertical asymptotes: $x = 2, x = -2$

Step 5: Horizontal asymptote: $y = 0$, not intersected



Step 7:



17. **Step 1:** $R(x) = \frac{x^2}{(x + 3)(x - 2)}$;

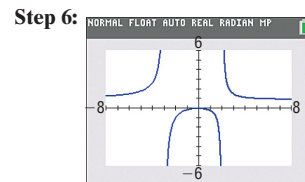
domain: $\{x \neq -3, x \neq 2\}$

Step 2: R is in lowest terms

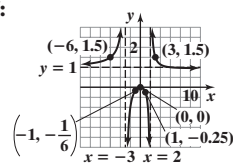
Step 3: y -intercept: 0 ; x -intercept: 0

Step 4: R is in lowest terms; vertical asymptotes: $x = 2, x = -3$

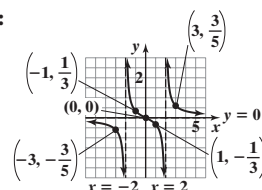
Step 5: Horizontal asymptote: $y = 1$,
intersected at $(6, 1)$



Step 7:



Step 7:



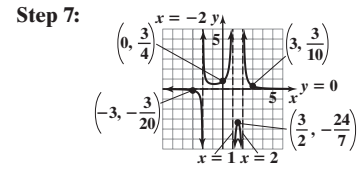
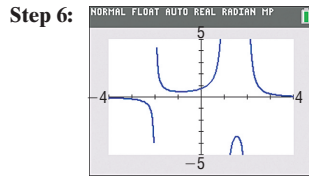
21. Step 1: $R(x) = \frac{3}{(x-1)(x+2)(x-2)}$;
domain: $\{x|x \neq 1, x \neq -2, x \neq 2\}$

Step 2: R is in lowest terms

Step 3: y-intercept: $\frac{3}{4}$; no x-intercept

Step 4: R is in lowest terms; vertical asymptotes: $x = -2, x = 1, x = 2$

Step 5: Horizontal asymptote: $y = 0$, not intersected



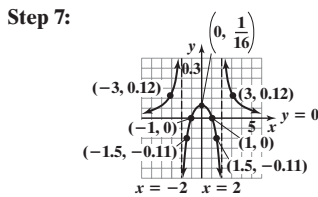
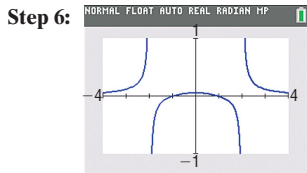
23. Step 1: $H(x) = \frac{(x+1)(x-1)}{(x^2+4)(x+2)(x-2)}$;
domain: $\{x|x \neq -2, x \neq 2\}$

Step 2: H is in lowest terms

Step 3: y-intercept: $\frac{1}{16}$; x-intercepts: $-1, 1$

Step 4: H is in lowest terms; vertical asymptotes: $x = -2, x = 2$

Step 5: Horizontal asymptote: $y = 0$, intersected at $(-1, 0)$ and $(1, 0)$



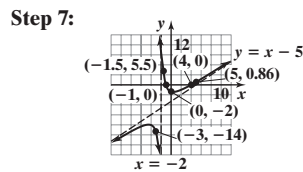
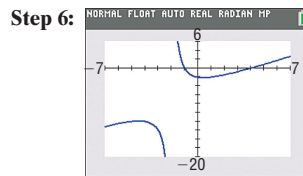
25. Step 1: $F(x) = \frac{(x+1)(x-4)}{x+2}$;
domain: $\{x|x \neq -2\}$

Step 2: F is in lowest terms

Step 3: y-intercept: -2 ; x-intercepts: $-1, 4$

Step 4: F is in lowest terms; vertical asymptote: $x = -2$

Step 5: Oblique asymptote: $y = x - 5$, not intersected



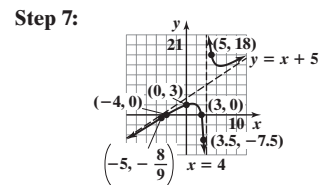
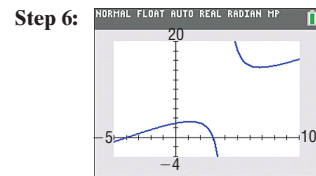
27. Step 1: $R(x) = \frac{(x+4)(x-3)}{x-4}$;
domain: $\{x|x \neq 4\}$

Step 2: R is in lowest terms

Step 3: y-intercept: 3 ; x-intercepts: $-4, 3$

Step 4: R is in lowest terms; vertical asymptote: $x = 4$

Step 5: Oblique asymptote: $y = x + 5$, not intersected



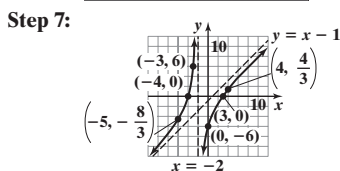
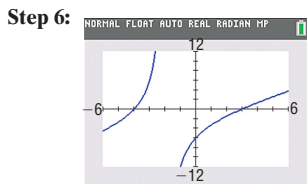
29. Step 1: $F(x) = \frac{(x+4)(x-3)}{x+2}$;
domain: $\{x|x \neq -2\}$

Step 2: F is in lowest terms

Step 3: y-intercept: -6 ; x-intercepts: $-4, 3$

Step 4: F is in lowest terms; vertical asymptote: $x = -2$

Step 5: Oblique asymptote: $y = x - 1$, not intersected



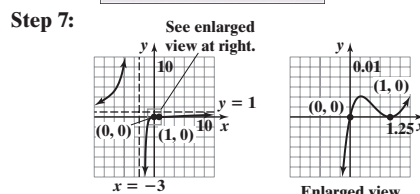
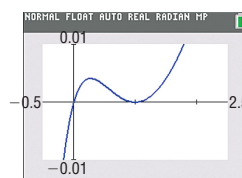
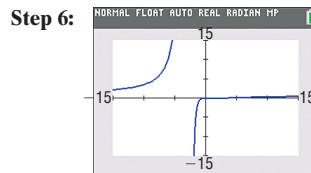
31. Step 1: Domain: $\{x|x \neq -3\}$

Step 2: R is in lowest terms

Step 3: y-intercept: 0 ; x-intercepts: $0, 1$

Step 4: Vertical asymptote: $x = -3$

Step 5: Horizontal asymptote: $y = 1$, not intersected



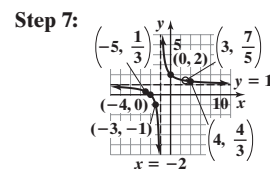
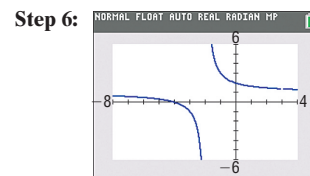
33. Step 1: $R(x) = \frac{(x+4)(x-3)}{(x+2)(x-3)}$;
domain: $\{x|x \neq -2, x \neq 3\}$

Step 2: In lowest terms, $R(x) = \frac{x+4}{x+2}$

Step 3: y-intercept: 2 ; x-intercept: -4

Step 4: Vertical asymptote: $x = -2$; hole at $(3, \frac{7}{5})$

Step 5: Horizontal asymptote: $y = 1$, not intersected



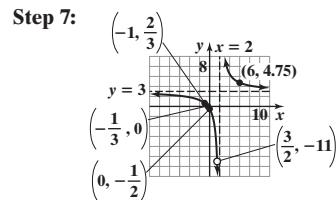
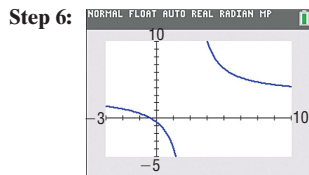
35. Step 1: $R(x) = \frac{(3x + 1)(2x - 3)}{(x - 2)(2x - 3)}$;
 domain: $\left\{x \mid x \neq \frac{3}{2}, x \neq 2\right\}$

Step 2: In lowest terms, $R(x) = \frac{3x + 1}{x - 2}$

Step 3: y-intercept: $-\frac{1}{2}$; x-intercept: $-\frac{1}{3}$

Step 4: Vertical asymptote: $x = 2$;
 hole at $\left(\frac{3}{2}, -11\right)$

Step 5: Horizontal asymptote: $y = 3$;
 not intersected



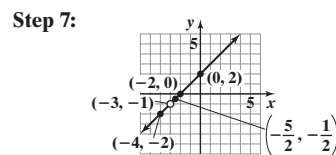
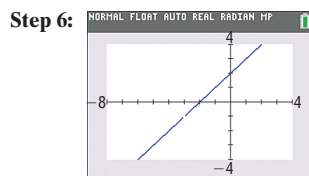
37. Step 1: $R(x) = \frac{(x + 3)(x + 2)}{x + 3}$;
 domain: $\{x \mid x \neq -3\}$

Step 2: In lowest terms, $R(x) = x + 2$

Step 3: y-intercept: 2; x-intercept: -2

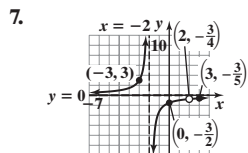
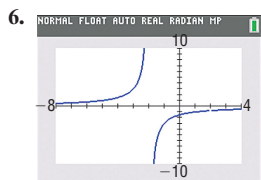
Step 4: Vertical asymptote: none;
 hole at $(-3, -1)$

Step 5: No horizontal or oblique asymptote



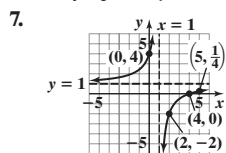
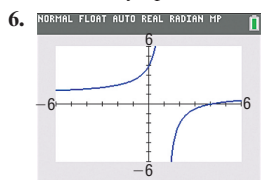
39. 1. $H(x) = \frac{-3(x - 2)}{(x - 2)(x + 2)}$; domain: $\{x \mid x \neq -2, x \neq 2\}$ **2.** In lowest terms, $H(x) = \frac{-3}{x + 2}$ **3.** y-intercept: $-\frac{3}{2}$; no x-intercept

4. Vertical asymptote: $x = -2$; hole at $\left(2, -\frac{3}{4}\right)$ **5.** Horizontal asymptote: $y = 0$; not intersected



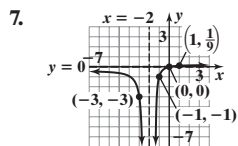
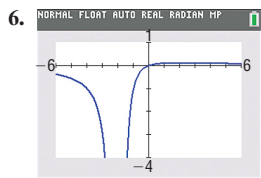
41. 1. $F(x) = \frac{(x - 1)(x - 4)}{(x - 1)^2}$; domain: $\{x \mid x \neq 1\}$ **2.** In lowest terms, $F(x) = \frac{x - 4}{x - 1}$ **3.** y-intercept: 4; x-intercept: 4

4. Vertical asymptote: $x = 1$ **5.** Horizontal asymptote: $y = 1$; not intersected



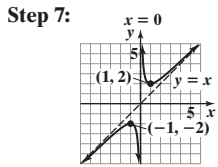
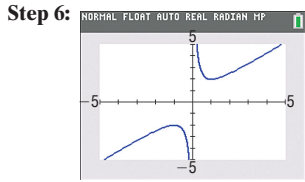
43. 1. $G(x) = \frac{x}{(x + 2)^2}$; domain: $\{x \mid x \neq -2\}$ **2.** G is in lowest terms **3.** y-intercept: 0; x-intercept: 0 **4.** Vertical asymptote: $x = -2$

5. Horizontal asymptote: $y = 0$; intersected at $(0, 0)$



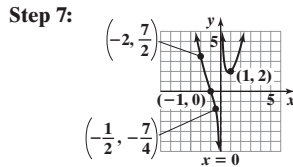
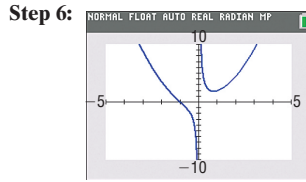
45. **Step 1:** $f(x) = \frac{x^2 + 1}{x}$; domain: $\{x|x \neq 0\}$

- Step 2:** f is in lowest terms
Step 3: no y -intercept; no x -intercepts
Step 4: f is in lowest terms;
 vertical asymptote: $x = 0$
Step 5: Oblique asymptote: $y = x$,
 not intersected



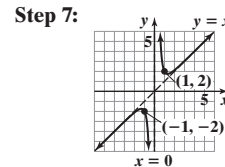
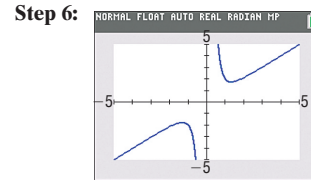
47. **Step 1:** $f(x) = \frac{x^3 + 1}{x} = \frac{(x + 1)(x^2 - x + 1)}{x}$;

- domain: $\{x|x \neq 0\}$
Step 2: f is in lowest terms
Step 3: no y -intercept; x -intercept: -1
Step 4: f is in lowest terms;
 vertical asymptote: $x = 0$
Step 5: No horizontal or oblique asymptote

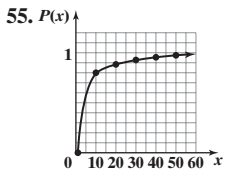


49. **Step 1:** $f(x) = \frac{x^4 + 1}{x^3}$; domain: $\{x|x \neq 0\}$

- Step 2:** f is in lowest terms
Step 3: no y -intercept; no x -intercepts
Step 4: f is in lowest terms;
 vertical asymptote: $x = 0$
Step 5: Oblique asymptote: $y = x$,
 not intersected

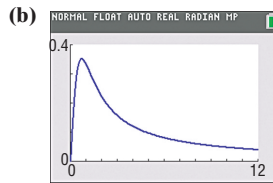


51. One possibility: $R(x) = \frac{x^2}{x^2 - 4}$ 53. One possibility: $R(x) = \frac{(x - 1)(x - 3)(x^2 + \frac{4}{3})}{(x + 1)^2(x - 2)^2}$



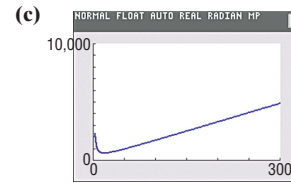
The likelihood of your ball being chosen decreases very quickly and approaches 0 as the number of attendees, x , increases.

57. (a) t -axis; $C(t) \rightarrow 0$



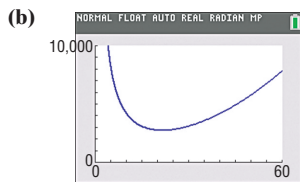
(c) 0.71 h after injection

59. (a) $C(x) = 16x + \frac{5000}{x} + 100$ (b) $x > 0$



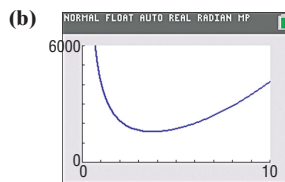
(d) Approximately 177 ft by 56.6 ft (longer side parallel to river)

61. (a) $S(x) = 2x^2 + \frac{40,000}{x}$



- (c) 2784.95 in.²
 (d) 21.54 in. \times 21.54 in. \times 21.54 in.
 (e) To minimize the cost of materials needed for construction

63. (a) $C(r) = 12\pi r^2 + \frac{4000}{r}$



The cost is smallest when $r = 3.76$ cm.

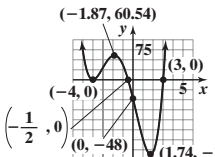
65. No. Each function is a quotient of polynomials, but it is not written in lowest terms. Each function is undefined for $x = 1$; each graph has a hole at $x = 1$. 71. If there is a common factor between the numerator and the denominator, and the factor yields a real zero, then the graph will have a hole.

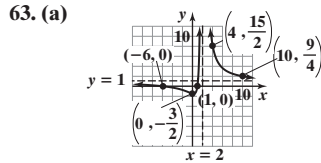
72. $(2, -5)$ 73. $y = |x| - 4$ 74. $\frac{17}{2}$ 75. $g(3) = 6$

4.6 Assess Your Understanding (page 248)

3. c 4. F 5. (a) $\{x|0 < x < 1 \text{ or } x > 2\}$; $(0, 1) \cup (2, \infty)$ (b) $\{x|x \leq 0 \text{ or } 1 \leq x \leq 2\}$; $(-\infty, 0] \cup [1, 2]$
 7. (a) $\{x|-1 < x < 0 \text{ or } x > 1\}$; $(-1, 0) \cup (1, \infty)$ (b) $\{x|x < -1 \text{ or } 0 \leq x < 1\}$; $(-\infty, -1) \cup [0, 1)$
 9. $\{x|x < 0 \text{ or } 0 < x < 3\}$; $(-\infty, 0) \cup (0, 3)$ 11. $\{x|x \leq 1\}$; $(-\infty, 1]$ 13. $\{x|x \leq -2 \text{ or } x \geq 2\}$; $(-\infty, -2] \cup [2, \infty)$
 15. $\{x|-4 < x < -1 \text{ or } x > 0\}$; $(-4, -1) \cup (0, \infty)$ 17. $\{x|-2 < x \leq -1\}$; $(-2, -1]$ 19. $\{x|x < -2\}$; $(-\infty, -2)$ 21. $\{x|x > 4\}$; $(4, \infty)$
 23. $\{x|-4 < x < 0 \text{ or } x > 0\}$; $(-4, 0) \cup (0, \infty)$ 25. $\{x|x \leq 1 \text{ or } 2 \leq x \leq 3\}$; $(-\infty, 1] \cup [2, 3]$ 27. $\{x|-1 < x < 0 \text{ or } x > 3\}$; $(-1, 0) \cup (3, \infty)$
 29. $\{x|x < -1 \text{ or } x > 1\}$; $(-\infty, -1) \cup (1, \infty)$ 31. $\{x|x < -1 \text{ or } x > 1\}$; $(-\infty, -1) \cup (1, \infty)$ 33. $\{x|x < -1 \text{ or } x > 1\}$; $(-\infty, -1) \cup (1, \infty)$
 35. $\{x|x \leq -1 \text{ or } 0 < x \leq 1\}$; $(-\infty, -1] \cup (0, 1]$ 37. $\{x|x < -1 \text{ or } x > 1\}$; $(-\infty, -1) \cup (1, \infty)$ 39. $\{x|x < 2\}$; $(-\infty, 2)$
 41. $\{x|-2 < x \leq 9\}$; $(-2, 9]$ 43. $\{x|x < 2 \text{ or } 3 < x < 5\}$; $(-\infty, 2) \cup (3, 5)$

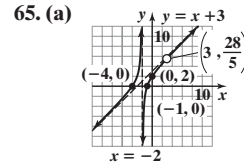
45. $\{x|x < -5 \text{ or } -4 \leq x \leq -3 \text{ or } x = 0 \text{ or } x > 1\}$; $(-\infty, -5) \cup [-4, -3] \cup \{0\} \cup (1, \infty)$
 47. $\left\{x \left| -\frac{1}{2} < x < 1 \text{ or } x > 3 \right.\right\}$; $\left(-\frac{1}{2}, 1\right) \cup (3, \infty)$ 49. $\{x|-1 < x < 3 \text{ or } x > 5\}$; $(-1, 3) \cup (5, \infty)$
 51. $\left\{x \left| x \leq -4 \text{ or } x \geq \frac{1}{2} \right.\right\}$; $(-\infty, -4] \cup \left[\frac{1}{2}, \infty\right)$ 53. $\{x|x < 3 \text{ or } x \geq 7\}$; $(-\infty, 3) \cup [7, \infty)$ 55. $\{x|x < 2\}$; $(-\infty, 2)$
 57. $\left\{x \left| x < -\frac{2}{3} \text{ or } 0 < x < \frac{3}{2} \right.\right\}$; $(-\infty, -\frac{2}{3}) \cup \left(0, \frac{3}{2}\right)$ 59. $\{x|x \leq -3 \text{ or } 0 \leq x \leq 3\}$; $(-\infty, -3] \cup [0, 3]$

61. (a) $-4, -\frac{1}{2}, 3$
 (b) $f(x) = (x + 4)^2(2x + 1)(x - 3)$
 (c) 
 (d) $\left(-\frac{1}{2}, 3\right)$

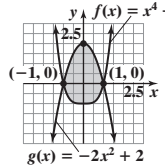


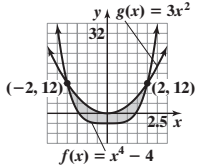
(b) $(-\infty, -6] \cup [1, 2) \cup (2, \infty)$

67. $\{x|x > 4\}$; $(4, \infty)$
 69. $\{x|x \leq -2 \text{ or } x \geq 2\}$; $(-\infty, -2] \cup [2, \infty)$
 71. $\{x|x < -4 \text{ or } x \geq 2\}$; $(-\infty, -4) \cup [2, \infty)$



(b) $[-4, -2) \cup [-1, 3) \cup (3, \infty)$

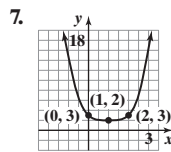
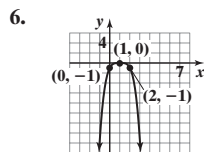
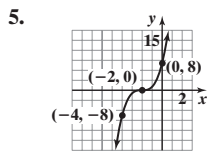
73. 
 $f(x) \leq g(x)$ if $-1 \leq x \leq 1$

75. 
 $f(x) \leq g(x)$ if $-2 \leq x \leq 2$

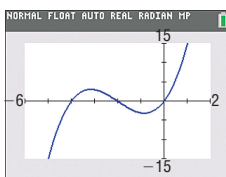
77. Produce at least 250 bicycles
 79. (a) The stretch is less than 39 ft.
 (b) The ledge should be at least 84 ft above the ground for a 150-lb jumper.
 81. At least 50 students must attend. 86. $(0, -4), (0, 4), (9, 0)$
 87. $x^2 - x - 4$ 88. $(-\infty, \infty)$ 89. $(0, 4), (1.33, 2.81)$

Review Exercises (page 252)

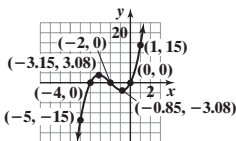
1. Polynomial of degree 5 2. Rational 3. Neither 4. Polynomial of degree 0



8. Step 1: $y = x^3$
 Step 2: x-intercepts: $-4, -2, 0$; y-intercept: 0
 Step 3: $-4, -2, 0$; multiplicity 1; crosses
 Step 4:

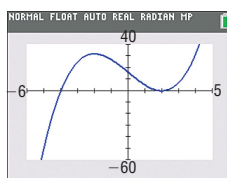


- Step 5: $(-3.15, 3.08), (-0.85, -3.08)$
 Step 6:

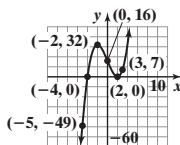


- Step 7: Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$
 Step 8: Increasing on $(-\infty, -3.15]$ and $[-0.85, \infty)$
 Decreasing on $[-3.15, -0.85]$

9. Step 1: $y = x^3$
 Step 2: x-intercepts: $-4, 2$; y-intercept: 16
 Step 3: -4 ; multiplicity 1; crosses;
 2; multiplicity 2; touches
 Step 4:

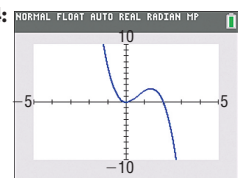


- Step 5: $(-2, 32), (2, 0)$
 Step 6:

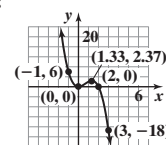


- Step 7: Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$
 Step 8: Increasing on $(-\infty, -2]$ and $[2, \infty)$
 Decreasing on $[-2, 2]$

10. Step 1: $y = -2x^3$
 Step 2: x-intercepts: 0, 2; y-intercept: 0
 Step 3: 0; multiplicity 2; touches;
 2; multiplicity 1; crosses
 Step 4:



- Step 5: $(0, 0), (1.33, 2.37)$
 Step 6:

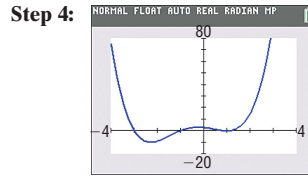


- Step 7: Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$
 Step 8: Increasing on $[0, 1.33]$
 Decreasing on $(-\infty, 0]$ and $[1.33, \infty)$

11. Step 1: $y = x^4$

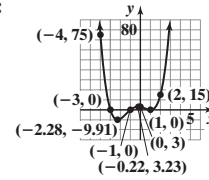
Step 2: x -intercepts: $-3, -1, 1$;
 y -intercept: 3

Step 3: $-3, -1$: multiplicity 1; crosses;
 1 : multiplicity 2; touches



Step 5: $(-2.28, -9.91), (-0.22, 3.23), (1, 0)$

Step 6:



Step 7: Domain: $(-\infty, \infty)$; Range: $[-9.91, \infty)$

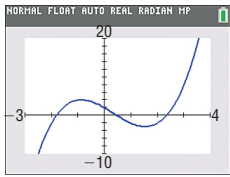
Step 8: Increasing on $[-2.28, -0.22]$ and $[1, \infty)$
Decreasing on $(-\infty, -2.28]$ and $[-0.22, 1]$

12. $R = 10$; g is not a factor of f . 13. $R = 0$; g is a factor of f . 14. $f(4) = 47,105$ 15. 4, 2, or 0 positive; 2 or 0 negative 16. 1 positive; 2 or 0 negative

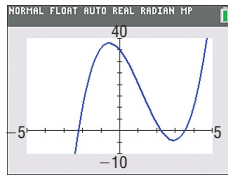
17. $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{3}, \pm \frac{1}{4}, \pm \frac{3}{4}, \pm \frac{1}{6}, \pm \frac{1}{12}$ 18. $-2, 1, 4$; $f(x) = (x + 2)(x - 1)(x - 4)$ 19. $\frac{1}{2}$, multiplicity 2; -2 ; $f(x) = 4\left(x - \frac{1}{2}\right)^2(x + 2)$

20. 2, multiplicity 2; $f(x) = (x - 2)^2(x^2 + 5)$ 21. $\{-3, 2\}$ 22. $\left\{-3, -1, -\frac{1}{2}, 1\right\}$

23. -2 and 3



24. -5 and 5



25. $f(0) = -1$; $f(1) = 1$ 26. $f(0) = -1$; $f(1) = 1$ 27. 1.52 28. 0.93

29. $4 - i$; $f(x) = x^3 - 14x^2 + 65x - 102$ 30. $-i, 1 - i$; $f(x) = x^4 - 2x^3 + 3x^2 - 2x + 2$

31. $-2, 1, 4$; $f(x) = (x + 2)(x - 1)(x - 4)$

32. $-2, \frac{1}{2}$ (multiplicity 2); $f(x) = 4(x + 2)\left(x - \frac{1}{2}\right)^2$

33. 2 (multiplicity 2), $-\sqrt{5}i, \sqrt{5}i$; $f(x) = (x + \sqrt{5}i)(x - \sqrt{5}i)(x - 2)^2$ 34. $-3, 2, -\frac{\sqrt{2}}{2}i, \frac{\sqrt{2}}{2}i$; $f(x) = 2(x + 3)(x - 2)\left(x + \frac{\sqrt{2}}{2}i\right)\left(x - \frac{\sqrt{2}}{2}i\right)$

35. Domain: $\{x \mid x \neq -3, x \neq 3\}$; horizontal asymptote: $y = 0$; vertical asymptotes: $x = -3, x = 3$

36. Domain: $\{x \mid x \neq -2\}$; horizontal asymptote: $y = 1$; vertical asymptote: $x = -2$

37. Step 1: $R(x) = \frac{2(x - 3)}{x}$;

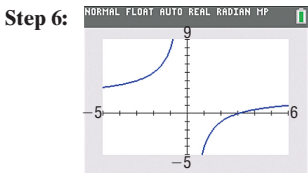
domain: $\{x \mid x \neq 0\}$

Step 2: R is in lowest terms

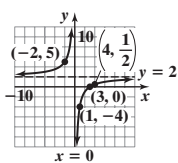
Step 3: no y -intercept; x -intercept: 3

Step 4: R is in lowest terms;
vertical asymptote: $x = 0$

Step 5: Horizontal asymptote: $y = 2$;
not intersected



Step 7:



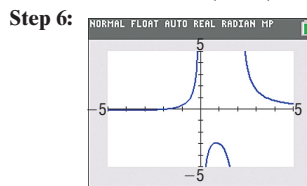
38. Step 1: Domain: $\{x \mid x \neq 0, x \neq 2\}$

Step 2: H is in lowest terms

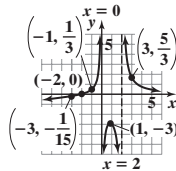
Step 3: no y -intercept; x -intercept: -2

Step 4: H is in lowest terms;
vertical asymptote: $x = 0, x = 2$

Step 5: Horizontal asymptote: $y = 0$;
intersected at $(-2, 0)$



Step 7:



39. Step 1: $R(x) = \frac{(x + 3)(x - 2)}{(x - 3)(x + 2)}$;

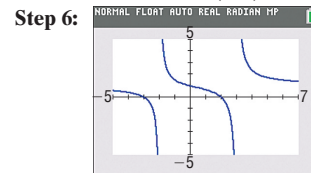
domain: $\{x \mid x \neq -2, x \neq 3\}$

Step 2: R is in lowest terms

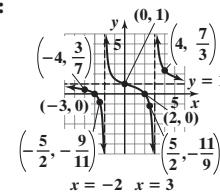
Step 3: y -intercept: 1 ; x -intercepts: $-3, 2$

Step 4: R is in lowest terms;
vertical asymptotes: $x = -2, x = 3$

Step 5: Horizontal asymptote: $y = 1$;
intersected at $(0, 1)$



Step 7:



40. Step 1: $F(x) = \frac{x^3}{(x + 2)(x - 2)}$;

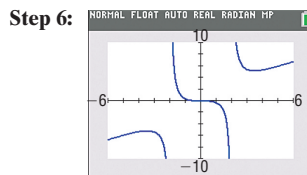
domain: $\{x \mid x \neq -2, x \neq 2\}$

Step 2: F is in lowest terms

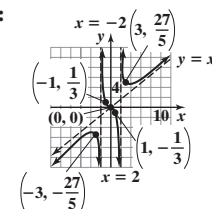
Step 3: y -intercept: 0 ; x -intercept: 0

Step 4: F is in lowest terms; vertical
asymptotes: $x = -2, x = 2$

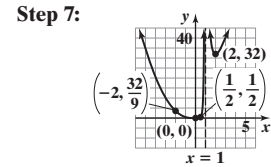
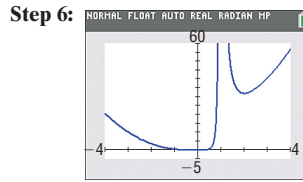
Step 5: Oblique asymptote: $y = x$;
intersected at $(0, 0)$



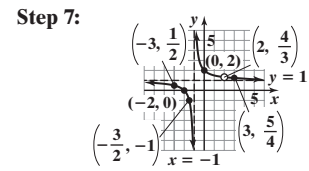
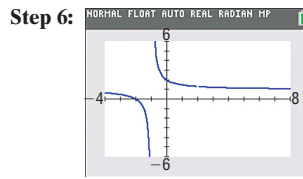
Step 7:



- 41. Step 1:** Domain: $\{x|x \neq 1\}$
Step 2: R is in lowest terms
Step 3: y -intercept: 0; x -intercept: 0
Step 4: R is in lowest terms;
 vertical asymptote: $x = 1$
Step 5: No oblique or horizontal asymptote



- 42. Step 1:** $G(x) = \frac{(x+2)(x-2)}{(x+1)(x-2)}$,
 domain: $\{x|x \neq -1, x \neq 2\}$
Step 2: In lowest terms, $G(x) = \frac{x+2}{x+1}$
Step 3: y -intercept: 2; x -intercept: -2
Step 4: Vertical asymptote: $x = -1$;
 hole at $(2, \frac{4}{3})$
Step 5: Horizontal asymptote: $y = 1$,
 not intersected



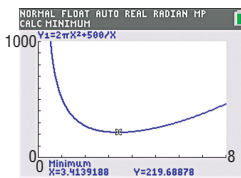
- 43. (a)** $\{-3, 2\}$ **(b)** $(-3, 2) \cup (2, \infty)$ **(c)** $(-\infty, -3] \cup \{2\}$ **(d)** $f(x) = (x-2)^2(x+3)$
44. (a) $y = 0.25$ **(b)** $x = -2, x = 2$ **(c)** $(-3, -2) \cup (-1, 2)$ **(d)** $(-\infty, -3] \cup (-2, -1] \cup (2, \infty)$ **(e)** $R(x) = \frac{x^2 + 4x + 3}{4x^2 - 16}$
45. $\{x|x < -2 \text{ or } -1 < x < 2\}; (-\infty, -2) \cup (-1, 2)$ **46.** $\{x|-4 \leq x \leq -1 \text{ or } x \geq 1\}; [-4, -1] \cup [1, \infty)$

47. $\{x|x < 1 \text{ or } x > 2\}; (-\infty, 1) \cup (2, \infty)$ **48.** $\{x|1 \leq x \leq 2 \text{ or } x > 3\}; [1, 2] \cup (3, \infty)$

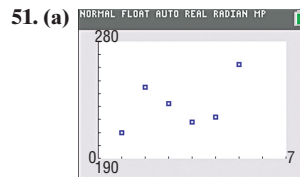
49. $\{x|x < -4 \text{ or } 2 < x < 4 \text{ or } x > 6\}; (-\infty, -4) \cup (2, 4) \cup (6, \infty)$

50. (a) $A(r) = 2\pi r^2 + \frac{500}{r}$

- (b)** 223.22 cm^2
(c) 257.08 cm^2
(d)

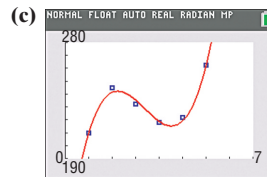


A is smallest when $r \approx 3.41$ cm.



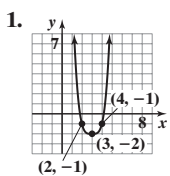
The relation appears to be cubic.

(b) $P(t) = 4.4926t^3 - 45.5294t^2 + 136.1209t + 115.4667; \approx \$928,000$

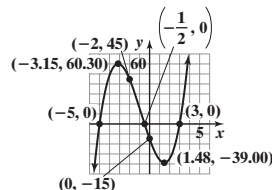


- 52. (a)** Even **(b)** Positive **(c)** Even **(d)** The graph touches the x -axis at $x = 0$, but does not cross it there. **(e)** 8

Chapter Test (page 254)

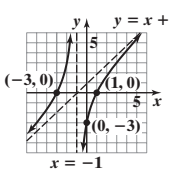


- 2. (a)** 3 **(b)** $\frac{p}{q}: \pm \frac{1}{2}, \pm 1, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm 3, \pm 5, \pm \frac{15}{2}, \pm 15$ **(c)** $-5, -\frac{1}{2}, 3; g(x) = (x+5)(2x+1)(x-3)$
(d) y -intercept: -15 ; x -intercepts: $-5, -\frac{1}{2}, 3$ **(e)** Crosses at $-5, -\frac{1}{2}, 3$ **(f)** $y = 2x^3$
(g) $(-3.15, 60.30), (1.48, -39.00)$ **(h)**



3. $4, -5i, 5i$ 4. $\left\{1, \frac{5 - \sqrt{61}}{6}, \frac{5 + \sqrt{61}}{6}\right\}$ 5. Domain: $\{x|x \neq -10, x \neq 4\}$; asymptotes: $x = -10, y = 2$

6. Domain: $\{x|x \neq -1\}$; asymptotes: $x = -1, y = x + 1$

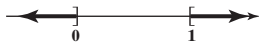
7.  8. Answers may vary. One possibility is $f(x) = x^4 - 4x^3 - 2x^2 + 20x$.

9. Answers may vary. One possibility is $r(x) = \frac{2(x-9)(x-1)}{(x-4)(x-9)}$.

10. $f(0) = 8; f(4) = -36$; Since $f(0) = 8 > 0$ and $f(4) = -36 < 0$, the Intermediate Value Theorem guarantees that there is at least one real zero between 0 and 4. 11. $\{x|x < 3 \text{ or } x > 8\}; (-\infty, 3) \cup (8, \infty)$

Cumulative Review (page 254)

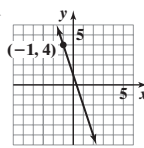
1. $\sqrt{26}$ 2. $\{x|x \leq 0 \text{ or } x \geq 1\}; (-\infty, 0] \text{ or } [1, \infty)$



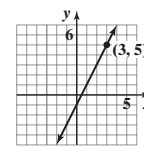
3. $\{x|-1 < x < 4\}; (-1, 4)$



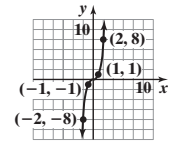
4. $f(x) = -3x + 1$



5. $y = 2x - 1$



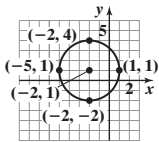
6.



7. Not a function; 3 has two images. 8. $\{0, 2, 4\}$ 9. $\left\{x \mid x \geq \frac{3}{2}\right\}; \left[\frac{3}{2}, \infty\right)$



10. Center: $(-2, 1)$; radius: 3

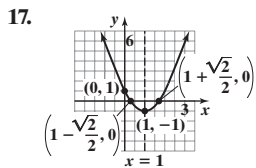
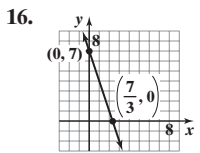


11. x-intercepts: $-3, 0, 3$; y-intercept: 0; symmetric with respect to the origin 12. $y = -\frac{2}{3}x + \frac{17}{3}$

13. Not a function; it fails the Vertical Line Test. 14. (a) 22 (b) $x^2 - 5x - 2$ (c) $-x^2 - 5x + 2$

(d) $9x^2 + 15x - 2$ (e) $2x + h + 5$ 15. (a) $\{x|x \neq 1\}$ (b) No; $(2, 7)$ is on the graph. (c) 4; $(3, 4)$ is on the graph.

(d) $\frac{7}{4}$; $\left(\frac{7}{4}, 9\right)$ is on the graph. (e) Rational



18. 6; $y = 6x - 1$ 19. (a) x-intercepts: $-5, -1, 5$; y-intercept: -3 (b) No symmetry

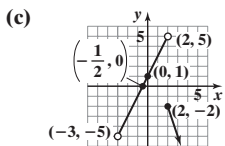
(c) Neither (d) Increasing: $(-\infty, -3]$ and $[2, \infty)$; decreasing: $[-3, 2]$

(e) Local maximum value is 5 and occurs at $x = -3$.

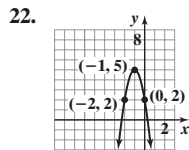
(f) Local minimum value is -6 and occurs at $x = 2$. 20. Odd

21. (a) Domain: $\{x|x > -3\}$ or $(-3, \infty)$

(b) x-intercept: $-\frac{1}{2}$; y-intercept: 1



(d) Range: $\{y|y < 5\}$ or $(-\infty, 5)$



23. (a) $(f + g)(x) = x^2 - 9x - 6$; domain: all real numbers

(b) $\left(\frac{f}{g}\right)(x) = \frac{x^2 - 5x + 1}{-4x - 7}$; domain: $\left\{x \mid x \neq -\frac{7}{4}\right\}$

24. (a) $R(x) = -\frac{1}{10}x^2 + 150x$ (b) \$14,000 (c) 750; \$56,250 (d) \$75

CHAPTER 5 Exponential and Logarithmic Functions

5.1 Assess Your Understanding (page 263)

4. composite function; $f(g(x))$ 5. F 6. c 7. a 8. F 9. (a) -1 (b) -1 (c) 8 (d) 0 (e) 8 (f) -7 11. (a) 4 (b) 5 (c) -1 (d) -2

13. (a) 98 (b) 49 (c) 4 (d) 4 15. (a) 97 (b) $-\frac{163}{2}$ (c) 1 (d) $-\frac{3}{2}$ 17. (a) $2\sqrt{2}$ (b) $2\sqrt{2}$ (c) 1 (d) 0 19. (a) $\frac{1}{17}$ (b) $\frac{1}{5}$ (c) 1 (d) $\frac{1}{2}$

21. (a) $\frac{3}{\sqrt[3]{4} + 1}$ (b) 1 (c) $\frac{6}{5}$ (d) 0 23. (a) $(f \circ g)(x) = 6x + 3$; all real numbers (b) $(g \circ f)(x) = 6x + 9$; all real numbers

(c) $(f \circ f)(x) = 4x + 9$; all real numbers (d) $(g \circ g)(x) = 9x$; all real numbers 25. (a) $(f \circ g)(x) = 3x^2 + 1$; all real numbers

(b) $(g \circ f)(x) = 9x^2 + 6x + 1$; all real numbers (c) $(f \circ f)(x) = 9x + 4$; all real numbers (d) $(g \circ g)(x) = x^4$; all real numbers

27. (a) $(f \circ g)(x) = x^4 + 8x^2 + 16$; all real numbers (b) $(g \circ f)(x) = x^4 + 4$; all real numbers (c) $(f \circ f)(x) = x^4$; all real numbers

(d) $(g \circ g)(x) = x^4 + 8x^2 + 20$; all real numbers 29. (a) $(f \circ g)(x) = \frac{3x}{2-x}$; $\{x|x \neq 0, x \neq 2\}$ (b) $(g \circ f)(x) = \frac{2(x-1)}{3}$; $\{x|x \neq 1\}$

(c) $(f \circ f)(x) = \frac{3(x-1)}{4-x}$; $\{x|x \neq 1, x \neq 4\}$ (d) $(g \circ g)(x) = x$; $\{x|x \neq 0\}$ 31. (a) $(f \circ g)(x) = \frac{4}{4+x}$; $\{x|x \neq -4, x \neq 0\}$

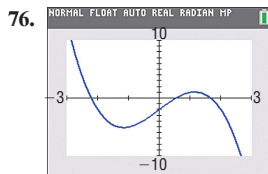
(b) $(g \circ f)(x) = \frac{-4(x-1)}{x}$; $\{x|x \neq 0, x \neq 1\}$ (c) $(f \circ f)(x) = x$; $\{x|x \neq 1\}$ (d) $(g \circ g)(x) = x$; $\{x|x \neq 0\}$

33. (a) $(f \circ g)(x) = \sqrt{2x+3}; \left\{x \mid x \geq -\frac{3}{2}\right\}$ (b) $(g \circ f)(x) = 2\sqrt{x} + 3; \{x \mid x \geq 0\}$ (c) $(f \circ f)(x) = \sqrt[3]{x}; \{x \mid x \geq 0\}$
 (d) $(g \circ g)(x) = 4x + 9$; all real numbers 35. (a) $(f \circ g)(x) = x; \{x \mid x \geq 1\}$ (b) $(g \circ f)(x) = |x|$; all real numbers
 (c) $(f \circ f)(x) = x^4 + 2x^2 + 2$; all real numbers (d) $(g \circ g)(x) = \sqrt{\sqrt{x-1}-1}; \{x \mid x \geq 2\}$ 37. (a) $(f \circ g)(x) = -\frac{4x-17}{2x-1}; \left\{x \mid x \neq 3; x \neq \frac{1}{2}\right\}$
 (b) $(g \circ f)(x) = -\frac{3x-3}{2x+8}; \{x \mid x \neq -4; x \neq -1\}$ (c) $(f \circ f)(x) = -\frac{2x+5}{x-2}; \{x \mid x \neq -1; x \neq 2\}$ (d) $(g \circ g)(x) = -\frac{3x-4}{2x-11}; \left\{x \mid x \neq \frac{11}{2}; x \neq 3\right\}$
 39. $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{2}x\right) = 2\left(\frac{1}{2}x\right) = x$; $(g \circ f)(x) = g(f(x)) = g(2x) = \frac{1}{2}(2x) = x$
 41. $(f \circ g)(x) = f(g(x)) = f(\sqrt[3]{x}) = (\sqrt[3]{x})^3 = x$; $(g \circ f)(x) = g(f(x)) = g(x^3) = \sqrt[3]{x^3} = x$
 43. $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{2}(x+6)\right) = 2\left[\frac{1}{2}(x+6)\right] - 6 = x + 6 - 6 = x$; $(g \circ f)(x) = g(f(x)) = g(2x-6) = \frac{1}{2}(2x-6+6) = \frac{1}{2}(2x) = x$
 45. $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{a}(x-b)\right) = a\left[\frac{1}{a}(x-b)\right] + b = x$; $(g \circ f)(x) = g(f(x)) = g(ax+b) = \frac{1}{a}(ax+b-b) = x$
 47. $f(x) = x^4; g(x) = 2x + 3$ (Other answers are possible.) 49. $f(x) = \sqrt{x}; g(x) = x^2 + 1$ (Other answers are possible.)
 51. $f(x) = |x|; g(x) = 2x + 1$ (Other answers are possible.) 53. $(f \circ g)(x) = 11; (g \circ f)(x) = 2$ 55. -3, 3 57. (a) $(f \circ g)(x) = acx + ad + b$
 (b) $(g \circ f)(x) = acx + bc + d$ (c) The domains of both $f \circ g$ and $g \circ f$ are all real numbers. (d) $f \circ g = g \circ f$ when $ad + b = bc + d$

59. $S(t) = \frac{16}{9}\pi t^6$ 61. $C(t) = 15,000 + 800,000t - 40,000t^2$ 63. $C(p) = \frac{2\sqrt{100-p}}{25} + 600, 0 \leq p \leq 100$ 65. $V(r) = 2\pi r^3$
 67. (a) $f(x) = 0.9428x$ (b) $g(x) = 126.457x$ (c) $g(f(x)) = g(0.9428x) = 119.2236596x$ (d) 119,223.6596 yen 69. (a) $f(p) = p - 200$
 (b) $g(p) = 0.8p$ (c) $(f \circ g)(p) = 0.8p - 200; (g \circ f)(p) = 0.8p - 160$; The 20% discount followed by the \$200 rebate is the better deal. 71. 15
 73. f is an odd function, so $f(-x) = -f(x)$. g is an even function, so $g(-x) = g(x)$. Then $(f \circ g)(-x) = f(g(-x)) = f(g(x)) = (f \circ g)(x)$. So $f \circ g$ is even. Also, $(g \circ f)(-x) = g(f(-x)) = g(-f(x)) = g(f(x)) = (g \circ f)(x)$, so $g \circ f$ is even.

74. $(f+g)(x) = 4x + 3$; Domain: all real numbers
 $(f-g)(x) = 2x + 13$; Domain: all real numbers
 $(f \cdot g)(x) = 3x^2 - 7x - 40$; Domain: all real numbers
 $\left(\frac{f}{g}\right)(x) = \frac{3x+8}{x-5}$; Domain: $\{x \mid x \neq 5\}$

75. -5, -3, 3



Local minimum: -5.08 at $x = -1.15$
 Local maximum: 1.08 at $x = 1.15$
 Decreasing: $[-3, -1.15]$; $[1.15, 3]$
 Increasing: $[-1.15, 1.15]$

77. Domain: $\{x \mid x \neq 3\}$
 Vertical asymptote: $x = 3$
 Oblique asymptote: $y = x + 9$

5.2 Assess Your Understanding (page 274)

5. $f(x_1) \neq f(x_2)$ 6. one-to-one 7. 3 8. $y = x$ 9. $[4, \infty)$ 10. T 11. a 12. d 13. one-to-one 15. not one-to-one
 17. not one-to-one 19. one-to-one 21. one-to-one 23. not one-to-one 25. one-to-one

27. Annual Rainfall (inches)

Annual Rainfall (inches)	Location
49.7	Atlanta, Georgia
43.8	Boston, Massachusetts
4.2	Las Vegas, Nevada
61.9	Miami, Florida
12.8	Los Angeles, California

Domain: {49.7, 43.8, 4.2, 61.9, 12.8}
 Range: {Atlanta, Boston, Las Vegas, Miami, Los Angeles}

29. Monthly Cost of Life Insurance

Monthly Cost of Life Insurance	Age
\$10.59	30
\$12.52	40
\$15.94	45

Domain: {\$10.59, \$12.52, \$15.94}
 Range: {30, 40, 45}

31. $\{(5, -3), (9, -2), (2, -1), (11, 0), (-5, 1)\}$
 Domain: {5, 9, 2, 11, -5}
 Range: $\{-3, -2, -1, 0, 1\}$
 33. $\{(1, -2), (2, -3), (0, -10), (9, 1), (4, 2)\}$
 Domain: {1, 2, 0, 9, 4}
 Range: $\{-2, -3, -10, 1, 2\}$
 35. $f(g(x)) = f\left(\frac{1}{3}(x-4)\right) = 3\left[\frac{1}{3}(x-4)\right] + 4 = (x-4) + 4 = x$
 $g(f(x)) = g(3x+4) = \frac{1}{3}[(3x+4)-4] = \frac{1}{3}(3x) = x$
 37. $f(g(x)) = f\left(\frac{x}{4} + 2\right) = 4\left[\frac{x}{4} + 2\right] - 8 = (x+8) - 8 = x$
 $g(f(x)) = g(4x-8) = \frac{4x-8}{4} + 2 = (x-2) + 2 = x$
 39. $f(g(x)) = f(\sqrt[3]{x+8}) = (\sqrt[3]{x+8})^3 - 8 = (x+8) - 8 = x$
 $g(f(x)) = g(x^3 - 8) = \sqrt[3]{(x^3 - 8) + 8} = \sqrt[3]{x^3} = x$
 41. $f(g(x)) = f\left(\frac{1}{x}\right) = \frac{1}{\left(\frac{1}{x}\right)} = x; x \neq 0, g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{\left(\frac{1}{x}\right)} = x, x \neq 0$

$$43. f(g(x)) = f\left(\frac{4x-3}{2-x}\right) = \frac{2\left(\frac{4x-3}{2-x}\right) + 3}{\frac{4x-3}{2-x} + 4}$$

$$= \frac{2(4x-3) + 3(2-x)}{4x-3 + 4(2-x)} = \frac{5x}{5} = x, x \neq 2$$

$$g(f(x)) = g\left(\frac{2x+3}{x+4}\right) = \frac{4\left(\frac{2x+3}{x+4}\right) - 3}{2 - \frac{2x+3}{x+4}}$$

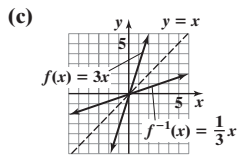
$$= \frac{4(2x+3) - 3(x+4)}{2(x+4) - (2x+3)} = \frac{5x}{5} = x, x \neq -4$$

51. (a) $f^{-1}(x) = \frac{1}{3}x$

$$f(f^{-1}(x)) = f\left(\frac{1}{3}x\right) = 3\left(\frac{1}{3}x\right) = x$$

$$f^{-1}(f(x)) = f^{-1}(3x) = \frac{1}{3}(3x) = x$$

(b) Domain of f = Range of f^{-1} = All real numbers;
Range of f = Domain of f^{-1} = All real numbers

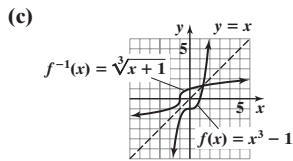


55. (a) $f^{-1}(x) = \sqrt[3]{x+1}$

$$f(f^{-1}(x)) = f(\sqrt[3]{x+1}) = (\sqrt[3]{x+1})^3 - 1 = x$$

$$f^{-1}(f(x)) = f^{-1}(x^3 - 1) = \sqrt[3]{(x^3 - 1) + 1} = x$$

(b) Domain of f = Range of f^{-1} = All real numbers;
Range of f = Domain of f^{-1} = All real numbers

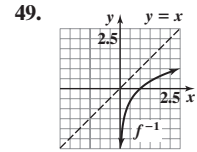
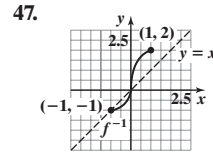
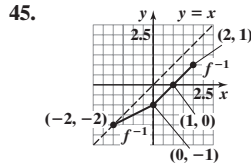
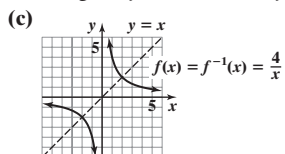


59. (a) $f^{-1}(x) = \frac{4}{x}$

$$f(f^{-1}(x)) = f\left(\frac{4}{x}\right) = \frac{4}{\left(\frac{4}{x}\right)} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{4}{x}\right) = \frac{4}{\left(\frac{4}{x}\right)} = x$$

(b) Domain of f = Range of $f^{-1} = \{x|x \neq 0\}$;
Range of f = Domain of $f^{-1} = \{x|x \neq 0\}$



53. (a) $f^{-1}(x) = \frac{x}{4} - \frac{1}{2}$

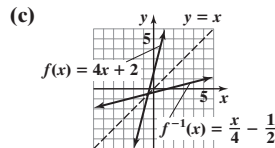
$$f(f^{-1}(x)) = f\left(\frac{x}{4} - \frac{1}{2}\right) = 4\left(\frac{x}{4} - \frac{1}{2}\right) + 2$$

$$= (x - 2) + 2 = x$$

$$f^{-1}(f(x)) = f^{-1}(4x + 2) = \frac{4x + 2}{4} - \frac{1}{2}$$

$$= \left(x + \frac{1}{2}\right) - \frac{1}{2} = x$$

(b) Domain of f = Range of f^{-1} = All real numbers;
Range of f = Domain of f^{-1} = All real numbers

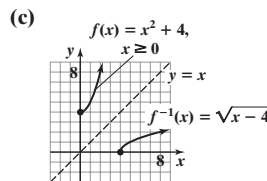


57. (a) $f^{-1}(x) = \sqrt{x-4}, x \geq 4$

$$f(f^{-1}(x)) = f(\sqrt{x-4}) = (\sqrt{x-4})^2 + 4 = x$$

$$f^{-1}(f(x)) = f^{-1}(x^2 + 4) = \sqrt{(x^2 + 4) - 4} = \sqrt{x^2} = x, x \geq 0$$

(b) Domain of f = Range of $f^{-1} = \{x|x \geq 0\}$;
Range of f = Domain of $f^{-1} = \{x|x \geq 4\}$

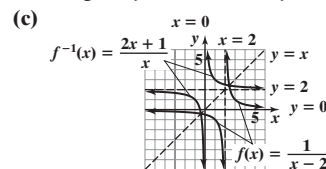


61. (a) $f^{-1}(x) = \frac{2x+1}{x}$

$$f(f^{-1}(x)) = f\left(\frac{2x+1}{x}\right) = \frac{1}{\frac{2x+1}{x} - 2} = \frac{x}{(2x+1) - 2x} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{1}{x-2}\right) = \frac{2\left(\frac{1}{x-2}\right) + 1}{\frac{1}{x-2}} = \frac{2 + (x-2)}{1} = x$$

(b) Domain of f = Range of $f^{-1} = \{x|x \neq 2\}$;
Range of f = Domain of $f^{-1} = \{x|x \neq 0\}$



$$63. \text{(a)} f^{-1}(x) = \frac{2-3x}{x}$$

$$f(f^{-1}(x)) = f\left(\frac{2-3x}{x}\right) = \frac{2}{3 + \frac{2-3x}{x}} = \frac{2x}{3x + 2 - 3x} = \frac{2x}{2} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{2}{3+x}\right) = \frac{2 - 3\left(\frac{2}{3+x}\right)}{\frac{2}{3+x}} = \frac{2(3+x) - 3 \cdot 2}{2} = \frac{2x}{2} = x$$

(b) Domain of f = Range of $f^{-1} = \{x \mid x \neq -3\}$; Range of f = Domain of $f^{-1} = \{x \mid x \neq 0\}$

$$65. \text{(a)} f^{-1}(x) = \frac{-2x}{x-3}$$

$$f(f^{-1}(x)) = f\left(\frac{-2x}{x-3}\right) = \frac{3\left(\frac{-2x}{x-3}\right)}{\frac{-2x}{x-3} + 2} = \frac{3(-2x)}{-2x + 2(x-3)} = \frac{-6x}{-6} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{3x}{x+2}\right) = \frac{-2\left(\frac{3x}{x+2}\right)}{\frac{3x}{x+2} - 3} = \frac{-2(3x)}{3x - 3(x+2)} = \frac{-6x}{-6} = x$$

(b) Domain of f = Range of $f^{-1} = \{x \mid x \neq -2\}$; Range of f = Domain of $f^{-1} = \{x \mid x \neq 3\}$

$$67. \text{(a)} f^{-1}(x) = \frac{x}{3x-2}$$

$$f(f^{-1}(x)) = f\left(\frac{x}{3x-2}\right) = \frac{2\left(\frac{x}{3x-2}\right)}{3\left(\frac{x}{3x-2}\right) - 1} = \frac{2x}{3x - (3x-2)} = \frac{2x}{2} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{2x}{3x-1}\right) = \frac{\frac{2x}{3x-1}}{3\left(\frac{2x}{3x-1}\right) - 2} = \frac{2x}{6x - 2(3x-1)} = \frac{2x}{2} = x$$

(b) Domain of f = Range of $f^{-1} = \left\{x \mid x \neq \frac{1}{3}\right\}$; Range of f = Domain of $f^{-1} = \left\{x \mid x \neq \frac{2}{3}\right\}$

$$69. \text{(a)} f^{-1}(x) = \frac{3x+4}{2x-3}$$

$$f(f^{-1}(x)) = f\left(\frac{3x+4}{2x-3}\right) = \frac{3\left(\frac{3x+4}{2x-3}\right) + 4}{2\left(\frac{3x+4}{2x-3}\right) - 3} = \frac{3(3x+4) + 4(2x-3)}{2(3x+4) - 3(2x-3)} = \frac{17x}{17} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{3x+4}{2x-3}\right) = \frac{3\left(\frac{3x+4}{2x-3}\right) + 4}{2\left(\frac{3x+4}{2x-3}\right) - 3} = \frac{3(3x+4) + 4(2x-3)}{2(3x+4) - 3(2x-3)} = \frac{17x}{17} = x$$

(b) Domain of f = Range of $f^{-1} = \left\{x \mid x \neq \frac{3}{2}\right\}$; Range of f = Domain of $f^{-1} = \left\{x \mid x \neq \frac{3}{2}\right\}$

$$71. \text{(a)} f^{-1}(x) = \frac{-2x+3}{x-2}$$

$$f(f^{-1}(x)) = f\left(\frac{-2x+3}{x-2}\right) = \frac{2\left(\frac{-2x+3}{x-2}\right) + 3}{\frac{-2x+3}{x-2} + 2} = \frac{2(-2x+3) + 3(x-2)}{-2x+3 + 2(x-2)} = \frac{-x}{-1} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{2x+3}{x+2}\right) = \frac{-2\left(\frac{2x+3}{x+2}\right) + 3}{\frac{2x+3}{x+2} - 2} = \frac{-2(2x+3) + 3(x+2)}{2x+3 - 2(x+2)} = \frac{-x}{-1} = x$$

(b) Domain of f = Range of $f^{-1} = \{x \mid x \neq -2\}$; Range of f = Domain of $f^{-1} = \{x \mid x \neq 2\}$

73. (a) $f^{-1}(x) = \frac{2}{\sqrt{1-2x}}$

$$f(f^{-1}(x)) = f\left(\frac{2}{\sqrt{1-2x}}\right) = \frac{\frac{4}{1-2x} - 4}{2 \cdot \frac{4}{1-2x}} = \frac{4 - 4(1-2x)}{2 \cdot 4} = \frac{8x}{8} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{x^2-4}{2x^2}\right) = \frac{2}{\sqrt{1-2\left(\frac{x^2-4}{2x^2}\right)}} = \frac{2}{\sqrt{\frac{4}{x^2}}} = \sqrt{x^2} = x, \text{ since } x > 0$$

(b) Domain of $f =$ Range of $f^{-1} = \{x|x > 0\}$; Range of $f =$ Domain of $f^{-1} = \left\{x \mid x < \frac{1}{2}\right\}$

75. (a) 0 (b) 2 (c) 0 (d) 1 77. 7 79. Domain of $f^{-1}: [-2, \infty)$; range of $f^{-1}: [5, \infty)$ 81. Domain of $g^{-1}: [0, \infty)$; range of $g^{-1}: (-\infty, 0]$

83. Increasing on the interval $[f(0), f(5)]$ 85. $f^{-1}(x) = \frac{1}{m}(x-b), m \neq 0$ 87. Quadrant I

89. Possible answer: $f(x) = |x|, x \geq 0$, is one-to-one; $f^{-1}(x) = x, x \geq 0$

91. (a) $r(d) = \frac{d + 90.39}{6.97}$

(b) $r(d(r)) = \frac{6.97r - 90.39 + 90.39}{6.97} = \frac{6.97r}{6.97} = r$

$$d(r(d)) = 6.97\left(\frac{d + 90.39}{6.97}\right) - 90.39 = d + 90.39 - 90.39 = d$$

(c) 56 miles per hour

93. (a) 77.6 kg

(b) $h(W) = \frac{W-50}{2.3} + 60 = \frac{W+88}{2.3}$

(c) $h(W(h)) = \frac{50 + 2.3(h-60) + 88}{2.3} = \frac{2.3h}{2.3} = h$

$$W(h(W)) = 50 + 2.3\left(\frac{W+88}{2.3} - 60\right) = 50 + W + 88 - 138 = W$$

(d) 73 inches

97. (a) t represents time, so $t \geq 0$.

(b) $t(H) = \sqrt{\frac{H-100}{-4.9}} = \sqrt{\frac{100-H}{4.9}}$

(c) 2.02 seconds

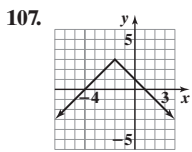
99. $f^{-1}(x) = \frac{-dx+b}{cx-a}; f = f^{-1}$ if $a = -d$ 103. No

108. Zeros: $\frac{-5-\sqrt{13}}{6}, \frac{-5+\sqrt{13}}{6}$, x-intercepts: $\frac{-5-\sqrt{13}}{6}, \frac{-5+\sqrt{13}}{6}$

109. Domain: $\left\{x \mid x \neq -\frac{3}{2}, x \neq 2\right\}$; Vertical asymptote: $x = -\frac{3}{2}$

Horizontal asymptote: $y = 3$

110. $6xh + 3h^2 - 7h$

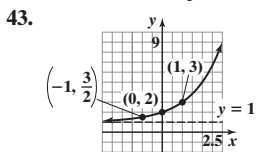


5.3 Assess Your Understanding (page 289)

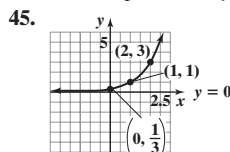
6. Exponential function; growth factor; initial value 7. a 8. T 9. T 10. $\left(-1, \frac{1}{a}\right); (0, 1); (1, a)$ 11. 4 12. F 13. b 14. c

15. (a) 8.815 (b) 8.821 (c) 8.824 (d) 8.825 17. (a) 21.217 (b) 22.217 (c) 22.440 (d) 22.459 19. 1.265 21. 0.347 23. 3.320 25. 149.952

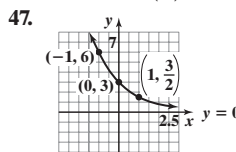
27. Neither 29. Exponential; $H(x) = 4^x$ 31. Exponential; $f(x) = 3(2^x)$ 33. Linear; $H(x) = 2x + 4$ 35. B 37. D 39. A 41. E



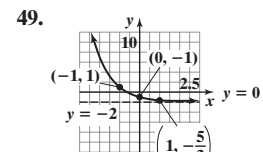
Domain: All real numbers
Range: $\{y|y > 1\}$ or $(1, \infty)$
Horizontal asymptote: $y = 1$



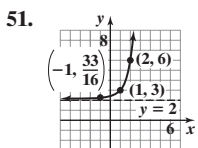
Domain: All real numbers
Range: $\{y|y > 0\}$ or $(0, \infty)$
Horizontal asymptote: $y = 0$



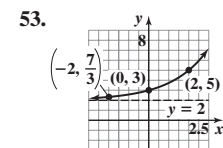
Domain: All real numbers
Range: $\{y|y > 0\}$ or $(0, \infty)$
Horizontal asymptote: $y = 0$



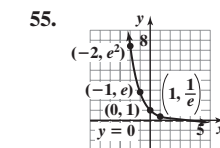
Domain: All real numbers
Range: $\{y|y > -2\}$ or $(-2, \infty)$
Horizontal asymptote: $y = -2$



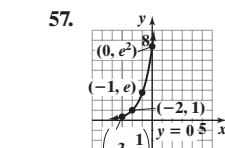
Domain: All real numbers
Range: $\{y|y > 2\}$ or $(2, \infty)$
Horizontal asymptote: $y = 2$



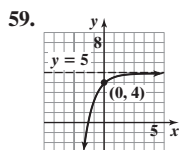
Domain: All real numbers
Range: $\{y|y > 2\}$ or $(2, \infty)$
Horizontal asymptote: $y = 2$



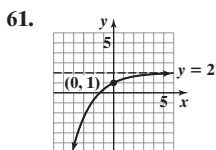
Domain: All real numbers
Range: $\{y|y > 0\}$ or $(0, \infty)$
Horizontal asymptote: $y = 0$



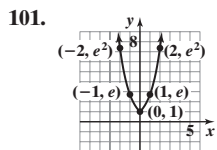
Domain: All real numbers
Range: $\{y|y > 0\}$ or $(0, \infty)$
Horizontal asymptote: $y = 0$



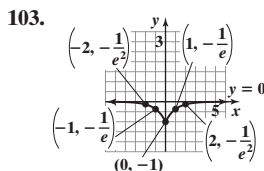
Domain: All real numbers
Range: $\{y|y < 5\}$ or $(-\infty, 5)$
Horizontal asymptote: $y = 5$



Domain: All real numbers
Range: $\{y|y < 2\}$ or $(-\infty, 2)$
Horizontal asymptote: $y = 2$



Domain: $(-\infty, \infty)$
Range: $[1, \infty)$
Intercept: $(0, 1)$



Domain: $(-\infty, \infty)$
Range: $[-1, 0)$
Intercept: $(0, -1)$

63. $\{3\}$ 65. $\{-4\}$ 67. $\{2\}$ 69. $\left\{\frac{3}{2}\right\}$ 71. $\{-\sqrt{2}, 0, \sqrt{2}\}$

73. $\{6\}$ 75. $\{-1, 7\}$ 77. $\{-4, 2\}$ 79. $\{-4\}$ 81. $\{1, 2\}$ 83. $\frac{1}{49}$

85. $\frac{1}{4}$ 87. 5 89. $f(x) = 3^x$ 91. $f(x) = -6^x$ 93. $f(x) = 3^x + 2$

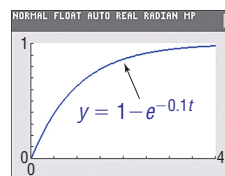
95. (a) 16; $(4, 16)$ (b) -4 ; $\left(-4, \frac{1}{16}\right)$ 97. (a) $\frac{9}{4}$; $\left(-1, \frac{9}{4}\right)$ (b) 3; $(3, 66)$

99. (a) 60; $(-6, 60)$ (b) -4 ; $(-4, 12)$ (c) -2

105. (a) 74% (b) 47% (c) Each pane allows only 97% of light to pass through.

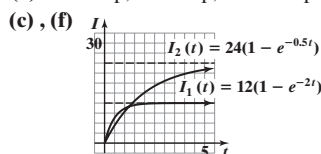
107. (a) \$16,231 (b) \$8626 (c) As each year passes, the sedan is worth 90% of its value the previous year. 109. (a) 30% (b) 9% (c) Each year only 30% of the previous survivors survive again. 111. 3.35 mg; 0.45 mg

113. (a) 0.632 (b) 0.982 (c) 1 (d) (e) About 7 min



115. (a) 0.0516 (b) 0.0888 117. (a) 70.95% (b) 72.62% (c) 100%

119. (a) 5.41 amp, 7.59 amp, 10.38 amp (b) 12 amp 121. 36 123.

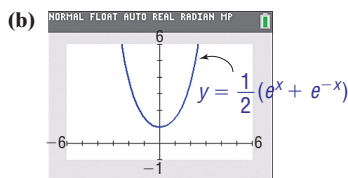


(d) 3.34 amp, 5.31 amp, 9.44 amp
(e) 24 amp

Final Denominator	Value of Expression	Compare Value to $e \approx 2.718281828$
1 + 1	2.5	$2.5 < e$
2 + 2	2.8	$2.8 > e$
3 + 3	2.7	$2.7 < e$
4 + 4	2.721649485	$2.721649485 > e$
5 + 5	2.717770035	$2.717770035 < e$
6 + 6	2.718348855	$2.718348855 > e$

125. $f(A + B) = a^{A+B} = a^A \cdot a^B = f(A) \cdot f(B)$ 127. $f(\alpha x) = a^{\alpha x} = (a^x)^\alpha = [f(x)]^\alpha$

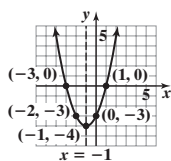
129. (a) $f(-x) = \frac{1}{2}(e^{-x} + e^{-(-x)}) = \frac{1}{2}(e^{-x} + e^x) = \frac{1}{2}(e^x + e^{-x}) = f(x)$



(c) $(\cosh x)^2 - (\sinh x)^2 = \left[\frac{1}{2}(e^x + e^{-x})\right]^2 - \left[\frac{1}{2}(e^x - e^{-x})\right]^2 = \frac{1}{4}[e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}] = \frac{1}{4}(4) = 1$

131. 59 minutes 135. $a^{-x} = (a^{-1})^x = \left(\frac{1}{a}\right)^x$ 136. $(-\infty, -5] \cup [-2, 2]$ 137. $(2, \infty)$ 138. $f(x) = -2x^2 + 12x - 13$

139. (a) (b) Domain: $(-\infty, \infty)$; Range: $[-4, \infty)$
(c) Decreasing: $(-\infty, -1]$; Increasing: $[-1, \infty)$

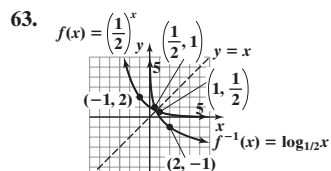
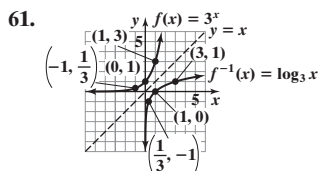


5.4 Assess Your Understanding (page 304)

4. $\{x|x > 0\}$ or $(0, \infty)$ 5. $\left(\frac{1}{a}, -1\right), (1, 0), (a, 1)$ 6. 1 7. F 8. T 9. a 10. c 11. $\log_3 9 = 2$ 13. $\log_a 1.6 = 2$ 15. $\log_2 7.2 = x$ 17. $\ln 8 = x$

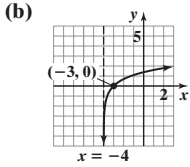
19. $2^3 = 8$ 21. $a^6 = 3$ 23. $3^x = 2$ 25. $e^x = 4$ 27. 0 29. 2 31. -4 33. $\frac{1}{2}$ 35. 4 37. $\frac{1}{2}$ 39. $\{x|x > 3\}; (3, \infty)$

41. All real numbers except 0; $\{x|x \neq 0\}; (-\infty, 0) \cup (0, \infty)$ 43. $\{x|x > 10\}; (10, \infty)$ 45. $\{x|x > -1\}; (-1, \infty)$
47. $\{x|x < -1 \text{ or } x > 0\}; (-\infty, -1) \cup (0, \infty)$ 49. $\{x|x \geq 1\}; [1, \infty)$ 51. 0.511 53. 30.099 55. 2.303 57. -53.991 59. $\sqrt{2}$



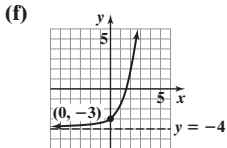
63. B 67. D 69. A 71. E

73. (a) Domain: $(-4, \infty)$

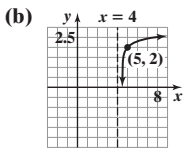


(c) Range: $(-\infty, \infty)$
Vertical asymptote: $x = -4$

(d) $f^{-1}(x) = e^x - 4$
(e) Domain of f^{-1} : $(-\infty, \infty)$
Range of f^{-1} : $(-4, \infty)$

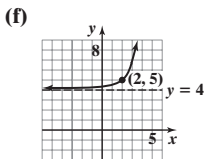


79. (a) Domain: $(4, \infty)$

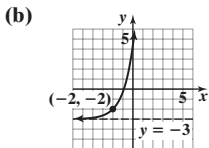


(c) Range: $(-\infty, \infty)$
Vertical asymptote: $x = 4$

(d) $f^{-1}(x) = 10^{x-2} + 4$
(e) Domain of f^{-1} : $(-\infty, \infty)$
Range of f^{-1} : $(4, \infty)$

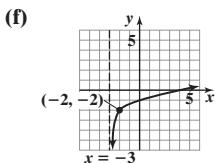


85. (a) Domain: $(-\infty, \infty)$

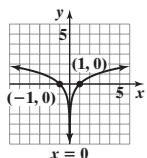


(c) Range: $(-3, \infty)$
Horizontal asymptote: $y = -3$

(d) $f^{-1}(x) = \ln(x + 3) - 2$
(e) Domain of f^{-1} : $(-3, \infty)$
Range of f^{-1} : $(-\infty, \infty)$

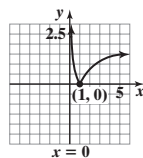


115.



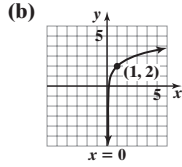
Domain: $\{x \mid x \neq 0\}$
Range: $(-\infty, \infty)$
Intercepts: $(-1, 0), (1, 0)$

117.



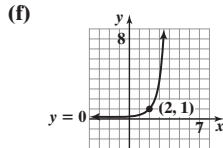
Domain: $\{x \mid x > 0\}$
Range: $\{y \mid y \geq 0\}$
Intercept: $(1, 0)$

75. (a) Domain: $(0, \infty)$

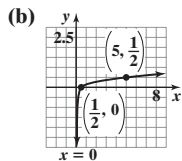


(c) Range: $(-\infty, \infty)$
Vertical asymptote: $x = 0$

(d) $f^{-1}(x) = e^{x-2}$
(e) Domain of f^{-1} : $(-\infty, \infty)$
Range of f^{-1} : $(0, \infty)$

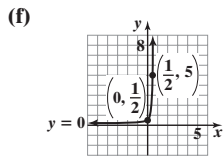


81. (a) Domain: $(0, \infty)$

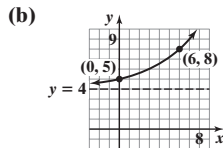


(c) Range: $(-\infty, \infty)$
Vertical asymptote: $x = 0$

(d) $f^{-1}(x) = \frac{1}{2} \cdot 10^{2x}$
(e) Domain of f^{-1} : $(-\infty, \infty)$
Range of f^{-1} : $(0, \infty)$

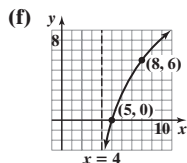


87. (a) Domain: $(-\infty, \infty)$

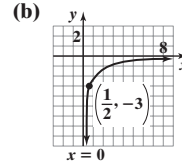


(c) Range: $(4, \infty)$
Horizontal asymptote: $y = 4$

(d) $f^{-1}(x) = 3 \log_2(x - 4)$
(e) Domain of f^{-1} : $(4, \infty)$
Range of f^{-1} : $(-\infty, \infty)$

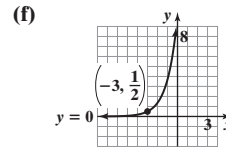


77. (a) Domain: $(0, \infty)$

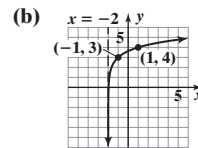


(c) Range: $(-\infty, \infty)$
Vertical asymptote: $x = 0$

(d) $f^{-1}(x) = \frac{1}{2} e^{x+3}$
(e) Domain of f^{-1} : $(-\infty, \infty)$
Range of f^{-1} : $(0, \infty)$

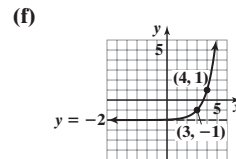


83. (a) Domain: $(-2, \infty)$



(c) Range: $(-\infty, \infty)$
Vertical asymptote: $x = -2$

(d) $f^{-1}(x) = 3^{x-3} - 2$
(e) Domain of f^{-1} : $(-\infty, \infty)$
Range of f^{-1} : $(-2, \infty)$



89. $\{9\}$ 91. $\left\{\frac{7}{2}\right\}$ 93. $\{2\}$ 95. $\{5\}$ 97. $\{3\}$

99. $\{2\}$ 101. $\left\{\frac{\ln 10}{3}\right\}$ 103. $\left\{\frac{\ln 8 - 5}{2}\right\}$

105. $\{-2\sqrt{2}, 2\sqrt{2}\}$

107. $\{-1\}$

109. $\left\{5 \ln \frac{7}{5}\right\}$

111. $\left\{2 - \log \frac{5}{2}\right\}$

113. (a) $\left\{x \mid x > -\frac{1}{2}\right\}; \left(-\frac{1}{2}, \infty\right)$

(b) 2; (40, 2) (c) 121; (121, 3) (d) 4

119. (a) 1 (b) 2 (c) 3

(d) It increases. (e) 0.000316

(f) 3.981×10^{-8}

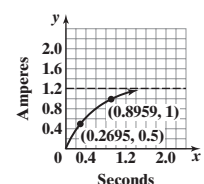
121. (a) 5.97 km (b) 0.90 km

123. (a) 6.93 min (b) 16.09 min

125. $h \approx 2.29$, so the time between injections is about 2 h, 17 min.

127. 0.2695 s

0.8959 s



129. 50 decibels (dB) 131. 90 dB 133. 8.1 135. (a) $k \approx 11.216$ (b) 6.73 (c) 0.41% (d) 0.14%

137. Because $y = \log_1 x$ means $1^y = 1 = x$, which cannot be true for $x \neq 1$ 139. Zeros: $-3, -\frac{1}{2}, \frac{1}{2}, 3$; x-intercepts: $-3, -\frac{1}{2}, \frac{1}{2}, 3$

140. 12 141. $f(1) = -5; f(2) = 17$ 142. $3 + i; f(x) = x^4 - 7x^3 + 14x^2 + 2x - 20; a = 1$

5.5 Assess Your Understanding (page 315)

1. 0 2. M 3. r 4. $\log_a M; \log_a N$ 5. $\log_a M; \log_a N$ 6. $r \log_a M$ 7. 7 8. F 9. F 10. F 11. b 12. b 13. 71 15. -4 17. 7 19. 1 21. 1

23. 3 25. $\frac{5}{4}$ 27. 4 29. $a + b$ 31. $b - a$ 33. $3a$ 35. $\frac{1}{5}(a + b)$ 37. $2 + \log_5 x$ 39. $3 \log_2 z$ 41. $1 + \ln x$ 43. $\ln x - x$ 45. $2 \log_a u + 3 \log_a v$

47. $2 \ln x + \frac{1}{2} \ln(1 - x)$ 49. $3 \log_2 x - \log_2(x - 3)$ 51. $\log x + \log(x + 2) - 2 \log(x + 3)$ 53. $\frac{1}{3} \ln(x - 2) + \frac{1}{3} \ln(x + 1) - \frac{2}{3} \ln(x + 4)$

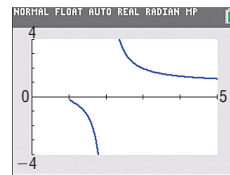
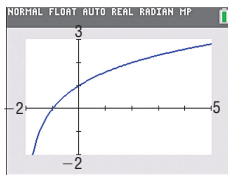
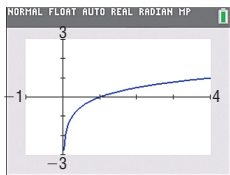
55. $\ln 5 + \ln x + \frac{1}{2} \ln(1 + 3x) - 3 \ln(x - 4)$ 57. $\log_5(u^3 v^4)$ 59. $\log_3\left(\frac{1}{x^{5/2}}\right)$ 61. $\log_4\left[\frac{x - 1}{(x + 1)^4}\right]$ 63. $-2 \ln(x - 1)$ 65. $\log_2[x(3x - 2)^4]$

67. $\log_a\left(\frac{25x^6}{\sqrt{2x + 3}}\right)$ 69. $\log_2\left[\frac{(x + 1)^2}{(x + 3)(x - 1)}\right]$ 71. 2.771 73. -3.880 75. 5.615 77. 0.874

79. $y = \frac{\log x}{\log 4}$

81. $y = \frac{\log(x + 2)}{\log 2}$

83. $y = \frac{\log(x + 1)}{\log(x - 1)}$



85. (a) $(f \circ g)(x) = x; \{x | x \text{ is any real number}\}$ or $(-\infty, \infty)$ 87. $y = Cx$ 89. $y = Cx(x + 1)$ 91. $y = Ce^{3x}$ 93. $y = Ce^{-4x} + 3$
 (b) $(g \circ f)(x) = x; \{x | x > 0\}$ or $(0, \infty)$ (c) 5
 (d) $(f \circ h)(x) = \ln x^2; \{x | x \neq 0\}$ or $(-\infty, 0) \cup (0, \infty)$ (e) 2

95. $y = \frac{\sqrt[3]{C}(2x + 1)^{1/6}}{(x + 4)^{1/9}}$ 97. 3 99. 1

101. $\log_a(x + \sqrt{x^2 - 1}) + \log_a(x - \sqrt{x^2 - 1}) = \log_a[(x + \sqrt{x^2 - 1})(x - \sqrt{x^2 - 1})] = \log_a[x^2 - (x^2 - 1)] = \log_a 1 = 0$

103. $\ln(1 + e^{2x}) = \ln[e^{2x}(e^{-2x} + 1)] = \ln e^{2x} + \ln(e^{-2x} + 1) = 2x + \ln(1 + e^{-2x})$

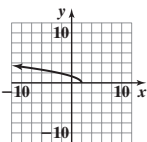
105. $y = f(x) = \log_a x; a^y = x$ implies $a^y = \left(\frac{1}{a}\right)^{-y} = x$, so $-y = \log_{1/a} x = -f(x)$.

107. $f(x) = \log_a x; f\left(\frac{1}{x}\right) = \log_a \frac{1}{x} = \log_a 1 - \log_a x = -f(x)$

109. $\log_a \frac{M}{N} = \log_a(M \cdot N^{-1}) = \log_a M + \log_a N^{-1} = \log_a M - \log_a N$, since $a^{\log_a N^{-1}} = N^{-1}$ implies $a^{-\log_a N} = N$; that is, $\log_a N = -\log_a N^{-1}$.

115. $\{-1.78, 1.29, 3.49\}$ 116. A repeated real solution (double root) 117. $-2, \frac{1}{5}, \frac{-5 - \sqrt{21}}{2}, \frac{-5 + \sqrt{21}}{2}$

118.



Domain: $\{x | x \leq 2\}$ or $(-\infty, 2]$

Range: $\{y | y \geq 0\}$ or $[0, \infty)$

5.6 Assess Your Understanding (page 322)

5. $\{16\}$ 7. $\left\{\frac{16}{5}\right\}$ 9. $\{6\}$ 11. $\{16\}$ 13. $\left\{\frac{1}{3}\right\}$ 15. $\{3\}$ 17. $\{5\}$ 19. $\left\{\frac{21}{8}\right\}$ 21. $\{-6\}$ 23. $\{-2\}$ 25. $\{-1 + \sqrt{1 + e^4}\} \approx \{6.456\}$

27. $\left\{\frac{-5 + 3\sqrt{5}}{2}\right\} \approx \{0.854\}$ 29. $\{2\}$ 31. $\left\{\frac{9}{2}\right\}$ 33. $\{7\}$ 35. $\{-2 + 4\sqrt{2}\}$ 37. $\{-\sqrt{3}, \sqrt{3}\}$ 39. $\left\{\frac{1}{3}, 729\right\}$ 41. $\{8\}$

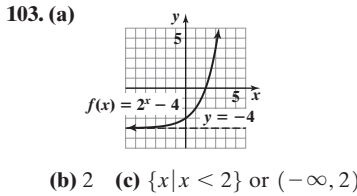
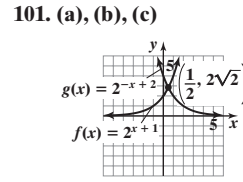
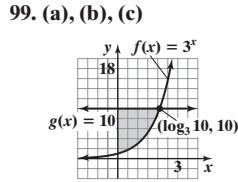
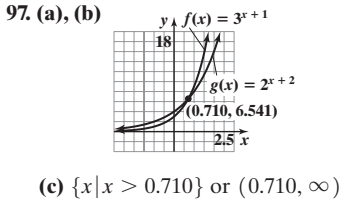
43. $\{\log_2 10\} = \left\{\frac{\ln 10}{\ln 2}\right\} \approx \{3.322\}$ 45. $\{-\log_8 1.2\} = \left\{-\frac{\ln 1.2}{\ln 8}\right\} \approx \{-0.088\}$ 47. $\left\{\frac{1}{3} \log_2 \frac{8}{5}\right\} = \left\{\frac{\ln \frac{8}{5}}{3 \ln 2}\right\} \approx \{0.226\}$

49. $\left\{\frac{\ln 3}{2 \ln 3 + \ln 4}\right\} \approx \{0.307\}$ 51. $\left\{\frac{\ln 7}{\ln 0.6 + \ln 7}\right\} \approx \{1.356\}$ 53. $\{0\}$ 55. $\left\{\frac{\ln \pi}{1 + \ln \pi}\right\} \approx \{0.534\}$ 57. $\left\{\frac{\ln 3}{\ln 2}\right\} \approx \{1.585\}$

59. $\{0\}$ 61. $\left\{\log_4(-2 + \sqrt{7})\right\} \approx \{-0.315\}$ 63. $\{\log_5 4\} \approx \{0.861\}$ 65. No real solution 67. $\{\log_4 5\} \approx \{1.161\}$ 69. $\{2.79\}$

71. $\{-0.57\}$ 73. $\{-0.70\}$ 75. $\{0.57\}$ 77. $\{0.39, 1.00\}$ 79. $\{1.32\}$ 81. $\{1.31\}$ 83. $\{1\}$ 85. $\{16\}$ 87. $\left\{-1, \frac{2}{3}\right\}$ 89. $\{0\}$

91. $\{\ln(2 + \sqrt{5})\} \approx \{1.444\}$ 93. $\left\{e^{\frac{\ln 5 \cdot \ln 3}{\ln 15}}\right\} \approx \{1.921\}$ 95. (a) $\{5\}; (5, 3)$ (b) $\{5\}; (5, 4)$ (c) $\{1\}$; yes, at $(1, 2)$ (d) $\{5\}$ (e) $\left\{-\frac{1}{11}\right\}$



105. (a) 2047 (b) 2059
107. (a) After 4.2 yr
(b) After 6.5 yr
(c) After 12.8 yr

110. $\left\{-3, \frac{1}{4}, 2\right\}$
111. one-to-one
112. $(f \circ g)(x) = \frac{x+5}{-x+11}$; $\{x | x \neq 3, x \neq 11\}$
113. $\{x | x \geq 1\}$, or $[1, \infty)$

5.7 Assess Your Understanding (page 331)

3. principal 4. $I; Prt$; simple interest 5. 4 6. effective rate of interest 7. \$108.29 9. \$609.50 11. \$697.09 13. \$1246.08 15. \$88.72 17. \$860.72
19. \$554.09 21. \$59.71 23. 5.095% 25. 5.127% 27. $6\frac{1}{4}\%$ compounded annually 29. 9% compounded monthly 31. 25.992% 33. 24.573%
35. (a) About 8.69 yr (b) About 8.66 yr 37. 6.823% 39. 10.15 yr; 10.14 yr 41. 15.27 yr or 15 yr, 3 mo 43. \$104,335 45. \$12,910.62
47. About \$30.17 per share or \$3017 49. Not quite. Jim will have \$1057.60. The second bank gives a better deal, since Jim will have \$1060.62 after 1 yr.
51. Will has \$11,632.73; Henry has \$10,947.89. 53. (a) \$64,589 (b) \$45,062 55. About \$1020 billion; about \$233 billion 57. \$940.90 59. 2.53%
61. 34.31 yr 63. (a) \$3686.45 (b) \$3678.79 65. \$6439.28

67. (a) 11.90 yr (b) 22.11 yr (c) $mP = P\left(1 + \frac{r}{n}\right)^{nt}$
 $m = \left(1 + \frac{r}{n}\right)^{nt}$
 $\ln m = \ln\left(1 + \frac{r}{n}\right)^{nt} = nt \ln\left(1 + \frac{r}{n}\right)$
 $t = \frac{\ln m}{n \ln\left(1 + \frac{r}{n}\right)}$
69. (a) 1.99% (b) In 2026 or after 17 yr 71. 22.7 yr 76. $R = 0$; yes
77. $f^{-1}(x) = \frac{2x}{x-1}$ 78. $-2, 5; f(x) = (x+2)^2(x-5)(x^2+1)$ 79. $\{6\}$

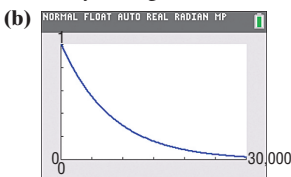
5.8 Assess Your Understanding (page 342)

1. (a) 500 insects (b) $0.02 = 2\%$ per day (c) (d) About 611 insects (e) After about 23.5 days (f) After about 34.7 days

3. (a) $-0.0244 = -2.44\%$ per year (b) (c) About 391.7 g (d) After about 9.1 yr (e) 28.4 yr

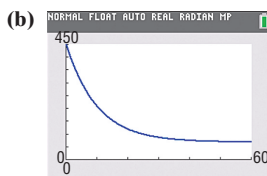
5. (a) $N(t) = N_0 e^{kt}$ (b) 5832 (c) 3.9 days

7. (a) $N(t) = N_0 e^{kt}$ (b) 25,198 9. 9.797 g
11. (a) 9953 years ago



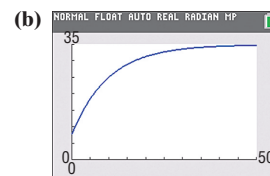
- (c) 5730 yr

13. (a) 5:18 PM



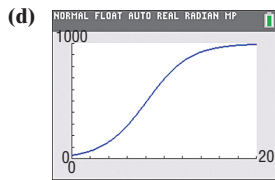
- (c) About 14.3 min (d) The temperature of the pizza approaches 70°F .

15. (a) $18.63^\circ\text{C}; 25.07^\circ\text{C}$



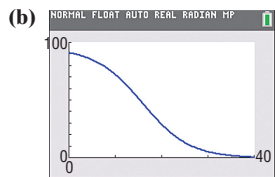
17. 1.7 ppm; 7.17 days, or 172 h 19. 0.26 M; 6.58 h, or 395 min 21. 26.6 days

23. (a) 1000 (b) 43.9% (c) 30 g

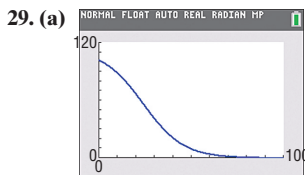


(e) 616.6 g (f) After 9.85 h (g) About 7.9 h

27. (a) In 1984, 91.8% of households did not own a personal computer.

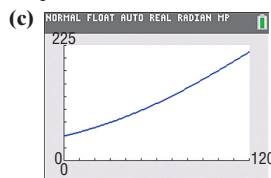


(c) 70.6% (d) During 2011



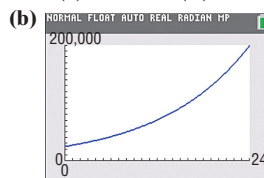
29. (a) (b) 0.78, or 78% (c) 50 people (d) As n increases, the probability decreases.

25. (a) $P(0) \approx 48$; In 1900, about 48 invasive species were present in the Great Lakes. (b) 1.7%



(d) About 176 (e) During 1999

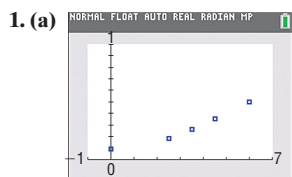
31. (a) $P(t) = 25,000(2)^{t/8}$



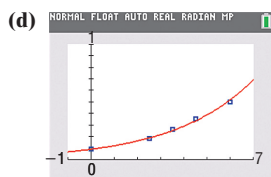
(c) 32,421 people (d) In about 13.42 yr (e) $P(t) = 25,000e^{0.087t}$

33. $f(x) = -\frac{3}{2}x + 7$ 34. Neither 35. $2 \ln x + \frac{1}{2} \ln y - \ln z$ 36. $2\sqrt[3]{5}$

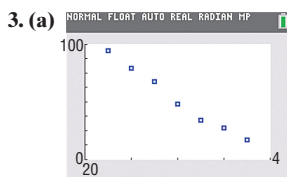
5.9 Assess Your Understanding (page 350)



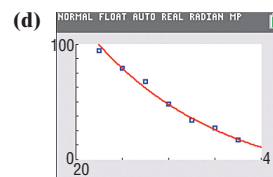
(b) $y = 0.0903(1.3384)^x$
(c) $N(t) = 0.0903e^{0.2915t}$



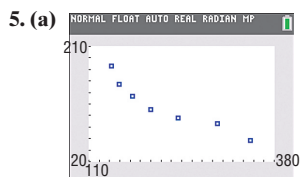
(e) 0.69
(f) After about 7.26 h



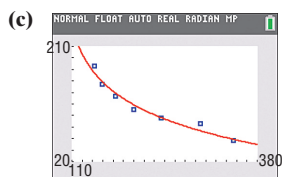
(b) $y = 118.7226(0.7013)^x$
(c) $A(t) = 118.7226e^{-0.3548t}$



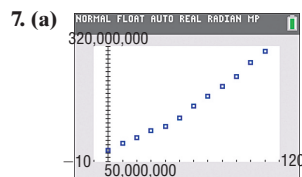
(e) 28.7%
(f) $k = -0.3548 = -35.48\%$ is the exponential growth rate. It represents the rate at which the percent of patients surviving advanced-stage breast cancer is decreasing.



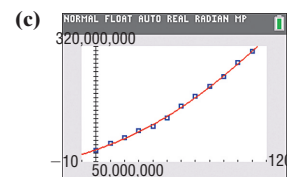
(b) $y = 330.0549 - 34.5008 \ln x$



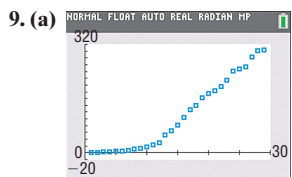
(d) 185 billion pounds
(e) Under by 5 billion pounds



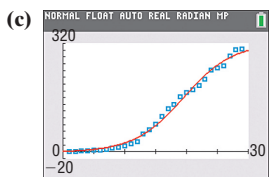
(b) $y = \frac{762,176,844.4}{1 + 8.7428e^{-0.0162x}}$



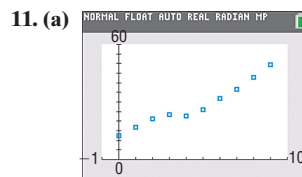
(d) 762,176,844
(e) Approximately 315,203,288 (f) 2023



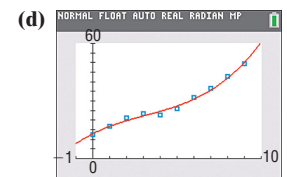
(b) $y = \frac{321.0384}{1 + 135.3081e^{-0.2516x}}$



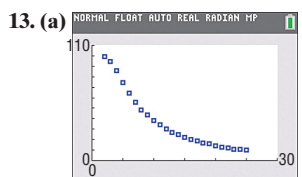
(d) 321.0 thousand cell sites
(e) 314.7 thousand cell sites



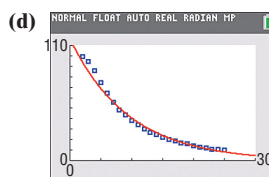
(b) Cubic
(c) $y = 0.0691x^3 - 0.6538x^2 + 4.4323x + 13.0352$



(e) About \$74.6 billion



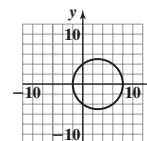
(b) Exponential
(c) $y = 115.5779(0.9012)^x$



(e) 5.1%

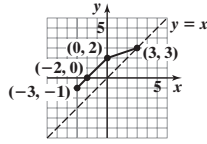
14. $f(x) = \frac{1}{3}(x+3)(x+1)^2(x-2)$

15. $\frac{3\sqrt{2}}{2}$ 16. $R = 7$; no 17.



Review Exercises (page 356)

1. (a) -4 (b) 1 (c) -6 (d) -6 2. (a) -26 (b) -241 (c) 16 (d) -1 3. (a) $\sqrt{11}$ (b) 1 (c) $\sqrt{\sqrt{6}+2}$ (d) 19
 4. (a) e^4 (b) $3e^{-2} - 2$ (c) e^{e^4} (d) -17
 5. $(f \circ g)(x) = 1 - 3x$, all real numbers; $(g \circ f)(x) = 7 - 3x$, all real numbers; $(f \circ f)(x) = x$, all real numbers; $(g \circ g)(x) = 9x + 4$, all real numbers
 6. $(f \circ g)(x) = \sqrt{3 + 3x + 3x^2}$, all real numbers; $(g \circ f)(x) = 1 + \sqrt{3x + 3x}$, $\{x|x \geq 0\}$;
 $(f \circ f)(x) = \sqrt{3\sqrt{3x}}$, $\{x|x \geq 0\}$; $(g \circ g)(x) = 3 + 3x + 4x^2 + 2x^3 + x^4$, all real numbers
 7. $(f \circ g)(x) = \frac{1+x}{1-x}$, $\{x|x \neq 0, x \neq 1\}$; $(g \circ f)(x) = \frac{x-1}{x+1}$, $\{x|x \neq -1, x \neq 1\}$; $(f \circ f)(x) = x$, $\{x|x \neq 1\}$; $(g \circ g)(x) = x$, $\{x|x \neq 0\}$
 8. (a) one-to-one (b) $\{(2, 1), (5, 3), (8, 5), (10, 6)\}$ 9.



10. $f^{-1}(x) = \frac{2x+3}{5x-2}$

$$f(f^{-1}(x)) = \frac{2\left(\frac{2x+3}{5x-2}\right) + 3}{5\left(\frac{2x+3}{5x-2}\right) - 2} = x$$

$$f^{-1}(f(x)) = \frac{2\left(\frac{2x+3}{5x-2}\right) + 3}{5\left(\frac{2x+3}{5x-2}\right) - 2} = x$$

Domain of f = range of f^{-1} = all real numbers except $\frac{2}{5}$

Range of f = domain of f^{-1} = all real numbers except $\frac{2}{5}$

11. $f^{-1}(x) = \frac{x+1}{x}$

$$f(f^{-1}(x)) = \frac{1}{\frac{x+1}{x} - 1} = x$$

$$f^{-1}(f(x)) = \frac{\frac{1}{x-1} + 1}{\frac{1}{x-1}} = x$$

Domain of f = range of f^{-1} = all real numbers except 1
 Range of f = domain of f^{-1} = all real numbers except 0

12. $f^{-1}(x) = x^2 + 2, x \geq 0$

$$f(f^{-1}(x)) = \sqrt{x^2 + 2} - 2 = |x| = x, x \geq 0$$

$$f^{-1}(f(x)) = (\sqrt{x-2})^2 + 2 = x$$

Domain of f = range of f^{-1} = $[2, \infty)$

Range of f = domain of f^{-1} = $[0, \infty)$

13. $f^{-1}(x) = (x-1)^3$;

$$f(f^{-1}(x)) = ((x-1)^3 + 1)^{1/3} + 1 = x$$

$$f^{-1}(f(x)) = (x^{1/3} + 1 - 1)^3 = x$$

Domain of f = range of f^{-1} = $(-\infty, \infty)$

Range of f = domain of f^{-1} = $(-\infty, \infty)$

14. (a) 81 (b) 2 (c) $\frac{1}{9}$ (d) -3

15. $\log_5 z = 2$ 16. $5^{13} = u$ 17. $\left\{x \mid x > \frac{2}{3}\right\}; \left(\frac{2}{3}, \infty\right)$

18. $\{x|x < 1 \text{ or } x > 2\}; (-\infty, 1) \cup (2, \infty)$

19. -3 20. $\sqrt{2}$ 21. 0.4

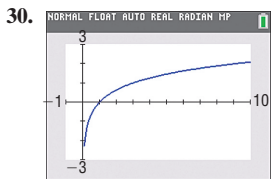
22. $\log_3 u + 2 \log_3 v - \log_3 w$ 23. $8 \log_2 a + 2 \log_2 b$

24. $2 \log x + \frac{1}{2} \log(x^3 + 1)$

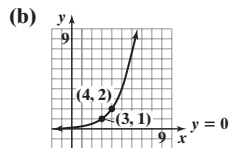
25. $2 \ln(2x + 3) - 2 \ln(x - 1) - 2 \ln(x - 2)$

26. $\frac{25}{4} \log_4 x$

27. $-2 \ln(x + 1)$ 28. $\ln \left[\frac{16\sqrt{x^2+1}}{\sqrt{x(x-4)}} \right]$ 29. 2.124



31. (a) Domain of f : $(-\infty, \infty)$



(c) Range of f : $(0, \infty)$
 Horizontal asymptote: $y = 0$

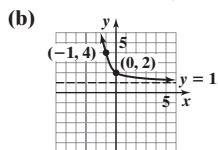
(d) $f^{-1}(x) = 3 + \log_2 x$

(e) Domain of f^{-1} : $(0, \infty)$

Range of f^{-1} : $(-\infty, \infty)$

(f)

32. (a) Domain of f : $(-\infty, \infty)$

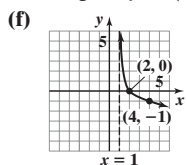


(c) Range of f : $(1, \infty)$
 Horizontal asymptote: $y = 1$

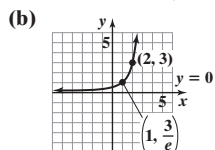
(d) $f^{-1}(x) = -\log_3(x - 1)$

(e) Domain of f^{-1} : $(1, \infty)$

Range of f^{-1} : $(-\infty, \infty)$



33. (a) Domain of f : $(-\infty, \infty)$

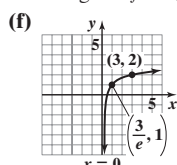


(c) Range of f : $(0, \infty)$
 Horizontal asymptote: $y = 0$

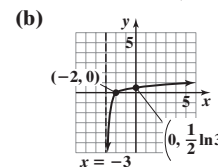
(d) $f^{-1}(x) = 2 + \ln\left(\frac{x}{3}\right)$

(e) Domain of f^{-1} : $(0, \infty)$

Range of f^{-1} : $(-\infty, \infty)$



34. (a) Domain of f : $(-3, \infty)$

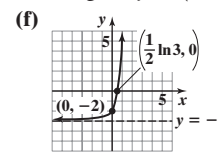


(c) Range of f : $(-\infty, \infty)$
 Vertical asymptote: $x = -3$

(d) $f^{-1}(x) = e^{2x} - 3$

(e) Domain of f^{-1} : $(-\infty, \infty)$

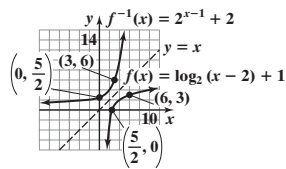
Range of f^{-1} : $(-3, \infty)$



35. $\left\{-\frac{16}{9}\right\}$ 36. $\left\{\frac{-1-\sqrt{3}}{2}, \frac{-1+\sqrt{3}}{2}\right\} \approx \{-1.366, 0.366\}$ 37. $\left\{\frac{1}{4}\right\}$ 38. $\left\{\frac{2 \ln 3}{\ln 5 - \ln 3}\right\} \approx \{4.301\}$ 39. $\{-2, 6\}$ 40. $\{83\}$ 41. $\left\{\frac{1}{2}, -3\right\}$

42. $\{1\}$ 43. $\{-1\}$ 44. $\{1 - \ln 5\} \approx \{-0.609\}$ 45. $\left\{\log_3(-2 + \sqrt{7})\right\} = \left\{\frac{\ln(-2 + \sqrt{7})}{\ln 3}\right\} \approx \{-0.398\}$

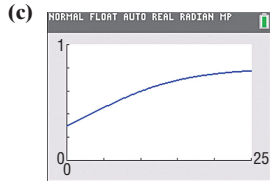
46. (a), (e)



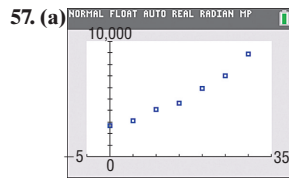
- (b) 3; (6, 3)
 (c) 10; (10, 4)
 (d) $\left\{x \mid x > \frac{5}{2}\right\}$ or $\left(\frac{5}{2}, \infty\right)$
 (e) $f^{-1}(x) = 2^{x-1} + 2$

47. (a) 37.3 W (b) 6.9 dB 48. (a) 11.77 (b) 9.56 in.
 49. (a) 9.85 yr (b) 4.27 yr 50. \$20,398.87; 4.04%; 17.5 yr
 51. \$41,668.97 52. 24,765 yr ago 53. 55.22 min, or 55 min, 13 sec
 54. 7,615,278,125 55. 7.204 g; 0.519 g

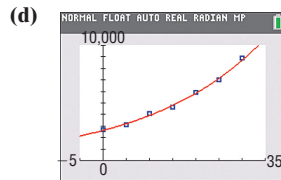
56. (a) 0.3 (b) 0.8



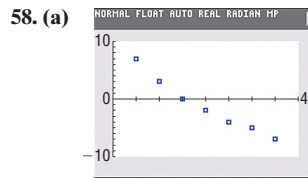
(d) In 2026



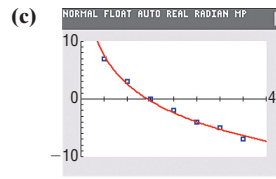
(b) $y = 2638.26(1.0407)^x$
 (c) $A(t) = 2638.26e^{0.0399t}$



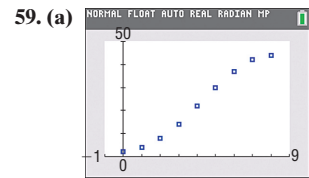
(e) 2021–22



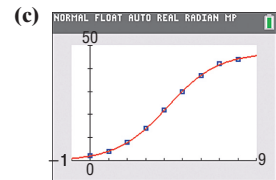
(b) $y = 18.921 - 7.096 \ln x$



(d) Approximately -3°F



(b) $C = \frac{46.9292}{1 + 21.2733e^{-0.7306t}}$



- (d) About 47 people; 50 people
 (e) 2.4 days; during the tenth hour of day 3
 (f) 9.5 days

Chapter Test (page 359)

1. (a) $f \circ g = \frac{2x+7}{2x+3}$; domain: $\left\{x \mid x \neq -\frac{3}{2}\right\}$ (b) $(g \circ f)(-2) = 5$ (c) $(f \circ g)(-2) = -3$

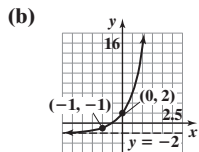
2. (a) The function is not one-to-one. (b) The function is one-to-one.

3. $f^{-1}(x) = \frac{2+5x}{3x}$; domain of $f = \left\{x \mid x \neq \frac{5}{3}\right\}$, range of $f = \{y \mid y \neq 0\}$; domain of $f^{-1} = \{x \mid x \neq 0\}$; range of $f^{-1} = \left\{y \mid y \neq \frac{5}{3}\right\}$

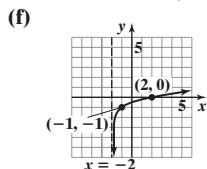
4. The point $(-5, 3)$ must be on the graph of f^{-1} . 5. $\{5\}$ 6. $\{4\}$ 7. $\{625\}$ 8. $e^3 + 2 \approx 22.086$ 9. $\log 20 \approx 1.301$

10. $\log_3 21 = \frac{\ln 21}{\ln 3} \approx 2.771$ 11. $\ln 133 \approx 4.890$

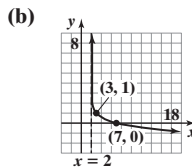
12. (a) Domain of $f: \{x \mid -\infty < x < \infty\}$ or $(-\infty, \infty)$



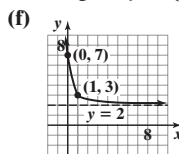
- (b) (c) Range of $f: \{y \mid y > -2\}$ or $(-2, \infty)$; Horizontal asymptote: $y = -2$
 (d) $f^{-1}(x) = \log_4(x+2) - 1$
 (e) Domain of $f^{-1}: \{x \mid x > -2\}$ or $(-2, \infty)$; Range of $f^{-1}: \{y \mid -\infty < y < \infty\}$ or $(-\infty, \infty)$



13. (a) Domain of $f: \{x \mid x > 2\}$ or $(2, \infty)$



- (b) (c) Range of $f: \{y \mid -\infty < y < \infty\}$ or $(-\infty, \infty)$; vertical asymptote: $x = 2$
 (d) $f^{-1}(x) = 5^{1-x} + 2$
 (e) Domain of $f^{-1}: \{x \mid -\infty < x < \infty\}$ or $(-\infty, \infty)$; Range of $f^{-1}: \{y \mid y > 2\}$ or $(2, \infty)$



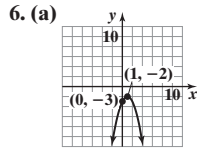
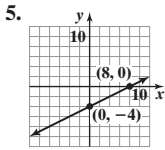
14. $\{1\}$ 15. $\{91\}$ 16. $\{-\ln 2\} \approx \{-0.693\}$ 17. $\left\{\frac{1-\sqrt{13}}{2}, \frac{1+\sqrt{13}}{2}\right\} \approx \{-1.303, 2.303\}$ 18. $\left\{\frac{3 \ln 7}{1 - \ln 7}\right\} \approx \{-6.172\}$

19. $\{2\sqrt{6}\} \approx \{4.899\}$ 20. $2 + 3 \log_2 x - \log_2(x-6) - \log_2(x+3)$ 21. About 250.39 days 22. (a) \$1033.82 (b) \$963.42 (c) 11.9 yr
 23. (a) About 83 dB (b) The pain threshold will be exceeded if 31,623 people shout at the same time.

Cumulative Review (page 360)

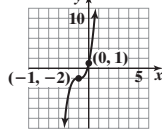
1. (a) Yes; no (b) Polynomial; the graph is smooth and continuous. 2. (a) 10 (b) $2x^2 + 3x + 1$ (c) $2x^2 + 4xh + 2h^2 - 3x - 3h + 1$

3. $(\frac{1}{2}, \frac{\sqrt{3}}{2})$ is on the graph. 4. $\{-26\}$



(b) All real numbers; $(-\infty, \infty)$ 7. $f(x) = 2(x - 4)^2 - 8 = 2x^2 - 16x + 24$

8. Exponential; $f(x) = 2 \cdot 3^x$ 9.



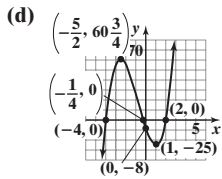
10. (a) $f(g(x)) = \frac{4}{(x - 3)^2} + 2$; domain: $\{x | x \neq 3\}$; 3

(b) $f(g(x)) = \log_2 x + 2$; domain: $\{x | x > 0\}$; $2 + \log_2 14$

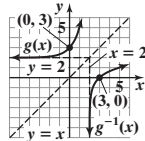
11. (a) Zeros: $-4, -\frac{1}{4}, 2$

(b) x -intercepts: $-4, -\frac{1}{4}, 2$;
 y -intercept: -8

(c) Local maximum value of 60.75 occurs at $x = -2.5$.
Local minimum value of -25 occurs at $x = 1$.



12. (a), (c)



Domain $g = \text{range } g^{-1} = (-\infty, \infty)$

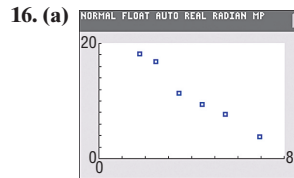
Range $g = \text{domain } g^{-1} = (2, \infty)$

(b) $g^{-1}(x) = \log_3(x - 2)$

13. $\{-\frac{3}{2}\}$ 14. $\{2\}$

15. (a) $\{-1\}$ (b) $\{x | x > -1\}$ or $(-1, \infty)$

(c) $\{25\}$



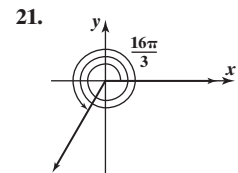
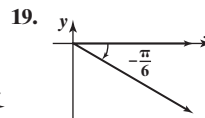
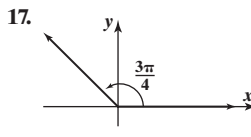
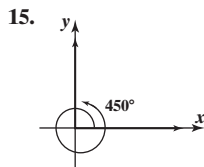
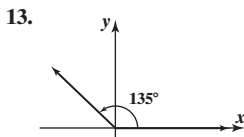
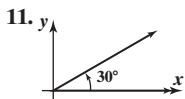
(b) Logarithmic; $y = 49.293 - 10.563 \ln x$

(c) Highest value of $|r|$

CHAPTER 6 Trigonometric Functions

6.1 Assess Your Understanding (page 372)

3. standard position 4. central angle 5. d 6. $r\theta; \frac{1}{2}r^2\theta$ 7. b 8. $\frac{s}{t}; \frac{\theta}{t}$ 9. T 10. F



23. $\frac{\pi}{6}$ 25. $\frac{4\pi}{3}$ 27. $-\frac{\pi}{3}$ 29. π 31. $-\frac{3\pi}{4}$ 33. $-\frac{\pi}{2}$ 35. 60° 37. -225° 39. 90° 41. 15° 43. -90° 45. -30° 47. 0.30 49. -0.70 51. 2.18

53. 179.91° 55. 114.59° 57. 362.11° 59. 40.17° 61. 50.24° 63. 9.15° 65. 40°19'12" 67. 18°15'18" 69. 19°59'24" 71. 5 m 73. 6 ft

75. 0.6 radian 77. $\frac{\pi}{3} \approx 1.047$ in. 79. 25 m² 81. $2\sqrt{3} \approx 3.464$ ft 83. 0.24 radian 85. $\frac{\pi}{3} \approx 1.047$ in.² 87. $s = 2.094$ ft; $A = 2.094$ ft²

89. $s = 14.661$ yd; $A = 87.965$ yd² 91. $3\pi \approx 9.42$ in; $5\pi \approx 15.71$ in. 93. $2\pi \approx 6.28$ m² 95. $\frac{675\pi}{2} \approx 1060.29$ ft² 97. $\frac{1075\pi}{3} \approx 1125.74$ in.²

99. $\omega = \frac{1}{60}$ radian/s; $v = \frac{1}{12}$ cm/s 101. ≈ 23.2 mph 103. ≈ 120.6 km/h 105. ≈ 452.5 rpm 107. ≈ 359 mi 109. ≈ 898 mi/h 111. ≈ 2292 mi/h

113. $\frac{3}{4}$ rpm 115. ≈ 2.86 mi/h 117. ≈ 31.47 rpm 119. 63π ft² 121. ≈ 1037 mi/h 123. Radius ≈ 3979 mi; circumference $\approx 25,000$ mi

125. $v_1 = r_1\omega_1, v_2 = r_2\omega_2$, and $v_1 = v_2$, so $r_1\omega_1 = r_2\omega_2$ and $\frac{r_1}{r_2} = \frac{\omega_2}{\omega_1}$ 134. $x = -\frac{7}{3}$ 135. $\{x | x \neq \pm 3\}$ 136. $y = -|x + 3| - 4$ 137. HA: $y = 3$; VA: $x = 7$

6.2 Assess Your Understanding (page 389)

7. b 8. (0, 1) 9. $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ 10. a 11. $\frac{y}{r}, \frac{x}{r}$ 12. F 13. $\sin t = \frac{1}{2}; \cos t = \frac{\sqrt{3}}{2}; \tan t = \frac{\sqrt{3}}{3}; \csc t = 2; \sec t = \frac{2\sqrt{3}}{3}; \cot t = \sqrt{3}$

15. $\sin t = \frac{\sqrt{21}}{5}; \cos t = -\frac{2}{5}; \tan t = -\frac{\sqrt{21}}{2}; \csc t = \frac{5\sqrt{21}}{21}; \sec t = -\frac{5}{2}; \cot t = -\frac{2\sqrt{21}}{21}$ 17. $\sin t = \frac{\sqrt{2}}{2}; \cos t = -\frac{\sqrt{2}}{2};$

$\tan t = -1; \csc t = \sqrt{2}; \sec t = -\sqrt{2}; \cot t = -1$ 19. $\sin t = -\frac{1}{3}; \cos t = \frac{2\sqrt{2}}{3}; \tan t = -\frac{\sqrt{2}}{4}; \csc t = -3; \sec t = \frac{3\sqrt{2}}{4}; \cot t = -2\sqrt{2}$

21. -1 23. 0 25. -1 27. 0 29. -1 31. $\frac{1}{2}(\sqrt{2} + 1)$ 33. 2 35. $\frac{1}{2}$ 37. $\sqrt{6}$ 39. 4 41. 0 43. $2\sqrt{2} + \frac{4\sqrt{3}}{3}$ 45. 1

47. $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}; \cos \frac{2\pi}{3} = -\frac{1}{2}; \tan \frac{2\pi}{3} = -\sqrt{3}; \csc \frac{2\pi}{3} = \frac{2\sqrt{3}}{3}; \sec \frac{2\pi}{3} = -2; \cot \frac{2\pi}{3} = -\frac{\sqrt{3}}{3}$

49. $\sin 210^\circ = -\frac{1}{2}; \cos 210^\circ = -\frac{\sqrt{3}}{2}; \tan 210^\circ = \frac{\sqrt{3}}{3}; \csc 210^\circ = -2; \sec 210^\circ = -\frac{2\sqrt{3}}{3}; \cot 210^\circ = \sqrt{3}$

51. $\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}; \cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}; \tan \frac{3\pi}{4} = -1; \csc \frac{3\pi}{4} = \sqrt{2}; \sec \frac{3\pi}{4} = -\sqrt{2}; \cot \frac{3\pi}{4} = -1$

53. $\sin \frac{8\pi}{3} = \frac{\sqrt{3}}{2}; \cos \frac{8\pi}{3} = -\frac{1}{2}; \tan \frac{8\pi}{3} = -\sqrt{3}; \csc \frac{8\pi}{3} = \frac{2\sqrt{3}}{3}; \sec \frac{8\pi}{3} = -2; \cot \frac{8\pi}{3} = -\frac{\sqrt{3}}{3}$

55. $\sin 405^\circ = \frac{\sqrt{2}}{2}; \cos 405^\circ = \frac{\sqrt{2}}{2}; \tan 405^\circ = 1; \csc 405^\circ = \sqrt{2}; \sec 405^\circ = \sqrt{2}; \cot 405^\circ = 1$

57. $\sin(-\frac{\pi}{6}) = -\frac{1}{2}; \cos(-\frac{\pi}{6}) = \frac{\sqrt{3}}{2}; \tan(-\frac{\pi}{6}) = -\frac{\sqrt{3}}{3}; \csc(-\frac{\pi}{6}) = -2; \sec(-\frac{\pi}{6}) = \frac{2\sqrt{3}}{3}; \cot(-\frac{\pi}{6}) = -\sqrt{3}$

59. $\sin(-135^\circ) = -\frac{\sqrt{2}}{2}; \cos(-135^\circ) = -\frac{\sqrt{2}}{2}; \tan(-135^\circ) = 1; \csc(-135^\circ) = -\sqrt{2}; \sec(-135^\circ) = -\sqrt{2}; \cot(-135^\circ) = 1$

61. $\sin \frac{5\pi}{2} = 1; \cos \frac{5\pi}{2} = 0; \tan \frac{5\pi}{2}$ is undefined; $\csc \frac{5\pi}{2} = 1; \sec \frac{5\pi}{2}$ is undefined; $\cot \frac{5\pi}{2} = 0$

63. $\sin(-\frac{14\pi}{3}) = -\frac{\sqrt{3}}{2}; \cos(-\frac{14\pi}{3}) = -\frac{1}{2}; \tan(-\frac{14\pi}{3}) = \sqrt{3}; \csc(-\frac{14\pi}{3}) = -\frac{2\sqrt{3}}{3}; \sec(-\frac{14\pi}{3}) = -2; \cot(-\frac{14\pi}{3}) = \frac{\sqrt{3}}{3}$

65. 0.47 67. 1.07 69. 0.32 71. 3.73 73. 0.84 75. 0.02 77. $\sin \theta = \frac{4}{5}; \cos \theta = -\frac{3}{5}; \tan \theta = -\frac{4}{3}; \csc \theta = \frac{5}{4}; \sec \theta = -\frac{5}{3}; \cot \theta = -\frac{3}{4}$

79. $\sin \theta = -\frac{3\sqrt{13}}{13}; \cos \theta = \frac{2\sqrt{13}}{13}; \tan \theta = -\frac{3}{2}; \csc \theta = -\frac{\sqrt{13}}{3}; \sec \theta = \frac{\sqrt{13}}{2}; \cot \theta = -\frac{2}{3}$

81. $\sin \theta = -\frac{\sqrt{2}}{2}; \cos \theta = -\frac{\sqrt{2}}{2}; \tan \theta = 1; \csc \theta = -\sqrt{2}; \sec \theta = -\sqrt{2}; \cot \theta = 1$

83. $\sin \theta = \frac{3}{5}; \cos \theta = \frac{4}{5}; \tan \theta = \frac{3}{4}; \csc \theta = \frac{5}{3}; \sec \theta = \frac{5}{4}; \cot \theta = \frac{4}{3}$ 85. 0 87. 0 89. -0.1 91. 3 93. 5 95. $\frac{\sqrt{3}}{2}$ 97. $\frac{1}{2}$ 99. $\frac{3}{4}$ 101. $\frac{\sqrt{3}}{2}$

103. $\sqrt{3}$ 105. $-\frac{\sqrt{3}}{2}$ 107. $\frac{1 + \sqrt{3}}{2}$ 109. $-\frac{1}{2}$ 111. $\frac{\sqrt{3}}{2}$ 113. $\frac{\sqrt{2}}{4}$ 115. (a) $\frac{\sqrt{2}}{2}; (\frac{\pi}{4}, \frac{\sqrt{2}}{2})$ (b) $(\frac{\sqrt{2}}{2}, \frac{\pi}{4})$ (c) $(\frac{\pi}{4}, -2)$

117. Answers may vary. One set of possible answers is $-\frac{11\pi}{3}, -\frac{5\pi}{3}, \frac{\pi}{3}, \frac{7\pi}{3}, \frac{13\pi}{3}$.

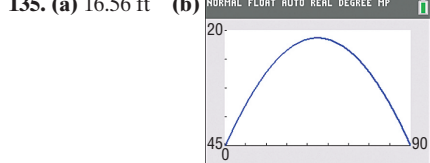
119.

θ	0.5	0.4	0.2	0.1	0.01	0.001	0.0001	0.00001
$\sin \theta$	0.4794	0.3894	0.1987	0.0998	0.0100	0.0010	0.0001	0.00001
$\frac{\sin \theta}{\theta}$	0.9589	0.9735	0.9933	0.9983	1.0000	1.0000	1.0000	1.0000

$\frac{\sin \theta}{\theta}$ approaches 1 as θ approaches 0.

121. $R \approx 310.56$ ft; $H \approx 77.64$ ft 123. $R \approx 19,541.95$ m; $H \approx 2278.14$ m 125. (a) 1.20 sec (b) 1.12 sec (c) 1.20 sec

127. (a) 1.9 hr; 0.57 hr (b) 1.69 hr; 0.75 hr (c) 1.63 hr; 0.86 hr (d) 1.67 hr; $\tan 90^\circ$ is undefined 129. 15.4 ft 131. 71 ft 133. $y = -\frac{4\sqrt{3}}{7}$



(c) 67.5° 137. (a) values estimated to the nearest tenth:

$\sin 1 \approx 0.8; \cos 1 \approx 0.5; \tan 1 \approx 1.6; \csc 1 \approx 1.3; \sec 1 \approx 2.0; \cot 1 \approx 0.6$; actual values to the nearest tenth: $\sin 1 \approx 0.8; \cos 1 \approx 0.5; \tan 1 \approx 1.6; \csc 1 \approx 1.2; \sec 1 \approx 1.9; \cot 1 \approx 0.6$

(b) values estimated to the nearest tenth: $\sin 5.1 \approx -0.9; \cos 5.1 \approx 0.4; \tan 5.1 \approx -2.3; \csc 5.1 \approx -1.1; \sec 5.1 \approx 2.5; \cot 5.1 \approx -0.4$; actual values to the nearest tenth: $\sin 5.1 \approx -0.9; \cos 5.1 \approx 0.4; \tan 5.1 \approx -2.4; \csc 5.1 \approx -1.1; \sec 5.1 \approx 2.6; \cot 5.1 \approx -0.4$

143. $\{x \mid x > -\frac{2}{5}\}$ or $(-\frac{2}{5}, \infty)$ 144. $4 - 3i, -5$ 145. $R = 134$ 146. $\{-10\}$

6.3 Assess Your Understanding (page 405)

5. $2\pi; \pi$ 6. All real numbers except odd multiples of $\frac{\pi}{2}$ 7. b 8. a 9. 1 10. F 11. $\frac{\sqrt{2}}{2}$ 13. 1 15. 1 17. $\sqrt{3}$ 19. $\frac{\sqrt{2}}{2}$ 21. 0 23. $\sqrt{2}$ 25. $\frac{\sqrt{3}}{3}$
 27. II 29. IV 31. IV 33. II 35. $\tan \theta = -\frac{3}{4}; \cot \theta = -\frac{4}{3}; \sec \theta = \frac{5}{4}; \csc \theta = -\frac{5}{3}$ 37. $\tan \theta = 2; \cot \theta = \frac{1}{2}; \sec \theta = \sqrt{5}; \csc \theta = \frac{\sqrt{5}}{2}$
 39. $\tan \theta = \frac{\sqrt{3}}{3}; \cot \theta = \sqrt{3}; \sec \theta = \frac{2\sqrt{3}}{3}; \csc \theta = 2$ 41. $\tan \theta = -\frac{\sqrt{2}}{4}; \cot \theta = -2\sqrt{2}; \sec \theta = \frac{3\sqrt{2}}{4}; \csc \theta = -3$
 43. $\cos \theta = -\frac{5}{13}; \tan \theta = -\frac{12}{5}; \csc \theta = \frac{13}{12}; \sec \theta = -\frac{13}{5}; \cot \theta = -\frac{5}{12}$ 45. $\sin \theta = -\frac{3}{5}; \tan \theta = \frac{3}{4}; \csc \theta = -\frac{5}{3}; \sec \theta = -\frac{5}{4}; \cot \theta = \frac{4}{3}$
 47. $\cos \theta = -\frac{12}{13}; \tan \theta = -\frac{5}{12}; \csc \theta = \frac{13}{5}; \sec \theta = -\frac{13}{12}; \cot \theta = -\frac{12}{5}$ 49. $\sin \theta = \frac{2\sqrt{2}}{3}; \tan \theta = -2\sqrt{2}; \csc \theta = \frac{3\sqrt{2}}{4}; \sec \theta = -3; \cot \theta = -\frac{\sqrt{2}}{4}$
 51. $\cos \theta = -\frac{\sqrt{5}}{3}; \tan \theta = -\frac{2\sqrt{5}}{5}; \csc \theta = \frac{3}{2}; \sec \theta = -\frac{3\sqrt{5}}{5}; \cot \theta = -\frac{\sqrt{5}}{2}$ 53. $\sin \theta = -\frac{\sqrt{3}}{2}; \cos \theta = \frac{1}{2}; \tan \theta = -\sqrt{3};$
 $\csc \theta = -\frac{2\sqrt{3}}{3}; \cot \theta = -\frac{\sqrt{3}}{3}$ 55. $\sin \theta = -\frac{3}{5}; \cos \theta = -\frac{4}{5}; \csc \theta = -\frac{5}{3}; \sec \theta = -\frac{5}{4}; \cot \theta = \frac{4}{3}$ 57. $\sin \theta = \frac{\sqrt{10}}{10}; \cos \theta = -\frac{3\sqrt{10}}{10};$
 $\csc \theta = \sqrt{10}; \sec \theta = -\frac{\sqrt{10}}{3}; \cot \theta = -3$ 59. $-\frac{\sqrt{3}}{2}$ 61. $-\frac{\sqrt{3}}{3}$ 63. 2 65. -1 67. -1 69. $\frac{\sqrt{2}}{2}$ 71. 0 73. $-\sqrt{2}$ 75. $\frac{2\sqrt{3}}{3}$

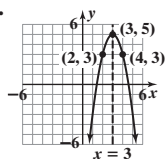
77. 1 79. 1 81. 0 83. 1 85. -1 87. 0 89. 0.9 91. 9 93. 0 95. All real numbers 97. Odd multiples of $\frac{\pi}{2}$ 99. Odd multiples of $\frac{\pi}{2}$
 101. $-1 \leq y \leq 1$ 103. All real numbers 105. $|y| \geq 1$ 107. Odd; yes; origin 109. Odd; yes; origin 111. Even; yes; y-axis
 113. (a) $-\frac{1}{3}$ (b) 1 115. (a) -2 (b) 6 117. (a) -4 (b) -12 119. ≈ 15.81 min 121. Let a be a real number and $P = (x, y)$ be the point on the unit circle that corresponds to t . Consider the equation $\tan t = \frac{y}{x} = a$. Then $y = ax$. But $x^2 + y^2 = 1$, so $x^2 + a^2x^2 = 1$. So $x = \pm \frac{1}{\sqrt{1+a^2}}$ and $y = \pm \frac{a}{\sqrt{1+a^2}}$; that is, for any real number a , there is a point $P = (x, y)$ on the unit circle for which $\tan t = a$. In other words, the range of the tangent function is the set of all real numbers. 123. Suppose that there is a number $p, 0 < p < 2\pi$, for which $\sin(\theta + p) = \sin \theta$ for all θ . If $\theta = 0$, then $\sin(0 + p) = \sin p = \sin 0 = 0$, so $p = \pi$. If $\theta = \frac{\pi}{2}$, then $\sin(\frac{\pi}{2} + p) = \sin(\frac{\pi}{2})$. But $p = \pi$. Thus, $\sin(\frac{3\pi}{2}) = -1 = \sin(\frac{\pi}{2}) = 1$. This is impossible. Therefore, the smallest positive number p for which $\sin(\theta + p) = \sin \theta$ for all θ is 2π . 125. $\sec \theta = \frac{1}{\cos \theta}$; since $\cos \theta$ has period 2π , so does $\sec \theta$. 127. If $P = (a, b)$ is the point on the unit circle corresponding to θ , then $Q = (-a, -b)$ is the point on the unit circle corresponding to $\theta + \pi$. Thus, $\tan(\theta + \pi) = \frac{-b}{-a} = \frac{b}{a} = \tan \theta$. Suppose that there exists a number $p, 0 < p < \pi$, for which $\tan(\theta + p) = \tan \theta$ for all θ . Then, if $\theta = 0$, then $\tan p = \tan 0 = 0$. But this means that p is a multiple of π . Since no multiple of π exists in the interval $(0, \pi)$, this is a contradiction. Therefore, the period of $f(\theta) = \tan \theta$ is π .

129. Let $P = (a, b)$ be the point on the unit circle corresponding to θ . Then $\csc \theta = \frac{1}{b} = \frac{1}{\sin \theta}; \sec \theta = \frac{1}{a} = \frac{1}{\cos \theta}; \cot \theta = \frac{a}{b} = \frac{1}{b/a} = \frac{1}{\tan \theta}$.

131. $(\sin \theta \cos \phi)^2 + (\sin \theta \sin \phi)^2 + \cos^2 \theta = \sin^2 \theta \cos^2 \phi + \sin^2 \theta \sin^2 \phi + \cos^2 \theta = \sin^2 \theta (\cos^2 \phi + \sin^2 \phi) + \cos^2 \theta = \sin^2 \theta + \cos^2 \theta = 1$

137. $(f \circ g)(x) = x^2 - 14x + 46$

138.



Vertex: (3, 5)
axis of symmetry: $x = 3$

139. $\{\ln 6 + 4\}$

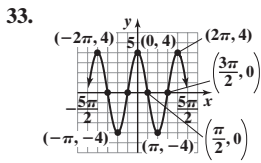
140. $y = \frac{1}{4}x + \frac{31}{4}$

6.4 Assess Your Understanding (page 418)

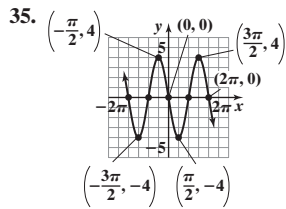
3. 1; $\frac{\pi}{2}$ 4. 3; π 5. 3; $\frac{\pi}{3}$ 6. T 7. F 8. T 9. d 10. d 11. (a) 0 (b) $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ (c) 1 (d) $0, \pi, 2\pi$

(e) $f(x) = 1$ for $x = -\frac{3\pi}{2}, \frac{\pi}{2}; f(x) = -1$ for $x = -\frac{\pi}{2}, \frac{3\pi}{2}$ (f) $-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ (g) $\{x | x = k\pi, k \text{ an integer}\}$ 13. Amplitude = 2; period = 2π

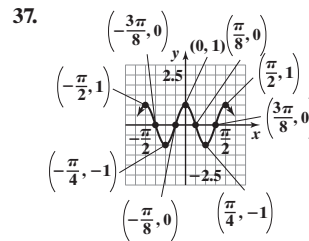
15. Amplitude = 4; period = π 17. Amplitude = 6; period = 2 19. Amplitude = $\frac{1}{2}$; period = $\frac{4\pi}{3}$ 21. Amplitude = $\frac{5}{3}$; period = 3 23. F 25. A 27. H 29. C 31. J



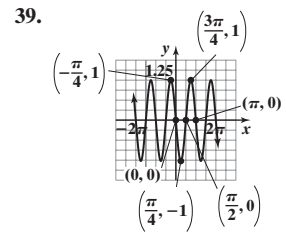
Domain: $(-\infty, \infty)$
Range: $[-4, 4]$



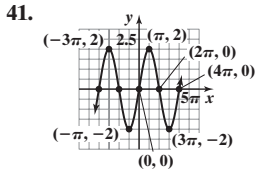
Domain: $(-\infty, \infty)$
Range: $[-4, 4]$



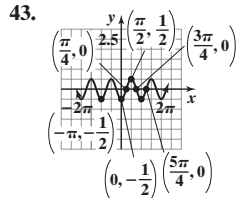
Domain: $(-\infty, \infty)$
Range: $[-1, 1]$



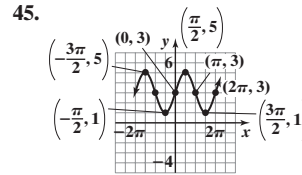
Domain: $(-\infty, \infty)$
Range: $[-1, 1]$



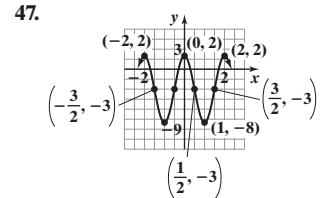
Domain: $(-\infty, \infty)$
Range: $[-2, 2]$



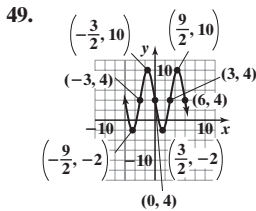
Domain: $(-\infty, \infty)$
Range: $[-\frac{1}{2}, \frac{1}{2}]$



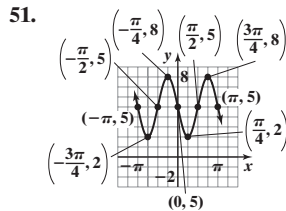
Domain: $(-\infty, \infty)$
Range: $[1, 5]$



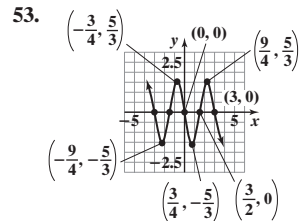
Domain: $(-\infty, \infty)$
Range: $[-8, 2]$



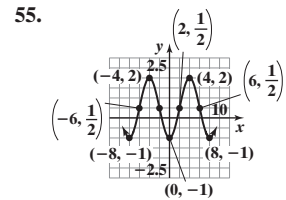
Domain: $(-\infty, \infty)$
Range: $[-2, 10]$



Domain: $(-\infty, \infty)$
Range: $[2, 8]$



Domain: $(-\infty, \infty)$
Range: $[-\frac{5}{3}, \frac{5}{3}]$

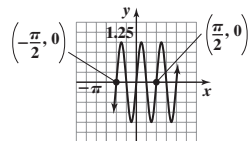


Domain: $(-\infty, \infty)$
Range: $[-1, 2]$

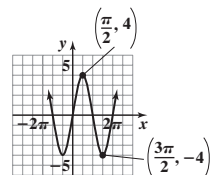
57. $y = \pm 3 \sin(2x)$ 59. $y = \pm 3 \sin(\pi x)$ 61. $y = 5 \cos(\frac{\pi}{4}x)$ 63. $y = -3 \cos(\frac{1}{2}x)$ 65. $y = \frac{3}{4} \sin(2\pi x)$ 67. $y = -\sin(\frac{3}{2}x)$

69. $y = -\cos(\frac{4\pi}{3}x) + 1$ 71. $y = 3 \sin(\frac{\pi}{2}x)$ 73. $y = -4 \cos(3x)$ 75. $\frac{2}{\pi}$ 77. $\frac{\sqrt{2}}{\pi}$

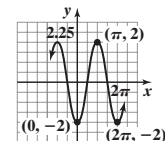
79. $(f \circ g)(x) = \sin(4x)$



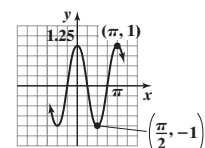
$(g \circ f)(x) = 4 \sin x$



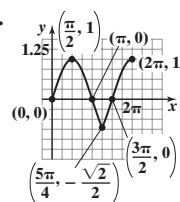
81. $(f \circ g)(x) = -2 \cos x$



$(g \circ f)(x) = \cos(-2x)$



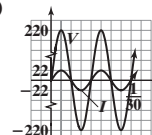
83.



87. (a) Amplitude = 220 V

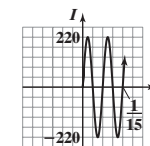
Period = $\frac{1}{60}$ s

(b), (e)



85. Period = $\frac{1}{30}$ s

Amplitude = 220 amp

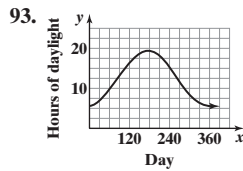
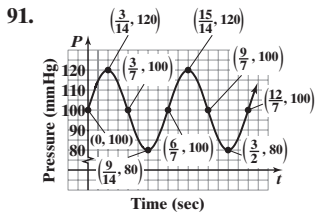


(c) $I(t) = 22 \sin(120\pi t)$

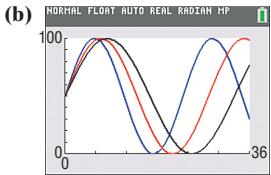
(d) Amplitude = 22 amp

Period = $\frac{1}{60}$ s

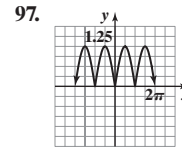
89. (a) $P(t) = \frac{[V_0 \sin(2\pi ft)]^2}{R} = \frac{V_0^2}{R} \sin^2(2\pi ft)$ (b) Since the graph of P has amplitude $\frac{V_0^2}{2R}$ and period $\frac{1}{2f}$ and is of the form $y = A \cos(\omega t) + B$, then $A = -\frac{V_0^2}{2R}$ and $B = \frac{V_0^2}{2R}$. Since $\frac{1}{2f} = \frac{2\pi}{\omega}$, then $\omega = 4\pi f$. Therefore, $P(t) = -\frac{V_0^2}{2R} \cos(4\pi ft) + \frac{V_0^2}{2R} = \frac{V_0^2}{2R} [1 - \cos(4\pi ft)]$.



95. (a) Physical potential: $\omega = \frac{2\pi}{23}$; emotional potential: $\omega = \frac{\pi}{14}$; intellectual potential: $\omega = \frac{2\pi}{33}$



(c) No (d) Physical potential peaks at 15 days after 20th birthday. Emotional potential is 50% at 17 days, with a maximum at 10 days and a minimum at 24 days. Intellectual potential starts fairly high, drops to a minimum at 13 days, and rises to a maximum at 29 days.



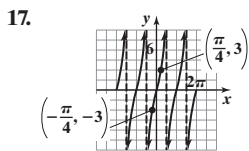
99. Answers may vary. $(-\frac{5\pi}{3}, \frac{1}{2}), (-\frac{\pi}{3}, \frac{1}{2}), (\frac{\pi}{3}, \frac{1}{2}), (\frac{5\pi}{3}, \frac{1}{2})$ 101. Answers may vary. $(-\frac{3\pi}{4}, 1), (\frac{\pi}{4}, 1), (\frac{5\pi}{4}, 1), (\frac{9\pi}{4}, 1)$ 107. $2x + h - 5$

108. (2, 5) 109. (0, 5), $(-\frac{5}{3}, 0), (-\frac{7}{3}, 0)$ 110. {5.85}

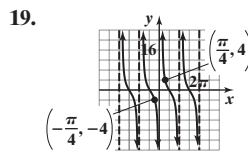
6.5 Assess Your Understanding (page 429)

3. origin; odd multiples of $\frac{\pi}{2}$ 4. y-axis; odd multiples of $\frac{\pi}{2}$ 5. b 6. T 7. 0 9. 1

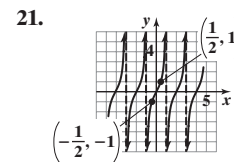
11. $\sec x = 1$ for $x = -2\pi, 0, 2\pi$; $\sec x = -1$ for $x = -\pi, \pi$ 13. $-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$ 15. $-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$



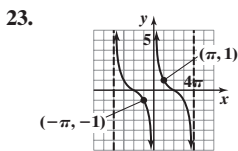
Domain: $\{x \mid x \neq \frac{k\pi}{2}, k \text{ is an odd integer}\}$
Range: $(-\infty, \infty)$



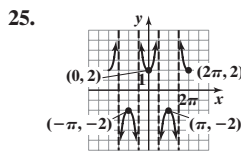
Domain: $\{x \mid x \neq k\pi, k \text{ is an integer}\}$
Range: $(-\infty, \infty)$



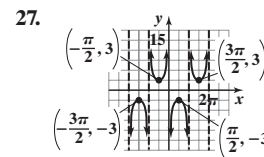
Domain: $\{x \mid x \text{ does not equal an odd integer}\}$
Range: $(-\infty, \infty)$



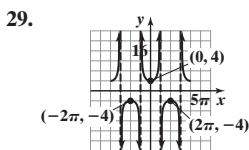
Domain: $\{x \mid x \neq 4k\pi, k \text{ is an integer}\}$
Range: $(-\infty, \infty)$



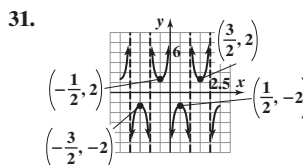
Domain: $\{x \mid x \neq \frac{k\pi}{2}, k \text{ is an odd integer}\}$
Range: $\{y \mid y \leq -2 \text{ or } y \geq 2\}$



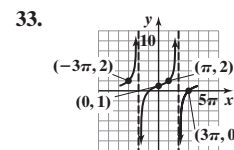
Domain: $\{x \mid x \neq k\pi, k \text{ is an integer}\}$
Range: $\{y \mid y \leq -3 \text{ or } y \geq 3\}$



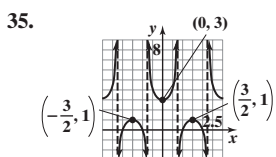
Domain: $\{x \mid x \neq k\pi, k \text{ is an odd integer}\}$
Range: $\{y \mid y \leq -4 \text{ or } y \geq 4\}$



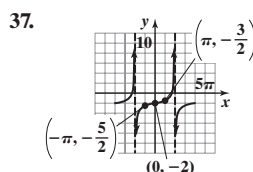
Domain: $\{x \mid x \text{ does not equal an integer}\}$
Range: $\{y \mid y \leq -2 \text{ or } y \geq 2\}$



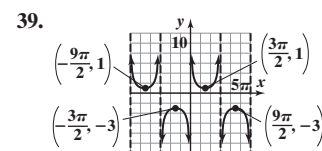
Domain: $\{x \mid x \neq 2\pi k, k \text{ is an odd integer}\}$
Range: $(-\infty, \infty)$



Domain: $\{x \mid x \neq \frac{3}{4}k, k \text{ is an odd integer}\}$
Range: $\{y \mid y \leq 1 \text{ or } y \geq 3\}$

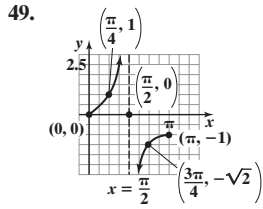
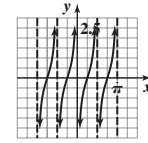
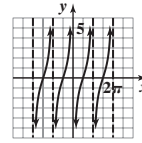
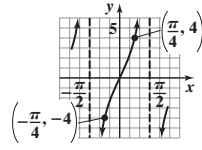
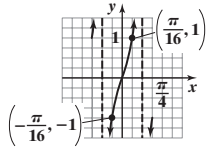


Domain: $\{x \mid x \neq 2\pi k, k \text{ is an odd integer}\}$
Range: $(-\infty, \infty)$

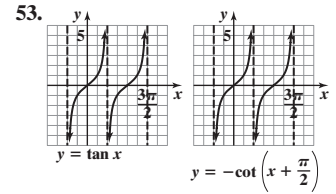
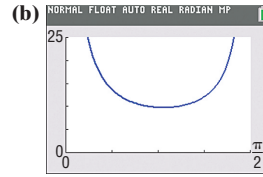


Domain: $\{x \mid x \neq 3\pi k, k \text{ is an integer}\}$
Range: $\{y \mid y \leq -3 \text{ or } y \geq 1\}$

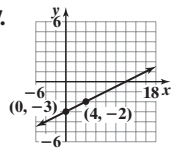
41. $\frac{2\sqrt{3}}{\pi}$ 43. $\frac{6\sqrt{3}}{\pi}$ 45. $(f \circ g)(x) = \tan(4x)$ $(g \circ f)(x) = 4 \tan x$ 47. $(f \circ g)(x) = -2 \cot x$ $(g \circ f)(x) = \cot(-2x)$



51. (a) $L(\theta) = \frac{3}{\cos \theta} + \frac{4}{\sin \theta}$
 $= 3 \sec \theta + 4 \csc \theta$
 (c) ≈ 0.83 (d) ≈ 9.86 ft

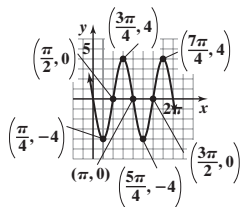


54. \$10,580.11 55. -1, 0 56. {-1, 3} 57.

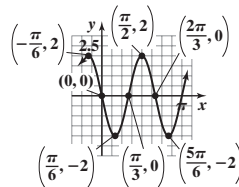


6.6 Assess Your Understanding (page 439)

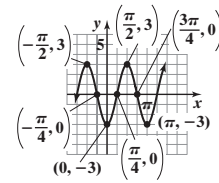
1. phase shift 2. False
 3. Amplitude = 4
 Period = π
 Phase shift = $\frac{\pi}{2}$



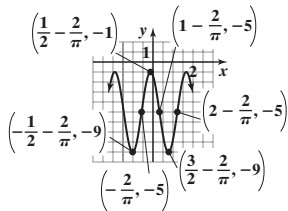
5. Amplitude = 2
 Period = $\frac{2\pi}{3}$
 Phase shift = $-\frac{\pi}{6}$



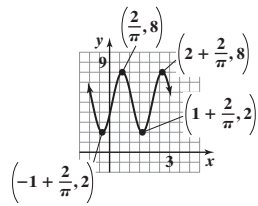
7. Amplitude = 3
 Period = π
 Phase shift = $-\frac{\pi}{4}$



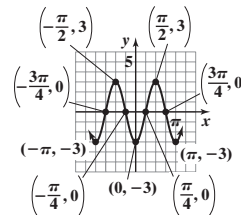
9. Amplitude = 4
 Period = 2
 Phase shift = $-\frac{2}{\pi}$



11. Amplitude = 3
 Period = 2
 Phase shift = $\frac{2}{\pi}$

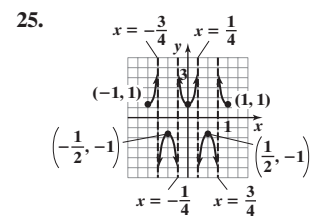
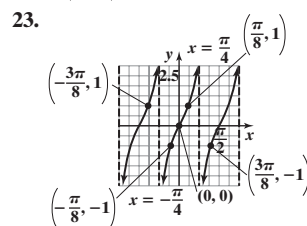
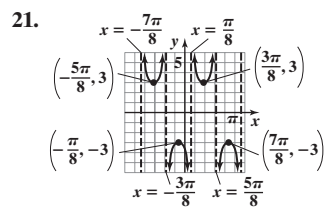
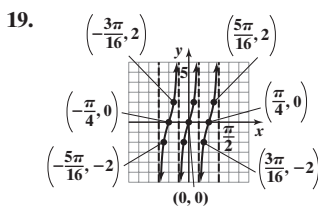


13. Amplitude = 3
 Period = π
 Phase shift = $\frac{\pi}{4}$

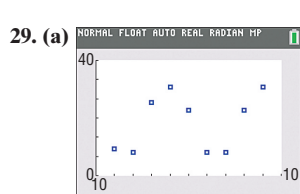
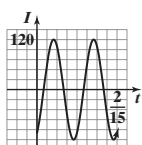


15. $y = 2 \sin\left[2\left(x - \frac{1}{2}\right)\right]$ or
 $y = 2 \sin(2x - 1)$

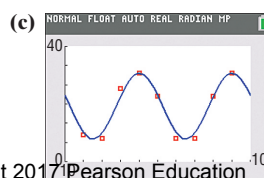
17. $y = 3 \sin\left[\frac{2}{3}\left(x + \frac{1}{3}\right)\right]$ or
 $y = 3 \sin\left(\frac{2}{3}x + \frac{2}{9}\right)$



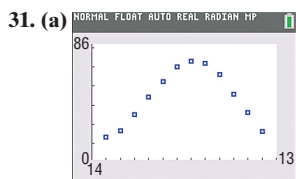
27. Period = $\frac{1}{15}$ s
 Amplitude = 120 amp
 Phase shift = $\frac{1}{90}$ s



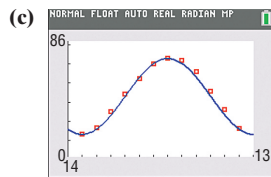
(b) $y = 8.5 \sin\left[\frac{2\pi}{5}\left(x - \frac{11}{4}\right)\right] + 24.5$
 or
 $y = 8.5 \sin\left(\frac{2\pi}{5}x - \frac{11\pi}{10}\right) + 24.5$



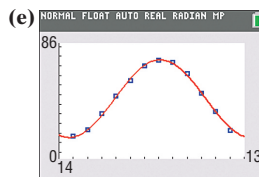
(d) $y = 9.46 \sin(1.247x + 2.096) + 24.088$
 (e)



(b) $y = 23.65 \sin\left[\frac{\pi}{6}(x - 4)\right] + 51.75$ or $y = 23.65 \sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 51.75$



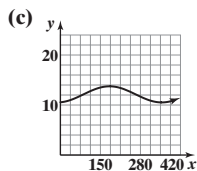
(d) $y = 24.25 \sin(0.493x - 1.927) + 51.61$



33. (a) 2:37 PM (b) $y = 2.2 \sin\left[\frac{24\pi}{149}(x + 0.9042)\right] + 3.07$ or $y = 2.2 \sin\left[\frac{24\pi}{149}x + 0.4575\right] + 3.07$ (c) 2.51 ft

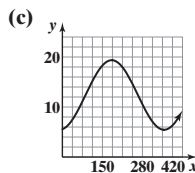
35. (a) $y = 1.615 \sin\left(\frac{2\pi}{365}x - 1.39\right) + 12.135$

(b) 12.42 h



37. (a) $y = 6.96 \sin\left(\frac{2\pi}{365}x - 1.39\right) + 12.41$

(b) 13.63 h



(d) The actual hours of sunlight on April 1, 2015, were 12.43 hours. This is close to the predicted amount of 12.42 hours.

(d) The actual hours of sunlight on April 1, 2015, were 13.35 hours. This is close to the predicted amount of 13.63 hours.

41. $f^{-1}(x) = \frac{2x - 9}{4}$ 42. 6.12% 43. $y = \frac{2}{3}x - 3$ or $2x - 3y = 9$ 44. $2\sqrt{13}$

Review Exercises (page 446)

1. $\frac{3\pi}{4}$ 2. $\frac{\pi}{10}$ 3. 135° 4. -450° 5. $\frac{1}{2}$ 6. $\frac{3\sqrt{2}}{2} - \frac{4\sqrt{3}}{3}$ 7. $-3\sqrt{2} - 2\sqrt{3}$ 8. 3 9. 0 10. 0 11. 1 12. 1 13. 1 14. -1 15. 1

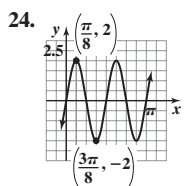
16. $\cos \theta = \frac{3}{5}$; $\tan \theta = \frac{4}{3}$; $\csc \theta = \frac{5}{4}$; $\sec \theta = \frac{5}{3}$; $\cot \theta = \frac{3}{4}$ 17. $\sin \theta = -\frac{12}{13}$; $\cos \theta = -\frac{5}{13}$; $\csc \theta = -\frac{13}{5}$; $\sec \theta = -\frac{13}{12}$; $\cot \theta = \frac{5}{12}$

18. $\sin \theta = \frac{3}{5}$; $\cos \theta = -\frac{4}{5}$; $\tan \theta = -\frac{3}{4}$; $\csc \theta = \frac{5}{3}$; $\cot \theta = -\frac{4}{3}$ 19. $\cos \theta = -\frac{5}{13}$; $\tan \theta = -\frac{12}{5}$; $\csc \theta = \frac{13}{12}$; $\sec \theta = -\frac{13}{5}$; $\cot \theta = -\frac{5}{12}$

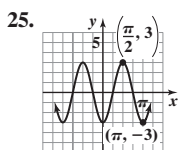
20. $\cos \theta = \frac{12}{13}$; $\tan \theta = -\frac{5}{12}$; $\csc \theta = -\frac{13}{5}$; $\sec \theta = \frac{13}{12}$; $\cot \theta = -\frac{12}{5}$ 21. $\sin \theta = -\frac{\sqrt{10}}{10}$; $\cos \theta = -\frac{3\sqrt{10}}{10}$; $\csc \theta = -\sqrt{10}$; $\sec \theta = -\frac{\sqrt{10}}{3}$; $\cot \theta = 3$

22. $\sin \theta = -\frac{2\sqrt{2}}{3}$; $\cos \theta = \frac{1}{3}$; $\tan \theta = -2\sqrt{2}$; $\csc \theta = -\frac{3\sqrt{2}}{4}$; $\cot \theta = -\frac{\sqrt{2}}{4}$

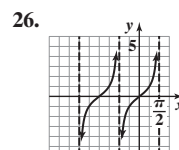
23. $\sin \theta = \frac{\sqrt{5}}{5}$; $\cos \theta = -\frac{2\sqrt{5}}{5}$; $\tan \theta = -\frac{1}{2}$; $\csc \theta = \sqrt{5}$; $\sec \theta = -\frac{\sqrt{5}}{2}$



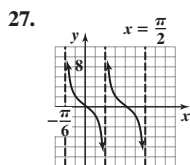
Domain: $(-\infty, \infty)$
Range: $[-2, 2]$



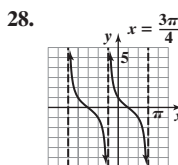
Domain: $(-\infty, \infty)$
Range: $[-3, 3]$



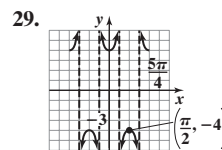
Domain: $\left\{x \mid x \neq \frac{k\pi}{2}, k \text{ is an odd integer}\right\}$
Range: $(-\infty, \infty)$



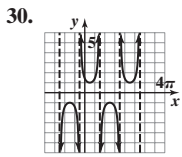
Domain: $\left\{x \mid x \neq \frac{\pi}{6} + k \cdot \frac{\pi}{3}, k \text{ is an integer}\right\}$
Range: $(-\infty, \infty)$



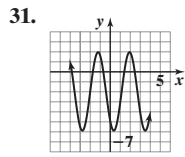
Domain: $\left\{x \mid x \neq -\frac{\pi}{4} + k\pi, k \text{ is an integer}\right\}$
Range: $(-\infty, \infty)$



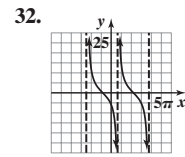
Domain: $\left\{x \mid x \neq \frac{k\pi}{4}, k \text{ is an odd integer}\right\}$
Range: $\{y \mid y \leq -4 \text{ or } y \geq 4\}$



30. Domain: $\left\{x \mid x \neq -\frac{\pi}{4} + k\pi, k \text{ is an integer}\right\}$
 Range: $\{y \mid y \leq -1 \text{ or } y \geq 1\}$



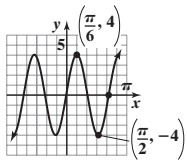
31. Domain: $(-\infty, \infty)$
 Range: $[-6, 2]$



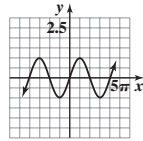
32. Domain: $\left\{x \mid x \neq \frac{3\pi}{4} + k \cdot 3\pi, k \text{ is an integer}\right\}$
 Range: $(-\infty, \infty)$

33. Amplitude = 1; Period = π 34. Amplitude = 2; Period = $\frac{2}{3}$

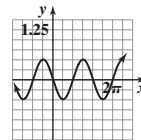
35. Amplitude = 4
 Period = $\frac{2\pi}{3}$
 Phase shift = 0



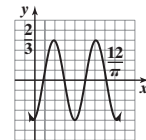
36. Amplitude = 1
 Period = 4π
 Phase shift = $-\pi$



37. Amplitude = $\frac{1}{2}$
 Period = $\frac{4\pi}{3}$
 Phase shift = $\frac{2\pi}{3}$



38. Amplitude = $\frac{2}{3}$
 Period = 2
 Phase shift = $\frac{6}{\pi}$



39. $y = 5 \cos \frac{x}{4}$ 40. $y = -7 \sin \left(\frac{\pi}{4}x\right)$ 41. 0.38 42. 1.02 43. Sine, cosine, cosecant, and secant: negative; tangent and cotangent: positive

44. IV 45. $\sin \theta = \frac{2\sqrt{2}}{3}$; $\cos \theta = -\frac{1}{3}$; $\tan \theta = -2\sqrt{2}$; $\csc \theta = \frac{3\sqrt{2}}{4}$; $\sec \theta = -3$; $\cot \theta = -\frac{\sqrt{2}}{4}$

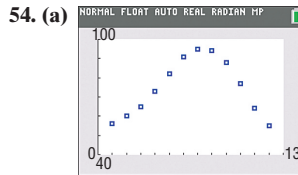
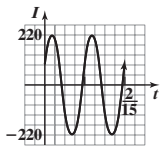
46. $\sin t = \frac{5\sqrt{29}}{29}$, $\cos t = -\frac{2\sqrt{29}}{29}$, $\tan t = -\frac{5}{2}$ 47. Domain: $\left\{x \mid x \neq \text{odd multiple of } \frac{\pi}{2}\right\}$; range: $\{y \mid |y| \geq 1\}$; period = 2π

48. (a) 32.34° (b) $63^\circ 10' 48''$

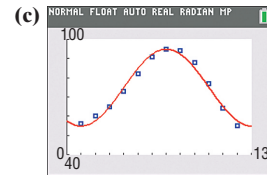
49. $\frac{\pi}{3} \approx 1.05 \text{ ft}$; $\frac{\pi}{3} \approx 1.05 \text{ ft}^2$ 50. $8\pi \approx 25.13 \text{ in.}$; $\frac{16\pi}{3} \approx 16.76 \text{ in.}$ 51. Approximately 114.59 revolutions/hr

52. $0.1 \text{ revolution/sec} = \frac{\pi}{5} \text{ radian/sec}$

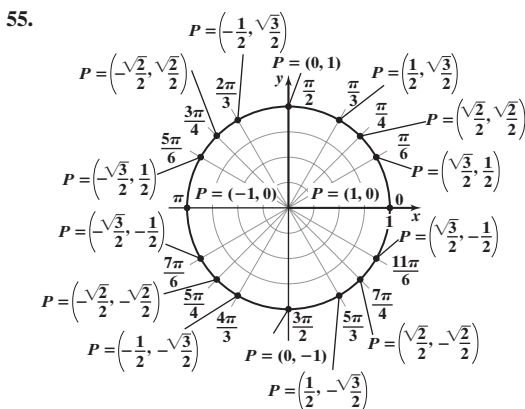
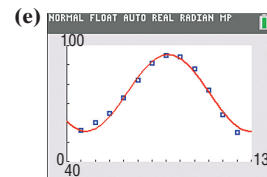
53. (a) $\frac{1}{15}$
 (b) 220
 (c) $-\frac{1}{180}$
 (d)



(b) $y = 20 \sin \left[\frac{\pi}{6}(x - 4)\right] + 75$ or
 $y = 20 \sin \left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 75$



(d) $y = 19.81 \sin(0.543x - 2.296) + 75.66$



Chapter Test (page 448)

1. $\frac{13\pi}{9}$ 2. $-\frac{20\pi}{9}$ 3. $\frac{13\pi}{180}$ 4. -22.5° 5. 810° 6. 135° 7. $\frac{1}{2}$ 8. 0 9. $-\frac{1}{2}$ 10. $-\frac{\sqrt{3}}{3}$ 11. 2 12. $\frac{3(1-\sqrt{2})}{2}$ 13. 0.292 14. 0.309

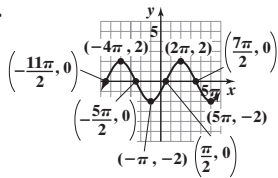
15. -1.524 16. 2.747 17.

	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\sec \theta$	$\csc \theta$	$\cot \theta$
θ in QI	+	+	+	+	+	+
θ in QII	+	-	-	-	+	-
θ in QIII	-	-	+	-	-	+
θ in QIV	-	+	-	+	-	-

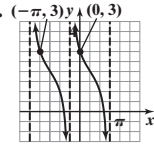
18. $-\frac{3}{5}$

19. $\cos \theta = -\frac{2\sqrt{6}}{7}$; $\tan \theta = -\frac{5\sqrt{6}}{12}$; $\csc \theta = \frac{7}{5}$; $\sec \theta = -\frac{7\sqrt{6}}{12}$; $\cot \theta = -\frac{2\sqrt{6}}{5}$ 20. $\sin \theta = -\frac{\sqrt{5}}{3}$; $\tan \theta = -\frac{\sqrt{5}}{2}$; $\csc \theta = -\frac{3\sqrt{5}}{5}$;
 $\sec \theta = \frac{3}{2}$; $\cot \theta = -\frac{2\sqrt{5}}{5}$ 21. $\sin \theta = \frac{12}{13}$; $\cos \theta = -\frac{5}{13}$; $\csc \theta = \frac{13}{12}$; $\sec \theta = -\frac{13}{5}$; $\cot \theta = -\frac{5}{12}$ 22. $\frac{7\sqrt{53}}{53}$ 23. $-\frac{5\sqrt{146}}{146}$ 24. $-\frac{1}{2}$

25.



26.



27. $y = -3 \sin\left(3x + \frac{3\pi}{4}\right)$

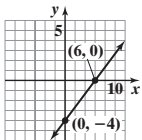
28. 78.93 ft²

29. 143.5 rpm

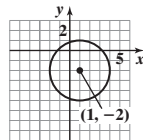
Cumulative Review (page 449)

1. $\left\{-1, \frac{1}{2}\right\}$ 2. $y - 5 = -3(x + 2)$ or $y = -3x - 1$ 3. $x^2 + (y + 2)^2 = 16$

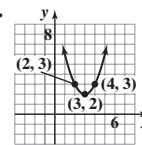
4. A line; slope $\frac{2}{3}$; intercepts (6, 0) and (0, -4)



5. A circle; center (1, -2); radius 3



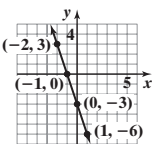
6.



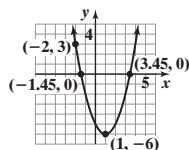
7. (a) (b) (c) (d) (e) (f)

8. $f^{-1}(x) = \frac{1}{3}(x + 2)$ 9. -2 10. 11. $3 - \frac{3\sqrt{3}}{2}$ 12. $y = 2(3^x)$ 13. $y = 3 \cos\left(\frac{\pi}{6}x\right)$

14. (a) $f(x) = -3x - 3$;
 $m = -3$; (-1, 0), (0, -3)

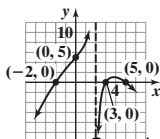
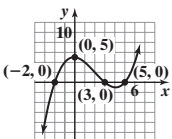


- (b) $f(x) = (x - 1)^2 - 6$; (0, -5),
 $(-\sqrt{6} + 1, 0)$, $(\sqrt{6} + 1, 0)$



- (c) We have that $y = 3$ when $x = -2$ and $y = -6$ when $x = 1$. Both points satisfy $y = ae^x$. Therefore, for (-2, 3) we have $3 = ae^{-2}$, which implies that $a = 3e^2$. But for (1, -6) we have $-6 = ae^1$, which implies that $a = -6e^{-1}$. Therefore, there is no exponential function $y = ae^x$ that contains (-2, 3) and (1, -6).

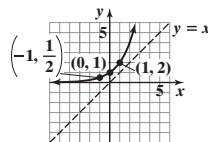
15. (a) $f(x) = \frac{1}{6}(x + 2)(x - 3)(x - 5)$ (b) $R(x) = -\frac{(x + 2)(x - 3)(x - 5)}{3(x - 2)}$



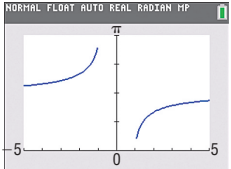
CHAPTER 7 Analytic Trigonometry

7.1 Assess Your Understanding (page 462)

7. $x = \sin y$ 8. $0 \leq x \leq \pi$ 9. $-\infty < x < \infty$ 10. F 11. T 12. T 13. d 14. a 15. 0 17. $-\frac{\pi}{2}$ 19. 0 21. $\frac{\pi}{4}$ 23. $\frac{\pi}{3}$ 25. $\frac{5\pi}{6}$ 27. 0.10 29. 1.37
 31. 0.51 33. -0.38 35. -0.12 37. 1.08 39. $\frac{4\pi}{5}$ 41. $-\frac{3\pi}{8}$ 43. $-\frac{\pi}{8}$ 45. $\frac{\pi}{3}$ 47. $-\frac{\pi}{5}$ 49. $\frac{\pi}{3}$ 51. $\frac{\pi}{4}$ 53. Not defined 55. $\frac{1}{4}$ 57. 4 59. Not defined 61. π
63. $f^{-1}(x) = \sin^{-1} \frac{x-2}{5}$
 Range of f = Domain of $f^{-1} = [-3, 7]$
 Range of $f^{-1} = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
65. $f^{-1}(x) = \frac{1}{3} \cos^{-1} \left(-\frac{x}{2}\right)$
 Range of f = Domain of $f^{-1} = [-2, 2]$
 Range of $f^{-1} = \left[0, \frac{\pi}{3}\right]$
67. $f^{-1}(x) = -\tan^{-1}(x+3) - 1$
 Range of f = Domain of $f^{-1} = (-\infty, \infty)$
 Range of $f^{-1} = \left(-1 - \frac{\pi}{2}, \frac{\pi}{2} - 1\right)$
69. $f^{-1}(x) = \frac{1}{2} \left[\sin^{-1} \left(\frac{x}{3}\right) - 1 \right]$
 Range of f = Domain of $f^{-1} = [-3, 3]$
 Range of $f^{-1} = \left[-\frac{1}{2} - \frac{\pi}{4}, -\frac{1}{2} + \frac{\pi}{4}\right]$
71. $\left\{\frac{\sqrt{2}}{2}\right\}$ 73. $\left\{-\frac{1}{4}\right\}$ 75. $\{\sqrt{3}\}$ 77. $\{-1\}$
79. (a) 13.92 h or 13 h, 55 min (b) 12 h (c) 13.85 h or 13 h, 51 min
 81. (a) 13.3 h or 13 h, 18 min (b) 12 h (c) 13.26 h or 13 h, 15 min
 83. (a) 12 h (b) 12 h (c) 12 h (d) It is 12 h. 85. 3.35 min
87. (a) $\frac{\pi}{3}$ square units (b) $\frac{5\pi}{12}$ square units 89. 4250 mi 91. $\left\{\frac{\log 7}{3}\right\}$ 92. The graph passes the horizontal-line test.
93. $f^{-1}(x) = \log_2(x-1)$ 94. $\frac{\sqrt{3}}{4}$



7.2 Assess Your Understanding (page 469)

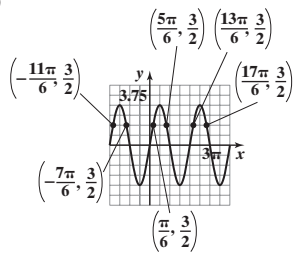
4. $x = \sec y; \geq 1; 0; \pi$ 5. cosine 6. F 7. T 8. T 9. $\frac{\sqrt{2}}{2}$ 11. $-\frac{\sqrt{3}}{3}$ 13. 2 15. $\sqrt{2}$ 17. $-\frac{\sqrt{2}}{2}$ 19. $\frac{2\sqrt{3}}{3}$ 21. $\frac{3\pi}{4}$ 23. $-\frac{\pi}{3}$ 25. $\frac{\sqrt{2}}{4}$ 27. $\frac{\sqrt{5}}{2}$
29. $-\frac{\sqrt{14}}{2}$ 31. $-\frac{3\sqrt{10}}{10}$ 33. $\sqrt{5}$ 35. $-\frac{\pi}{4}$ 37. $\frac{\pi}{6}$ 39. $-\frac{\pi}{2}$ 41. $\frac{\pi}{6}$ 43. $\frac{2\pi}{3}$ 45. 1.32 47. 0.46 49. -0.34 51. 2.72 53. -0.73 55. 2.55
57. $\frac{1}{\sqrt{1+u^2}}$ 59. $\frac{u}{\sqrt{1-u^2}}$ 61. $\frac{\sqrt{u^2-1}}{|u|}$ 63. $\frac{\sqrt{u^2-1}}{|u|}$ 65. $\frac{1}{u}$ 67. $\frac{5}{13}$ 69. $\frac{3\pi}{4}$ 71. $-\frac{3}{4}$ 73. $\frac{5}{13}$ 75. $\frac{5\pi}{6}$ 77. $-\sqrt{15}$
79. (a) $\theta = 31.89^\circ$ (b) 54.64 ft in diameter (c) 3796 ft high 81. (a) $\theta = 22.3^\circ$ (b) $v_0 = 2940.23$ ft/s
83.  87. $-5i, 5i, -2, 2$ 88. Neither 89. $\frac{7\pi}{4}$ 90. $\frac{5\pi}{2} \approx 7.85$ in.

7.3 Assess Your Understanding (page 477)

7. F 8. T 9. T 10. F 11. d 12. a 13. $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ 15. $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ 17. $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$ 19. $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$ 21. $\left\{\frac{3\pi}{4}, \frac{5\pi}{4}\right\}$ 23. $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$
25. $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$ 27. $\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ 29. $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$ 31. $\left\{\frac{4\pi}{9}, \frac{8\pi}{9}, \frac{16\pi}{9}\right\}$ 33. $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$ 35. $\left\{\frac{11\pi}{6}\right\}$
37. $\left\{\theta \mid \theta = \frac{\pi}{6} + 2k\pi, \theta = \frac{5\pi}{6} + 2k\pi\right\}; \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}$ 39. $\left\{\theta \mid \theta = \frac{5\pi}{6} + k\pi\right\}; \frac{5\pi}{6}, \frac{11\pi}{6}, \frac{17\pi}{6}, \frac{23\pi}{6}, \frac{29\pi}{6}, \frac{35\pi}{6}$
41. $\left\{\theta \mid \theta = \frac{\pi}{2} + 2k\pi, \theta = \frac{3\pi}{2} + 2k\pi\right\}; \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}$ 43. $\left\{\theta \mid \theta = \frac{\pi}{3} + k\pi, \theta = \frac{2\pi}{3} + k\pi\right\}; \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$
45. $\left\{\theta \mid \theta = \frac{8\pi}{3} + 4k\pi, \theta = \frac{10\pi}{3} + 4k\pi\right\}; \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{20\pi}{3}, \frac{22\pi}{3}, \frac{32\pi}{3}, \frac{34\pi}{3}$ 47. {0.41, 2.73} 49. {1.37, 4.51} 51. {2.69, 3.59} 53. {1.82, 4.46}
55. {2.08, 5.22} 57. {0.73, 2.41} 59. $\left\{\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}\right\}$ 61. $\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ 63. $\left\{0, \frac{\pi}{4}, \frac{5\pi}{4}\right\}$ 65. $\left\{\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}\right\}$ 67. $\{\pi\}$ 69. $\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$
71. $\left\{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\}$ 73. $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$ 75. $\left\{\frac{\pi}{2}\right\}$ 77. {0} 79. $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ 81. No real solution 83. {-1.31, 1.98, 3.84} 85. {0.52}

87. {1.26} 89. {-1.02, 1.02} 91. {0, 2.15} 93. {0.76, 1.35} 95. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

97. (a) $-2\pi, -\pi, 0, \pi, 2\pi, 3\pi, 4\pi$ (b)

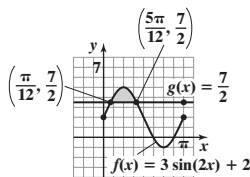


(c) $\left\{-\frac{11\pi}{6}, -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}\right\}$

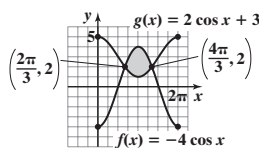
(d) $\left\{x \mid -\frac{11\pi}{6} < x < -\frac{7\pi}{6} \text{ or } \frac{\pi}{6} < x < \frac{5\pi}{6} \text{ or } \frac{13\pi}{6} < x < \frac{17\pi}{6}\right\}$

99. (a) $\left\{x \mid x = -\frac{\pi}{4} + k\pi, k \text{ is any integer}\right\}$ (b) $-\frac{\pi}{2} < x < -\frac{\pi}{4}$ or $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right)$

101. (a), (d)



(b) $\left\{\frac{\pi}{12}, \frac{5\pi}{12}\right\}$ 103. (a), (d)



(b) $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

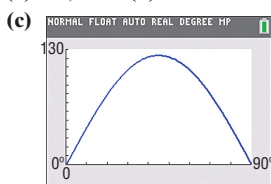
(c) $\left\{x \mid \frac{\pi}{12} < x < \frac{5\pi}{12}\right\}$ or $\left(\frac{\pi}{12}, \frac{5\pi}{12}\right)$

(c) $\left\{x \mid \frac{2\pi}{3} < x < \frac{4\pi}{3}\right\}$ or $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$

105. (a) 0 s, 0.43 s, 0.86 s (b) 0.21 s (c) $[0, 0.03] \cup [0.39, 0.43] \cup [0.86, 0.89]$ 107. (a) 150 mi (b) 6.06, 8.44, 15.72, 18.11 min

(c) Before 6.06 min, between 8.44 and 15.72 min, and after 18.11 min (d) No 109. 2.03, 4.91

111. (a) 30°, 60° (b) 123.6 m 113. 28.90° 115. Yes; it varies from 1.25 to 1.34. 117. 1.47

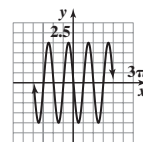


119. If θ is the original angle of incidence and ϕ is the angle of refraction, then $\frac{\sin \theta}{\sin \phi} = n_2$.

The angle of incidence of the emerging beam is also ϕ , and the index of refraction is $\frac{1}{n_2}$. Thus, θ is the angle of refraction of the emerging beam.

123. $x = \log_6 y$ 124. $\frac{9 - \sqrt{17}}{4}, \frac{9 + \sqrt{17}}{4}$ 125. $\tan \theta = -\frac{1}{3}; \csc \theta = -\sqrt{10}; \sec \theta = \frac{\sqrt{10}}{3}; \cot \theta = -3$ 126. Amplitude: 2

Period: π
Phase shift: $\frac{\pi}{2}$



7.4 Assess Your Understanding (page 487)

3. identity; conditional 4. -1 5. 0 6. T 7. F 8. T 9. c 10. b 11. $\frac{1}{\cos \theta}$ 13. $\frac{1 + \sin \theta}{\cos \theta}$ 15. $\frac{1}{\sin \theta \cos \theta}$ 17. 2 19. $\frac{3 \sin \theta + 1}{\sin \theta + 1}$

21. $\csc \theta \cdot \cos \theta = \frac{1}{\sin \theta} \cdot \cos \theta = \frac{\cos \theta}{\sin \theta} = \cot \theta$ 23. $1 + \tan^2(-\theta) = 1 + (-\tan \theta)^2 = 1 + \tan^2 \theta = \sec^2 \theta$

25. $\cos \theta (\tan \theta + \cot \theta) = \cos \theta \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right) = \cos \theta \left(\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \right) = \cos \theta \left(\frac{1}{\cos \theta \sin \theta} \right) = \frac{1}{\sin \theta} = \csc \theta$

27. $\tan u \cot u - \cos^2 u = \tan u \cdot \frac{1}{\tan u} - \cos^2 u = 1 - \cos^2 u = \sin^2 u$ 29. $(\sec \theta - 1)(\sec \theta + 1) = \sec^2 \theta - 1 = \tan^2 \theta$

31. $(\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = \sec^2 \theta - \tan^2 \theta = 1$ 33. $\cos^2 \theta (1 + \tan^2 \theta) = \cos^2 \theta \sec^2 \theta = \cos^2 \theta \cdot \frac{1}{\cos^2 \theta} = 1$

35. $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta + \sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta = \sin^2 \theta + \cos^2 \theta + \sin^2 \theta + \cos^2 \theta = 1 + 1 = 2$

37. $\sec^4 \theta - \sec^2 \theta = \sec^2 \theta (\sec^2 \theta - 1) = (1 + \tan^2 \theta) \tan^2 \theta = \tan^4 \theta + \tan^2 \theta$

39. $\sec u - \tan u = \frac{1}{\cos u} - \frac{\sin u}{\cos u} = \frac{1 - \sin u}{\cos u} \cdot \frac{1 + \sin u}{1 + \sin u} = \frac{1 - \sin^2 u}{\cos u (1 + \sin u)} = \frac{\cos^2 u}{\cos u (1 + \sin u)} = \frac{\cos u}{1 + \sin u}$

41. $3 \sin^2 \theta + 4 \cos^2 \theta = 3 \sin^2 \theta + 3 \cos^2 \theta + \cos^2 \theta = 3(\sin^2 \theta + \cos^2 \theta) + \cos^2 \theta = 3 + \cos^2 \theta$

43. $1 - \frac{\cos^2 \theta}{1 + \sin \theta} = 1 - \frac{1 - \sin^2 \theta}{1 + \sin \theta} = 1 - \frac{(1 + \sin \theta)(1 - \sin \theta)}{1 + \sin \theta} = 1 - (1 - \sin \theta) = \sin \theta$

45. $\frac{1 + \tan v}{1 - \tan v} = \frac{1 + \frac{1}{\cot v}}{1 - \frac{1}{\cot v}} = \frac{\frac{\cot v + 1}{\cot v}}{\frac{\cot v - 1}{\cot v}} = \frac{\cot v + 1}{\cot v - 1}$ 47. $\frac{\sec \theta}{\csc \theta} + \frac{\sin \theta}{\cos \theta} = \frac{\frac{1}{\cos \theta}}{\frac{1}{\sin \theta}} + \tan \theta = \frac{\sin \theta}{\cos \theta} + \tan \theta = \tan \theta + \tan \theta = 2 \tan \theta$
49. $\frac{1 + \sin \theta}{1 - \sin \theta} = \frac{1 + \frac{1}{\csc \theta}}{1 - \frac{1}{\csc \theta}} = \frac{\frac{\csc \theta + 1}{\csc \theta}}{\frac{\csc \theta - 1}{\csc \theta}} = \frac{\csc \theta + 1}{\csc \theta - 1}$
51. $\frac{1 - \sin v}{\cos v} + \frac{\cos v}{1 - \sin v} = \frac{(1 - \sin v)^2 + \cos^2 v}{\cos v(1 - \sin v)} = \frac{1 - 2 \sin v + \sin^2 v + \cos^2 v}{\cos v(1 - \sin v)} = \frac{2 - 2 \sin v}{\cos v(1 - \sin v)} = \frac{2(1 - \sin v)}{\cos v(1 - \sin v)} = \frac{2}{\cos v} = 2 \sec v$
53. $\frac{\sin \theta}{\sin \theta - \cos \theta} = \frac{1}{\frac{\sin \theta - \cos \theta}{\sin \theta}} = \frac{1}{1 - \frac{\cos \theta}{\sin \theta}} = \frac{1}{1 - \cot \theta}$
55. $(\sec \theta - \tan \theta)^2 = \sec^2 \theta - 2 \sec \theta \tan \theta + \tan^2 \theta = \frac{1}{\cos^2 \theta} - \frac{2 \sin \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1 - 2 \sin \theta + \sin^2 \theta}{\cos^2 \theta} = \frac{(1 - \sin \theta)^2}{1 - \sin^2 \theta} = \frac{(1 - \sin \theta)^2}{(1 - \sin \theta)(1 + \sin \theta)}$
 $= \frac{1 - \sin \theta}{1 + \sin \theta}$
57. $\frac{\cos \theta}{1 - \tan \theta} + \frac{\sin \theta}{1 - \cot \theta} = \frac{\cos \theta}{1 - \frac{\sin \theta}{\cos \theta}} + \frac{\sin \theta}{1 - \frac{\cos \theta}{\sin \theta}} = \frac{\cos \theta}{\frac{\cos \theta - \sin \theta}{\cos \theta}} + \frac{\sin \theta}{\frac{\sin \theta - \cos \theta}{\sin \theta}} = \frac{\cos^2 \theta}{\cos \theta - \sin \theta} + \frac{\sin^2 \theta}{\sin \theta - \cos \theta}$
 $= \frac{\cos^2 \theta - \sin^2 \theta}{\cos \theta - \sin \theta} = \frac{(\cos \theta - \sin \theta)(\cos \theta + \sin \theta)}{\cos \theta - \sin \theta} = \sin \theta + \cos \theta$
59. $\tan \theta + \frac{\cos \theta}{1 + \sin \theta} = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} = \frac{\sin \theta(1 + \sin \theta) + \cos^2 \theta}{\cos \theta(1 + \sin \theta)} = \frac{\sin \theta + \sin^2 \theta + \cos^2 \theta}{\cos \theta(1 + \sin \theta)} = \frac{\sin \theta + 1}{\cos \theta(1 + \sin \theta)} = \frac{1}{\cos \theta} = \sec \theta$
61. $\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{\tan \theta + (\sec \theta - 1)}{\tan \theta - (\sec \theta - 1)} \cdot \frac{\tan \theta + (\sec \theta - 1)}{\tan \theta + (\sec \theta - 1)} = \frac{\tan^2 \theta + 2 \tan \theta(\sec \theta - 1) + \sec^2 \theta - 2 \sec \theta + 1}{\tan^2 \theta - (\sec^2 \theta - 2 \sec \theta + 1)}$
 $= \frac{\sec^2 \theta - 1 + 2 \tan \theta(\sec \theta - 1) + \sec^2 \theta - 2 \sec \theta + 1}{\sec^2 \theta - 1 - \sec^2 \theta + 2 \sec \theta - 1} = \frac{2 \sec^2 \theta - 2 \sec \theta + 2 \tan \theta(\sec \theta - 1)}{-2 + 2 \sec \theta}$
 $= \frac{2 \sec \theta(\sec \theta - 1) + 2 \tan \theta(\sec \theta - 1)}{2(\sec \theta - 1)} = \frac{2(\sec \theta - 1)(\sec \theta + \tan \theta)}{2(\sec \theta - 1)} = \tan \theta + \sec \theta$
63. $\frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = \frac{\frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta}}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}} = \frac{\frac{\sin^2 \theta - \cos^2 \theta}{\cos \theta \sin \theta}}{\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}} = \frac{\sin^2 \theta - \cos^2 \theta}{1} = \sin^2 \theta - \cos^2 \theta$
65. $\frac{\tan u - \cot u}{\tan u + \cot u} + 1 = \frac{\frac{\sin u}{\cos u} - \frac{\cos u}{\sin u}}{\frac{\sin u}{\cos u} + \frac{\cos u}{\sin u}} + 1 = \frac{\frac{\sin^2 u - \cos^2 u}{\cos u \sin u}}{\frac{\sin^2 u + \cos^2 u}{\cos u \sin u}} + 1 = \frac{\sin^2 u - \cos^2 u}{\sin^2 u + \cos^2 u} + 1 = \sin^2 u - \cos^2 u + 1 = \sin^2 u + (1 - \cos^2 u) = 2 \sin^2 u$
67. $\frac{\sec \theta + \tan \theta}{\cot \theta + \cos \theta} = \frac{\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta}}{\frac{\cos \theta}{\sin \theta} + \cos \theta} = \frac{\frac{1 + \sin \theta}{\cos \theta}}{\frac{\cos \theta + \cos \theta \sin \theta}{\sin \theta}} = \frac{1 + \sin \theta}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta(1 + \sin \theta)} = \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta} = \tan \theta \sec \theta$
69. $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} + 1 = \frac{1 - \tan^2 \theta + 1 + \tan^2 \theta}{1 + \tan^2 \theta} = \frac{2}{1 + \tan^2 \theta} = \frac{2}{\sec^2 \theta} = 2 \cos^2 \theta$
71. $\frac{\sec \theta - \csc \theta}{\sec \theta \csc \theta} = \frac{\sec \theta}{\sec \theta \csc \theta} - \frac{\csc \theta}{\sec \theta \csc \theta} = \frac{1}{\csc \theta} - \frac{1}{\sec \theta} = \sin \theta - \cos \theta$
73. $\sec \theta - \cos \theta = \frac{1}{\cos \theta} - \cos \theta = \frac{1 - \cos^2 \theta}{\cos \theta} = \frac{\sin^2 \theta}{\cos \theta} = \sin \theta \cdot \frac{\sin \theta}{\cos \theta} = \sin \theta \tan \theta$
75. $\frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} = \frac{1 + \sin \theta + 1 - \sin \theta}{(1 + \sin \theta)(1 - \sin \theta)} = \frac{2}{1 - \sin^2 \theta} = \frac{2}{\cos^2 \theta} = 2 \sec^2 \theta$
77. $\frac{\sec \theta}{1 - \sin \theta} = \frac{\sec \theta}{1 - \sin \theta} \cdot \frac{1 + \sin \theta}{1 + \sin \theta} = \frac{\sec \theta(1 + \sin \theta)}{1 - \sin^2 \theta} = \frac{\sec \theta(1 + \sin \theta)}{\cos^2 \theta} = \frac{1 + \sin \theta}{\cos^3 \theta}$
79. $\frac{(\sec v - \tan v)^2 + 1}{\csc v(\sec v - \tan v)} = \frac{\sec^2 v - 2 \sec v \tan v + \tan^2 v + 1}{\frac{1}{\sin v}(\sec v - \tan v)} = \frac{2 \sec^2 v - 2 \sec v \tan v}{\frac{1}{\sin v}(\frac{1}{\cos v} - \frac{\sin v}{\cos v})} = \frac{2 \sec^2 v - 2 \sec v \tan v}{\frac{1}{\sin v}(\frac{1 - \sin v}{\cos v})} = \frac{2}{\cos^2 v} \cdot \frac{2 \sin v}{\cos^2 v} = \frac{2 - 2 \sin v}{\cos^2 v} \cdot \frac{\sin v \cos v}{1 - \sin v}$
 $= \frac{2(1 - \sin v)}{\cos v} \cdot \frac{\sin v}{1 - \sin v} = \frac{2 \sin v}{\cos v} = 2 \tan v$

$$81. \frac{\sin \theta + \cos \theta}{\cos \theta} - \frac{\sin \theta - \cos \theta}{\sin \theta} = \frac{\sin \theta}{\cos \theta} + 1 - 1 + \frac{\cos \theta}{\sin \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} = \frac{1}{\cos \theta \sin \theta} = \sec \theta \csc \theta$$

$$83. \frac{\sin^3 \theta + \cos^3 \theta}{\sin \theta + \cos \theta} = \frac{(\sin \theta + \cos \theta)(\sin^2 \theta - \sin \theta \cos \theta + \cos^2 \theta)}{\sin \theta + \cos \theta} = \sin^2 \theta + \cos^2 \theta - \sin \theta \cos \theta = 1 - \sin \theta \cos \theta$$

$$85. \frac{\cos^2 \theta - \sin^2 \theta}{1 - \tan^2 \theta} = \frac{\cos^2 \theta - \sin^2 \theta}{1 - \frac{\sin^2 \theta}{\cos^2 \theta}} = \frac{\cos^2 \theta - \sin^2 \theta}{\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta}} = \cos^2 \theta$$

$$87. \frac{(2 \cos^2 \theta - 1)^2}{\cos^4 \theta - \sin^4 \theta} = \frac{[2 \cos^2 \theta - (\sin^2 \theta + \cos^2 \theta)]^2}{(\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta)} = \frac{(\cos^2 \theta - \sin^2 \theta)^2}{\cos^2 \theta - \sin^2 \theta} = \cos^2 \theta - \sin^2 \theta = (1 - \sin^2 \theta) - \sin^2 \theta = 1 - 2 \sin^2 \theta$$

$$89. \frac{1 + \sin \theta + \cos \theta}{1 + \sin \theta - \cos \theta} = \frac{(1 + \sin \theta) + \cos \theta}{(1 + \sin \theta) - \cos \theta} \cdot \frac{(1 + \sin \theta) + \cos \theta}{(1 + \sin \theta) + \cos \theta} = \frac{1 + 2 \sin \theta + \sin^2 \theta + 2(1 + \sin \theta) \cos \theta + \cos^2 \theta}{1 + 2 \sin \theta + \sin^2 \theta - \cos^2 \theta}$$

$$= \frac{1 + 2 \sin \theta + \sin^2 \theta + 2(1 + \sin \theta)(\cos \theta) + (1 - \sin^2 \theta)}{1 + 2 \sin \theta + \sin^2 \theta - (1 - \sin^2 \theta)} = \frac{2 + 2 \sin \theta + 2(1 + \sin \theta)(\cos \theta)}{2 \sin \theta + 2 \sin^2 \theta}$$

$$= \frac{2(1 + \sin \theta) + 2(1 + \sin \theta)(\cos \theta)}{2 \sin \theta(1 + \sin \theta)} = \frac{2(1 + \sin \theta)(1 + \cos \theta)}{2 \sin \theta(1 + \sin \theta)} = \frac{1 + \cos \theta}{\sin \theta}$$

$$91. (a \sin \theta + b \cos \theta)^2 + (a \cos \theta - b \sin \theta)^2 = a^2 \sin^2 \theta + 2ab \sin \theta \cos \theta + b^2 \cos^2 \theta + a^2 \cos^2 \theta - 2ab \sin \theta \cos \theta + b^2 \sin^2 \theta$$

$$= a^2 (\sin^2 \theta + \cos^2 \theta) + b^2 (\cos^2 \theta + \sin^2 \theta) = a^2 + b^2$$

$$93. \frac{\tan \alpha + \tan \beta}{\cot \alpha + \cot \beta} = \frac{\tan \alpha + \tan \beta}{\frac{1}{\tan \alpha} + \frac{1}{\tan \beta}} = \frac{\tan \alpha + \tan \beta}{\frac{\tan \beta + \tan \alpha}{\tan \alpha \tan \beta}} = (\tan \alpha + \tan \beta) \cdot \frac{\tan \alpha \tan \beta}{\tan \alpha + \tan \beta} = \tan \alpha \tan \beta$$

$$95. (\sin \alpha + \cos \beta)^2 + (\cos \beta + \sin \alpha)(\cos \beta - \sin \alpha) = (\sin^2 \alpha + 2 \sin \alpha \cos \beta + \cos^2 \beta) + (\cos^2 \beta - \sin^2 \alpha)$$

$$= 2 \cos^2 \beta + 2 \sin \alpha \cos \beta = 2 \cos \beta (\cos \beta + \sin \alpha) = 2 \cos \beta (\sin \alpha + \cos \beta)$$

$$97. \ln |\sec \theta| = \ln |\cos \theta|^{-1} = -\ln |\cos \theta|$$

$$99. \ln |1 + \cos \theta| + \ln |1 - \cos \theta| = \ln (|1 + \cos \theta| |1 - \cos \theta|) = \ln |1 - \cos^2 \theta| = \ln |\sin^2 \theta| = 2 \ln |\sin \theta|$$

$$101. g(x) = \sec x - \cos x = \frac{1}{\cos x} - \cos x = \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} = \frac{1 - \cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x} = \sin x \cdot \frac{\sin x}{\cos x} = \sin x \cdot \tan x = f(x)$$

$$103. f(\theta) = \frac{1 - \sin \theta}{\cos \theta} - \frac{\cos \theta}{1 + \sin \theta} = \frac{1 - \sin \theta}{\cos \theta} \cdot \frac{1 + \sin \theta}{1 + \sin \theta} - \frac{\cos \theta}{1 + \sin \theta} \cdot \frac{\cos \theta}{\cos \theta} = \frac{1 - \sin^2 \theta}{\cos \theta(1 + \sin \theta)} - \frac{\cos^2 \theta}{\cos \theta(1 + \sin \theta)}$$

$$= \frac{\cos^2 \theta}{\cos \theta(1 + \sin \theta)} - \frac{\cos^2 \theta}{\cos \theta(1 + \sin \theta)} = 0 = g(\theta)$$

$$105. \sqrt{16 + 16 \tan^2 \theta} = \sqrt{16 \sqrt{1 + \tan^2 \theta}} = 4 \sqrt{\sec^2 \theta} = 4 \sec \theta, \text{ since } \sec \theta > 0 \text{ for } -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$107. 1200 \sec \theta (2 \sec^2 \theta - 1) = 1200 \frac{1}{\cos \theta} \left(\frac{2}{\cos^2 \theta} - 1 \right) = 1200 \frac{1}{\cos \theta} \left(\frac{2}{\cos^2 \theta} - \frac{\cos^2 \theta}{\cos^2 \theta} \right) = 1200 \frac{1}{\cos \theta} \left(\frac{2 - \cos^2 \theta}{\cos^2 \theta} \right) = \frac{1200 (1 + 1 - \cos^2 \theta)}{\cos^3 \theta}$$

$$= \frac{1200 (1 + \sin^2 \theta)}{\cos^3 \theta}$$

$$113. \text{Maximum, } 1250 \quad 114. (f \circ g)(x) = \frac{x-1}{x-2} \quad 115. \sin \theta = \frac{5}{13}; \cos \theta = -\frac{12}{13}; \tan \theta = -\frac{5}{12}; \csc \theta = \frac{13}{5}; \sec \theta = -\frac{13}{12}; \cot \theta = -\frac{12}{5} \quad 116. -\frac{2}{\pi}$$

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$$5. - \quad 6. - \quad 7. F \quad 8. F \quad 9. F \quad 10. T \quad 11. a \quad 12. d \quad 13. -\frac{1}{4}(\sqrt{2} + \sqrt{6}) \quad 15. 2 - \sqrt{3} \quad 17. \frac{1}{4}(\sqrt{6} + \sqrt{2}) \quad 19. \frac{1}{4}(\sqrt{2} - \sqrt{6})$$

$$21. -\frac{1}{4}(\sqrt{6} + \sqrt{2}) \quad 23. \sqrt{6} - \sqrt{2} \quad 25. \frac{1}{2} \quad 27. 0 \quad 29. 1 \quad 31. -1 \quad 33. \frac{1}{2} \quad 35. (a) \frac{2\sqrt{5}}{25} \quad (b) \frac{11\sqrt{5}}{25} \quad (c) \frac{2\sqrt{5}}{5} \quad (d) 2$$

$$37. (a) \frac{4-3\sqrt{3}}{10} \quad (b) \frac{-3-4\sqrt{3}}{10} \quad (c) \frac{4+3\sqrt{3}}{10} \quad (d) \frac{25\sqrt{3}+48}{39} \quad 39. (a) -\frac{5+12\sqrt{3}}{26} \quad (b) \frac{12-5\sqrt{3}}{26} \quad (c) \frac{-5+12\sqrt{3}}{26} \quad (d) \frac{-240+169\sqrt{3}}{69}$$

$$41. (a) -\frac{2\sqrt{2}}{3} \quad (b) \frac{-2\sqrt{2} + \sqrt{3}}{6} \quad (c) \frac{-2\sqrt{2} + \sqrt{3}}{6} \quad (d) \frac{9-4\sqrt{2}}{7} \quad 43. \frac{1-2\sqrt{6}}{6} \quad 45. \frac{\sqrt{3}-2\sqrt{2}}{6} \quad 47. \frac{8\sqrt{2}-9\sqrt{3}}{5}$$

$$49. \sin\left(\frac{\pi}{2} + \theta\right) = \sin \frac{\pi}{2} \cos \theta + \cos \frac{\pi}{2} \sin \theta = 1 \cdot \cos \theta + 0 \cdot \sin \theta = \cos \theta$$

$$51. \sin(\pi - \theta) = \sin \pi \cos \theta - \cos \pi \sin \theta = 0 \cdot \cos \theta - (-1) \sin \theta = \sin \theta$$

$$53. \sin(\pi + \theta) = \sin \pi \cos \theta + \cos \pi \sin \theta = 0 \cdot \cos \theta + (-1) \sin \theta = -\sin \theta$$

$$55. \tan(\pi - \theta) = \frac{\tan \pi - \tan \theta}{1 + \tan \pi \tan \theta} = \frac{0 - \tan \theta}{1 + 0 \cdot \tan \theta} = -\tan \theta$$

$$57. \sin\left(\frac{3\pi}{2} + \theta\right) = \sin \frac{3\pi}{2} \cos \theta + \cos \frac{3\pi}{2} \sin \theta = (-1) \cos \theta + 0 \cdot \sin \theta = -\cos \theta$$

$$59. \sin(\alpha + \beta) + \sin(\alpha - \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta + \sin \alpha \cos \beta - \cos \alpha \sin \beta = 2 \sin \alpha \cos \beta$$

$$61. \frac{\sin(\alpha + \beta)}{\sin \alpha \cos \beta} = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\sin \alpha \cos \beta} = \frac{\sin \alpha \cos \beta}{\sin \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\sin \alpha \cos \beta} = 1 + \cot \alpha \tan \beta$$

$$63. \frac{\cos(\alpha + \beta)}{\cos \alpha \cos \beta} = \frac{\cos \alpha \cos \beta - \sin \alpha \sin \beta}{\cos \alpha \cos \beta} = \frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta} = 1 - \tan \alpha \tan \beta$$

$$65. \frac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)} = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\sin \alpha \cos \beta - \cos \alpha \sin \beta} = \frac{\frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\cos \alpha \cos \beta}} = \frac{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}} = \frac{\tan \alpha + \tan \beta}{\tan \alpha - \tan \beta}$$

$$67. \cot(\alpha + \beta) = \frac{\cos(\alpha + \beta)}{\sin(\alpha + \beta)} = \frac{\cos \alpha \cos \beta - \sin \alpha \sin \beta}{\sin \alpha \cos \beta + \cos \alpha \sin \beta} = \frac{\frac{\cos \alpha \cos \beta - \sin \alpha \sin \beta}{\sin \alpha \sin \beta}}{\frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\sin \alpha \sin \beta}} = \frac{\frac{\cos \alpha \cos \beta}{\sin \alpha \sin \beta} - \frac{\sin \alpha \sin \beta}{\sin \alpha \sin \beta}}{\frac{\sin \alpha \cos \beta}{\sin \alpha \sin \beta} + \frac{\cos \alpha \sin \beta}{\sin \alpha \sin \beta}} = \frac{\cot \alpha \cot \beta - 1}{\cot \beta + \cot \alpha}$$

$$69. \sec(\alpha + \beta) = \frac{1}{\cos(\alpha + \beta)} = \frac{1}{\cos \alpha \cos \beta - \sin \alpha \sin \beta} = \frac{\frac{1}{\sin \alpha \sin \beta}}{\frac{\cos \alpha \cos \beta - \sin \alpha \sin \beta}{\sin \alpha \sin \beta}} = \frac{\frac{1}{\sin \alpha} \cdot \frac{1}{\sin \beta}}{\frac{\cos \alpha \cos \beta}{\sin \alpha \sin \beta} - \frac{\sin \alpha \sin \beta}{\sin \alpha \sin \beta}} = \frac{\csc \alpha \csc \beta}{\cot \alpha \cot \beta - 1}$$

$$71. \sin(\alpha - \beta) \sin(\alpha + \beta) = (\sin \alpha \cos \beta - \cos \alpha \sin \beta)(\sin \alpha \cos \beta + \cos \alpha \sin \beta) = \sin^2 \alpha \cos^2 \beta - \cos^2 \alpha \sin^2 \beta$$

$$= (\sin^2 \alpha)(1 - \sin^2 \beta) - (1 - \sin^2 \alpha)(\sin^2 \beta) = \sin^2 \alpha - \sin^2 \beta$$

$$73. \sin(\theta + k\pi) = \sin \theta \cos k\pi + \cos \theta \sin k\pi = (\sin \theta)(-1)^k + (\cos \theta)(0) = (-1)^k \sin \theta, k \text{ any integer}$$

$$75. \frac{\sqrt{3}}{2} \quad 77. -\frac{24}{25} \quad 79. -\frac{33}{65} \quad 81. \frac{63}{65} \quad 83. \frac{48 + 25\sqrt{3}}{39} \quad 85. \frac{4}{3} \quad 87. u\sqrt{1-v^2} - v\sqrt{1-u^2}; -1 \leq u \leq 1; -1 \leq v \leq 1$$

$$89. \frac{u\sqrt{1-v^2} - v}{\sqrt{1+u^2}}; -\infty < u < \infty; -1 \leq v \leq 1 \quad 91. \frac{uv - \sqrt{1-u^2}\sqrt{1-v^2}}{v\sqrt{1-u^2} + u\sqrt{1-v^2}}; -1 \leq u \leq 1; -1 \leq v \leq 1 \quad 93. \left\{ \frac{\pi}{2}, \frac{7\pi}{6} \right\} \quad 95. \left\{ \frac{\pi}{4} \right\} \quad 97. \left\{ \frac{11\pi}{6} \right\}$$

$$99. \text{Let } \alpha = \sin^{-1} v \text{ and } \beta = \cos^{-1} v. \text{ Then } \sin \alpha = \cos \beta = v, \text{ and since } \sin \alpha = \cos\left(\frac{\pi}{2} - \alpha\right), \cos\left(\frac{\pi}{2} - \alpha\right) = \cos \beta.$$

If $v \geq 0$, then $0 \leq \alpha \leq \frac{\pi}{2}$, so $\left(\frac{\pi}{2} - \alpha\right)$ and β both lie on $\left[0, \frac{\pi}{2}\right]$. If $v < 0$, then $-\frac{\pi}{2} \leq \alpha < 0$, so $\left(\frac{\pi}{2} - \alpha\right)$ and β both lie on $\left(\frac{\pi}{2}, \pi\right]$.

Either way, $\cos\left(\frac{\pi}{2} - \alpha\right) = \cos \beta$ implies $\frac{\pi}{2} - \alpha = \beta$, or $\alpha + \beta = \frac{\pi}{2}$.

$$101. \text{Let } \alpha = \tan^{-1} \frac{1}{v} \text{ and } \beta = \tan^{-1} v. \text{ Because } v \neq 0, \alpha, \beta \neq 0. \text{ Then } \tan \alpha = \frac{1}{v} = \frac{1}{\tan \beta} = \cot \beta, \text{ and since}$$

$$\tan \alpha = \cot\left(\frac{\pi}{2} - \alpha\right), \cot\left(\frac{\pi}{2} - \alpha\right) = \cot \beta. \text{ Because } v > 0, 0 < \alpha < \frac{\pi}{2}, \text{ and so } \left(\frac{\pi}{2} - \alpha\right) \text{ and } \beta \text{ both lie on } \left(0, \frac{\pi}{2}\right).$$

$$\text{Then } \cot\left(\frac{\pi}{2} - \alpha\right) = \cot \beta \text{ implies } \frac{\pi}{2} - \alpha = \beta, \text{ or } \alpha = \frac{\pi}{2} - \beta.$$

$$103. \sin(\sin^{-1} v + \cos^{-1} v) = \sin(\sin^{-1} v) \cos(\cos^{-1} v) + \cos(\sin^{-1} v) \sin(\cos^{-1} v) = (v)(v) + \sqrt{1-v^2}\sqrt{1-v^2} = v^2 + 1 - v^2 = 1$$

$$105. \frac{\sin(x+h) - \sin x}{h} = \frac{\sin x \cos h + \cos x \sin h - \sin x}{h} = \frac{\cos x \sin h - \sin x(1 - \cos h)}{h} = \cos x \cdot \frac{\sin h}{h} - \sin x \cdot \frac{1 - \cos h}{h}$$

$$107. (a) \tan(\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3) = \tan((\tan^{-1} 1 + \tan^{-1} 2) + \tan^{-1} 3) = \frac{\tan(\tan^{-1} 1 + \tan^{-1} 2) + \tan(\tan^{-1} 3)}{1 - \tan(\tan^{-1} 1 + \tan^{-1} 2) \tan(\tan^{-1} 3)}$$

$$= \frac{\frac{\tan(\tan^{-1} 1) + \tan(\tan^{-1} 2)}{1 - \tan(\tan^{-1} 1) \tan(\tan^{-1} 2)} + 3}{1 - \frac{\tan(\tan^{-1} 1) + \tan(\tan^{-1} 2)}{1 - \tan(\tan^{-1} 1) \tan(\tan^{-1} 2)} \cdot 3} = \frac{\frac{1+2}{1-1 \cdot 2} + 3}{1 - \frac{1+2}{1-1 \cdot 2} \cdot 3} = \frac{\frac{3}{-1} + 3}{1 - \frac{3}{-1} \cdot 3} = \frac{-3 + 3}{1 + 9} = \frac{0}{10} = 0$$

$$(b) \text{ From the definition of the inverse tangent function, } 0 < \tan^{-1} 1 < \frac{\pi}{2}, 0 < \tan^{-1} 2 < \frac{\pi}{2}, \text{ and } 0 < \tan^{-1} 3 < \frac{\pi}{2},$$

$$\text{so } 0 < \tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 < \frac{3\pi}{2}.$$

On the interval $\left(0, \frac{3\pi}{2}\right)$, $\tan \theta = 0$ if and only if $\theta = \pi$. Therefore, from part (a), $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$.

$$109. \tan \theta = \tan(\theta_2 - \theta_1) = \frac{\tan \theta_2 - \tan \theta_1}{1 + \tan \theta_1 \tan \theta_2} = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$111. 2 \cot(\alpha - \beta) = \frac{2}{\tan(\alpha - \beta)} = 2 \left(\frac{1 + \tan \alpha \tan \beta}{\tan \alpha - \tan \beta} \right) = 2 \left(\frac{1 + (x+1)(x-1)}{(x+1) - (x-1)} \right) = 2 \left(\frac{1 + x^2 - 1}{x+1 - x+1} \right) = \frac{2x^2}{2} = x^2$$

113. $\tan \frac{\pi}{2}$ is not defined; $\tan\left(\frac{\pi}{2} - \theta\right) = \frac{\sin\left(\frac{\pi}{2} - \theta\right)}{\cos\left(\frac{\pi}{2} - \theta\right)} = \frac{\cos \theta}{\sin \theta} = \cot \theta$.

114. $\left(-\frac{1}{3}, -\frac{5}{9}\right), (-5, 1)$ 115. 510° 116. $\frac{9\pi}{2} \text{ cm}^2 \approx 14.14 \text{ cm}^2$ 117. $\sin \theta = -\frac{2\sqrt{5}}{5}; \cos \theta = \frac{\sqrt{5}}{5}; \csc \theta = -\frac{\sqrt{5}}{2}; \sec \theta = \sqrt{5}; \cot \theta = -\frac{1}{2}$

7.6 Assess Your Understanding (page 509)

1. $\sin^2 \theta; 2 \cos^2 \theta; 2 \sin^2 \theta$ 2. $1 - \cos \theta$ 3. $\sin \theta$ 4. T 5. F 6. F 7. b 8. c

9. (a) $\frac{24}{25}$ (b) $\frac{7}{25}$ (c) $\frac{\sqrt{10}}{10}$ (d) $\frac{3\sqrt{10}}{10}$ (e) $\frac{24}{7}$ (f) $\frac{1}{3}$

11. (a) $\frac{24}{25}$ (b) $-\frac{7}{25}$ (c) $\frac{2\sqrt{5}}{5}$ (d) $-\frac{\sqrt{5}}{5}$ (e) $-\frac{24}{7}$ (f) -2

13. (a) $-\frac{2\sqrt{2}}{3}$ (b) $\frac{1}{3}$ (c) $\sqrt{\frac{3+\sqrt{6}}{6}}$ (d) $\sqrt{\frac{3-\sqrt{6}}{6}}$ (e) $-2\sqrt{2}$ (f) $\sqrt{\frac{3+\sqrt{6}}{3-\sqrt{6}}}$ or $\sqrt{5+2\sqrt{6}}$

15. (a) $\frac{4\sqrt{2}}{9}$ (b) $-\frac{7}{9}$ (c) $\frac{\sqrt{3}}{3}$ (d) $\frac{\sqrt{6}}{3}$ (e) $-\frac{4\sqrt{2}}{7}$ (f) $\frac{\sqrt{2}}{2}$

17. (a) $-\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\sqrt{\frac{5+2\sqrt{5}}{10}}$ (d) $\sqrt{\frac{5-2\sqrt{5}}{10}}$ (e) $-\frac{4}{3}$ (f) $\sqrt{\frac{5+2\sqrt{5}}{5-2\sqrt{5}}}$ or $\sqrt{9+4\sqrt{5}}$

19. (a) $-\frac{3}{5}$ (b) $-\frac{4}{5}$ (c) $\frac{1}{2}\sqrt{\frac{10-\sqrt{10}}{5}}$ (d) $-\frac{1}{2}\sqrt{\frac{10+\sqrt{10}}{5}}$ (e) $\frac{3}{4}$ (f) $-\sqrt{\frac{10-\sqrt{10}}{10+\sqrt{10}}}$ or $-\frac{\sqrt{11-2\sqrt{10}}}{3}$

21. $\frac{\sqrt{2-\sqrt{2}}}{2}$ 23. $1 - \sqrt{2}$ 25. $-\frac{\sqrt{2+\sqrt{3}}}{2}$ 27. $\frac{2}{\sqrt{2+\sqrt{2}}} = (2 - \sqrt{2})\sqrt{2+\sqrt{2}}$ 29. $-\frac{\sqrt{2-\sqrt{2}}}{2}$ 31. $-\frac{4}{5}$ 33. $\frac{\sqrt{10(5-\sqrt{5})}}{10}$

35. $\frac{4}{3}$ 37. $-\frac{7}{8}$ 39. $\frac{\sqrt{10}}{4}$ 41. $-\frac{\sqrt{15}}{3}$ 43. $\sin^4 \theta = (\sin^2 \theta)^2 = \left(\frac{1 - \cos(2\theta)}{2}\right)^2 = \frac{1}{4} [1 - 2 \cos(2\theta) + \cos^2(2\theta)] = \frac{1}{4} - \frac{1}{2} \cos(2\theta) + \frac{1}{4} \cos^2(2\theta)$
 $= \frac{1}{4} - \frac{1}{2} \cos(2\theta) + \frac{1}{4} \left(\frac{1 + \cos(4\theta)}{2}\right) = \frac{1}{4} - \frac{1}{2} \cos(2\theta) + \frac{1}{8} + \frac{1}{8} \cos(4\theta) = \frac{3}{8} - \frac{1}{2} \cos(2\theta) + \frac{1}{8} \cos(4\theta)$

45. $\cos(3\theta) = 4 \cos^3 \theta - 3 \cos \theta$ 47. $\sin(5\theta) = 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$ 49. $\cos^4 \theta - \sin^4 \theta = (\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta) = \cos(2\theta)$

51. $\cot(2\theta) = \frac{1}{\tan(2\theta)} = \frac{1 - \tan^2 \theta}{2 \tan \theta} = \frac{1 - \frac{1}{\cot^2 \theta}}{2\left(\frac{1}{\cot \theta}\right)} = \frac{\frac{\cot^2 \theta - 1}{\cot^2 \theta}}{\frac{2}{\cot \theta}} = \frac{\cot^2 \theta - 1}{\cot^2 \theta} \cdot \frac{\cot \theta}{2} = \frac{\cot^2 \theta - 1}{2 \cot \theta}$

53. $\sec(2\theta) = \frac{1}{\cos(2\theta)} = \frac{1}{2 \cos^2 \theta - 1} = \frac{1}{\frac{2}{\sec^2 \theta} - 1} = \frac{1}{\frac{2 - \sec^2 \theta}{\sec^2 \theta}} = \frac{\sec^2 \theta}{2 - \sec^2 \theta}$

55. $\cos^2(2u) - \sin^2(2u) = \cos[2(2u)] = \cos(4u)$

57. $\frac{\cos(2\theta)}{1 + \sin(2\theta)} = \frac{\cos^2 \theta - \sin^2 \theta}{1 + 2 \sin \theta \cos \theta} = \frac{(\cos \theta - \sin \theta)(\cos \theta + \sin \theta)}{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta} = \frac{(\cos \theta - \sin \theta)(\cos \theta + \sin \theta)}{(\sin \theta + \cos \theta)(\sin \theta + \cos \theta)} = \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta}$
 $= \frac{\frac{\cos \theta - \sin \theta}{\sin \theta}}{\frac{\cos \theta + \sin \theta}{\sin \theta}} = \frac{\frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\sin \theta}}{\frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\sin \theta}} = \frac{\cot \theta - 1}{\cot \theta + 1}$

59. $\sec^2 \frac{\theta}{2} = \frac{1}{\cos^2\left(\frac{\theta}{2}\right)} = \frac{1}{\frac{1 + \cos \theta}{2}} = \frac{2}{1 + \cos \theta}$

61. $\cot^2 \frac{v}{2} = \frac{1}{\tan^2\left(\frac{v}{2}\right)} = \frac{1}{\frac{1 - \cos v}{1 + \cos v}} = \frac{1 + \cos v}{1 - \cos v} = \frac{1 + \frac{1}{\sec v}}{1 - \frac{1}{\sec v}} = \frac{\frac{\sec v + 1}{\sec v}}{\frac{\sec v - 1}{\sec v}} = \frac{\sec v + 1}{\sec v - 1} \cdot \frac{\sec v}{\sec v} = \frac{\sec v + 1}{\sec v - 1}$

63. $\frac{1 - \tan^2\left(\frac{\theta}{2}\right)}{1 + \tan^2\left(\frac{\theta}{2}\right)} = \frac{1 - \frac{1 - \cos \theta}{1 + \cos \theta}}{1 + \frac{1 - \cos \theta}{1 + \cos \theta}} = \frac{\frac{1 + \cos \theta - (1 - \cos \theta)}{1 + \cos \theta}}{\frac{1 + \cos \theta + 1 - \cos \theta}{1 + \cos \theta}} = \frac{2 \cos \theta}{1 + \cos \theta} \cdot \frac{1 + \cos \theta}{2} = \cos \theta$

65. $\frac{\sin(3\theta)}{\sin\theta} - \frac{\cos(3\theta)}{\cos\theta} = \frac{\sin(3\theta)\cos\theta - \cos(3\theta)\sin\theta}{\sin\theta\cos\theta} = \frac{\sin(3\theta - \theta)}{\frac{1}{2}(2\sin\theta\cos\theta)} = \frac{2\sin(2\theta)}{\sin(2\theta)} = 2$

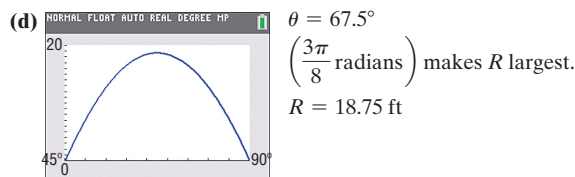
67. $\tan(3\theta) = \tan(\theta + 2\theta) = \frac{\tan\theta + \tan(2\theta)}{1 - \tan\theta\tan(2\theta)} = \frac{\tan\theta + \frac{2\tan\theta}{1 - \tan^2\theta}}{1 - \frac{\tan\theta(2\tan\theta)}{1 - \tan^2\theta}} = \frac{\tan\theta - \tan^3\theta + 2\tan\theta}{1 - \tan^2\theta - 2\tan^2\theta} = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

69. $\frac{1}{2}(\ln|1 - \cos(2\theta)| - \ln 2) = \ln\left(\frac{|1 - \cos(2\theta)|}{2}\right)^{1/2} = \ln|\sin^2\theta|^{1/2} = \ln|\sin\theta|$ 71. $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$ 73. $\left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

75. $\left\{0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}\right\}$ 77. No real solution 79. $\left\{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\}$ 81. $\frac{\sqrt{3}}{2}$ 83. $\frac{7}{25}$ 85. $\frac{24}{7}$ 87. $\frac{24}{25}$ 89. $\frac{1}{5}$ 91. $\frac{25}{7}$ 93. $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

95. $\frac{\pi}{2}, \frac{3\pi}{2}$ 97. (a) $W = 2D(\csc\theta - \cot\theta) = 2D\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right) = 2D\frac{1 - \cos\theta}{\sin\theta} = 2D\tan\frac{\theta}{2}$ (b) $\theta = 24.45^\circ$

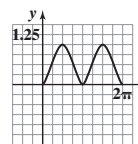
99. (a) $R = \frac{v_0^2\sqrt{2}}{16}\cos\theta(\sin\theta - \cos\theta)$ (b) $\frac{3\pi}{8}$ or 67.5°
 $= \frac{v_0^2\sqrt{2}}{32}(2\cos\theta\sin\theta - 2\cos^2\theta)$ (c) $32(2 - \sqrt{2}) \approx 18.75$ ft
 $= \frac{v_0^2\sqrt{2}}{32}[\sin(2\theta) - \cos(2\theta) - 1]$



101. $A = \frac{1}{2}h(\text{base}) = h\left(\frac{1}{2}\text{base}\right) = s\cos\frac{\theta}{2} \cdot s\sin\frac{\theta}{2} = \frac{1}{2}s^2\sin\theta$

103. (a) $150 + 75\sqrt{3}$ cm² (b) $\frac{10 + 5\sqrt{3}}{2}$ cm (c) $\frac{175\pi + 100\pi\sqrt{3}}{4}$ cm² (d) $\frac{600 + 300\sqrt{3} - 175\pi - 100\pi\sqrt{3}}{4}$ cm² 105. $-\frac{1}{4}$

107. $\frac{2z}{1+z^2} = \frac{2\tan\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)} = \frac{2\tan\left(\frac{\alpha}{2}\right)}{\sec^2\left(\frac{\alpha}{2}\right)} = \frac{2\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right)}{\frac{1}{\cos^2\left(\frac{\alpha}{2}\right)}} = 2\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right) = \sin\left(2 \cdot \frac{\alpha}{2}\right) = \sin\alpha$ 109.

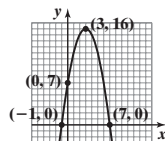


111. $\sin\frac{\pi}{24} = \frac{\sqrt{2}}{4}\sqrt{4 - \sqrt{6} - \sqrt{2}}$

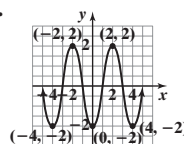
$\cos\frac{\pi}{24} = \frac{\sqrt{2}}{4}\sqrt{4 + \sqrt{6} + \sqrt{2}}$

113. $\sin^3\theta + \sin^3(\theta + 120^\circ) + \sin^3(\theta + 240^\circ) = \sin^3\theta + (\sin\theta\cos 120^\circ + \cos\theta\sin 120^\circ)^3 + (\sin\theta\cos 240^\circ + \cos\theta\sin 240^\circ)^3$
 $= \sin^3\theta + \left(-\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta\right)^3 + \left(-\frac{1}{2}\sin\theta - \frac{\sqrt{3}}{2}\cos\theta\right)^3$
 $= \sin^3\theta + \frac{1}{8}(3\sqrt{3}\cos^3\theta - 9\cos^2\theta\sin\theta + 3\sqrt{3}\cos\theta\sin^2\theta - \sin^3\theta) - \frac{1}{8}(\sin^3\theta + 3\sqrt{3}\sin^2\theta\cos\theta + 9\sin\theta\cos^2\theta + 3\sqrt{3}\cos^3\theta)$
 $= \frac{3}{4}\sin^3\theta - \frac{9}{4}\cos^2\theta\sin\theta = \frac{3}{4}[\sin^3\theta - 3\sin\theta(1 - \sin^2\theta)] = \frac{3}{4}(4\sin^3\theta - 3\sin\theta) = -\frac{3}{4}\sin(3\theta)$ (from Example 2)

115. $-\frac{1}{2}$ 117. $y = \frac{1}{2}x - 4$ 118.



119. $\frac{\sqrt{3} + 1}{2}$ 120.



7.7 Assess Your Understanding (page 515)

1. $\frac{1}{2}\left(\frac{\sqrt{3}}{2} - 1\right)$ 3. $-\frac{1}{2}\left(\frac{\sqrt{3}}{2} + 1\right)$ 5. $\frac{\sqrt{2}}{2}$ 7. $\frac{1}{2}[\cos(2\theta) - \cos(6\theta)]$ 9. $\frac{1}{2}[\sin(6\theta) + \sin(2\theta)]$ 11. $\frac{1}{2}[\cos(2\theta) + \cos(8\theta)]$

13. $\frac{1}{2}[\cos\theta - \cos(3\theta)]$ 15. $\frac{1}{2}[\sin(2\theta) + \sin\theta]$ 17. $2\sin\theta\cos(3\theta)$ 19. $2\cos(3\theta)\cos\theta$ 21. $2\sin(2\theta)\cos\theta$ 23. $2\sin\theta\sin\frac{\theta}{2}$

25. $\frac{\sin\theta + \sin(3\theta)}{2\sin(2\theta)} = \frac{2\sin(2\theta)\cos\theta}{2\sin(2\theta)} = \cos\theta$ 27. $\frac{\sin(4\theta) + \sin(2\theta)}{\cos(4\theta) + \cos(2\theta)} = \frac{2\sin(3\theta)\cos\theta}{2\cos(3\theta)\cos\theta} = \frac{\sin(3\theta)}{\cos(3\theta)} = \tan(3\theta)$

29. $\frac{\cos \theta - \cos(3\theta)}{\sin \theta + \sin(3\theta)} = \frac{2 \sin(2\theta) \sin \theta}{2 \sin(2\theta) \cos \theta} = \frac{\sin \theta}{\cos \theta} = \tan \theta$

31. $\sin \theta [\sin \theta + \sin(3\theta)] = \sin \theta [2 \sin(2\theta) \cos \theta] = \cos \theta [2 \sin(2\theta) \sin \theta] = \cos \theta \left[2 \cdot \frac{1}{2} [\cos \theta - \cos(3\theta)] \right] = \cos \theta [\cos \theta - \cos(3\theta)]$

33. $\frac{\sin(4\theta) + \sin(8\theta)}{\cos(4\theta) + \cos(8\theta)} = \frac{2 \sin(6\theta) \cos(2\theta)}{2 \cos(6\theta) \cos(2\theta)} = \frac{\sin(6\theta)}{\cos(6\theta)} = \tan(6\theta)$

35. $\frac{\sin(4\theta) + \sin(8\theta)}{\sin(4\theta) - \sin(8\theta)} = \frac{2 \sin(6\theta) \cos(-2\theta)}{2 \sin(-2\theta) \cos(6\theta)} = \frac{\sin(6\theta)}{\cos(6\theta)} \cdot \frac{\cos(2\theta)}{-\sin(2\theta)} = \tan(6\theta) [-\cot(2\theta)] = -\frac{\tan(6\theta)}{\tan(2\theta)}$

37. $\frac{\sin \alpha + \sin \beta}{\sin \alpha - \sin \beta} = \frac{2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}}{2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}} = \frac{\sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}}{\cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}} = \tan \frac{\alpha + \beta}{2} \cot \frac{\alpha - \beta}{2}$

39. $\frac{\sin \alpha + \sin \beta}{\cos \alpha + \cos \beta} = \frac{2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}}{2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}} = \frac{\sin \frac{\alpha + \beta}{2}}{\cos \frac{\alpha + \beta}{2}} = \tan \frac{\alpha + \beta}{2}$

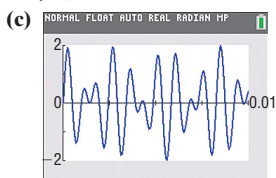
41. $1 + \cos(2\theta) + \cos(4\theta) + \cos(6\theta) = [1 + \cos(6\theta)] + [\cos(2\theta) + \cos(4\theta)] = 2 \cos^2(3\theta) + 2 \cos(3\theta) \cos(-\theta) = 2 \cos(3\theta) [\cos(3\theta) + \cos \theta] = 2 \cos(3\theta) [2 \cos(2\theta) \cos \theta] = 4 \cos \theta \cos(2\theta) \cos(3\theta)$

43. $\left\{ 0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3} \right\}$

45. $\left\{ 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi, \frac{6\pi}{5}, \frac{7\pi}{5}, \frac{8\pi}{5}, \frac{9\pi}{5} \right\}$

47. (a) $y = 2 \sin(2061\pi t) \cos(357\pi t)$

(b) $y_{\max} = 2$



49. $I_u = I_x \cos^2 \theta + I_y \sin^2 \theta - 2I_{xy} \sin \theta \cos \theta = I_x \cos^2 \theta + I_y \sin^2 \theta - I_{xy} \sin 2\theta$

$$= I_x \left(\frac{\cos 2\theta + 1}{2} \right) + I_y \left(\frac{1 - \cos 2\theta}{2} \right) - I_{xy} \sin 2\theta$$

$$= \frac{I_x}{2} \cos 2\theta + \frac{I_x}{2} + \frac{I_y}{2} - \frac{I_y}{2} \cos 2\theta - I_{xy} \sin 2\theta$$

$$= \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2} \cos 2\theta - I_{xy} \sin 2\theta$$

$$I_v = I_x \sin^2 \theta + I_y \cos^2 \theta + 2I_{xy} \sin \theta \cos \theta = I_x \left(\frac{1 - \cos 2\theta}{2} \right) + I_y \left(\frac{\cos 2\theta + 1}{2} \right) + I_{xy} \sin 2\theta$$

$$= \frac{I_x}{2} - \frac{I_x}{2} \cos 2\theta + \frac{I_y}{2} \cos 2\theta + \frac{I_y}{2} + I_{xy} \sin 2\theta$$

$$= \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2} \cos 2\theta + I_{xy} \sin 2\theta$$

51. $\sin(2\alpha) + \sin(2\beta) + \sin(2\gamma) = 2 \sin(\alpha + \beta) \cos(\alpha - \beta) + \sin(2\gamma) = 2 \sin(\alpha + \beta) \cos(\alpha - \beta) + 2 \sin \gamma \cos \gamma = 2 \sin(\pi - \gamma) \cos(\alpha - \beta) + 2 \sin \gamma \cos \gamma = 2 \sin \gamma \cos(\alpha - \beta) + 2 \sin \gamma \cos \gamma = 2 \sin \gamma [\cos(\alpha - \beta) + \cos \gamma]$

$$= 2 \sin \gamma \left(2 \cos \frac{\alpha - \beta + \gamma}{2} \cos \frac{\alpha - \beta - \gamma}{2} \right) = 4 \sin \gamma \cos \frac{\pi - 2\beta}{2} \cos \frac{2\alpha - \pi}{2} = 4 \sin \gamma \cos \left(\frac{\pi}{2} - \beta \right) \cos \left(\alpha - \frac{\pi}{2} \right)$$

$$= 4 \sin \gamma \sin \beta \sin \alpha = 4 \sin \alpha \sin \beta \sin \gamma$$

53. $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) + \sin(\alpha + \beta) = 2 \sin \alpha \cos \beta$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

55. $2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} = 2 \cdot \frac{1}{2} \left[\cos \left(\frac{\alpha + \beta}{2} + \frac{\alpha - \beta}{2} \right) + \cos \left(\frac{\alpha + \beta}{2} - \frac{\alpha - \beta}{2} \right) \right] = \cos \frac{2\alpha}{2} + \cos \frac{2\beta}{2} = \cos \alpha + \cos \beta$

57. {13} 58. Amplitude: 5; Period: $\frac{\pi}{2}$; Phase shift: $\frac{\pi}{4}$ 59. $\frac{2\sqrt{6}}{7}$

60. $f^{-1}(x) = \sin^{-1}\left(\frac{x+5}{3}\right)$; Range of $f =$ Domain of $f^{-1} = [-8, -2]$; Range of $f^{-1} = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Review Exercises (page 518)

1. $\frac{\pi}{2}$ 2. $\frac{\pi}{2}$ 3. $\frac{\pi}{4}$ 4. $-\frac{\pi}{6}$ 5. $\frac{5\pi}{6}$ 6. $-\frac{\pi}{3}$ 7. $\frac{\pi}{4}$ 8. $\frac{3\pi}{4}$ 9. $\frac{3\pi}{8}$ 10. $\frac{3\pi}{4}$ 11. $-\frac{\pi}{3}$ 12. $\frac{\pi}{7}$ 13. $-\frac{\pi}{9}$ 14. 0.9 15. 0.6 16. 5 17. Not defined

18. $-\frac{\pi}{6}$ 19. π 20. $-\sqrt{3}$ 21. $\frac{2\sqrt{3}}{3}$ 22. $\frac{4}{5}$ 23. $-\frac{4}{3}$ 24. $f^{-1}(x) = \frac{1}{3} \sin^{-1}\left(\frac{x}{2}\right)$; Range of $f =$ Domain of $f^{-1} = [-2, 2]$; Range of $f^{-1} = \left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$

25. $f^{-1}(x) = \cos^{-1}(3 - x)$; Range of $f = \text{Domain of } f^{-1} = [2, 4]$; Range of $f^{-1} = [0, \pi]$
26. $\sqrt{1 - u^2}$
27. $\frac{|u|}{u\sqrt{u^2 - 1}}$
28. $\tan \theta \cot \theta - \sin^2 \theta = 1 - \sin^2 \theta = \cos^2 \theta$
29. $\sin^2 \theta (1 + \cot^2 \theta) = \sin^2 \theta \csc^2 \theta = 1$
30. $5 \cos^2 \theta + 3 \sin^2 \theta = 2 \cos^2 \theta + 3(\cos^2 \theta + \sin^2 \theta) = 3 + 2 \cos^2 \theta$
31. $\frac{1 - \cos \theta}{\sin \theta} + \frac{\sin \theta}{1 - \cos \theta} = \frac{(1 - \cos \theta)^2 + \sin^2 \theta}{\sin \theta (1 - \cos \theta)} = \frac{1 - 2 \cos \theta + \cos^2 \theta + \sin^2 \theta}{\sin \theta (1 - \cos \theta)} = \frac{2(1 - \cos \theta)}{\sin \theta (1 - \cos \theta)} = 2 \csc \theta$
32. $\frac{\cos \theta}{\cos \theta - \sin \theta} = \frac{\frac{\cos \theta}{\cos \theta}}{\frac{\cos \theta - \sin \theta}{\cos \theta}} = \frac{1}{1 - \frac{\sin \theta}{\cos \theta}} = \frac{1}{1 - \tan \theta}$
33. $\frac{\csc \theta}{1 + \csc \theta} = \frac{\frac{1}{\sin \theta}}{1 + \frac{1}{\sin \theta}} = \frac{1}{1 + \sin \theta} = \frac{1}{1 + \sin \theta} \cdot \frac{1 - \sin \theta}{1 - \sin \theta} = \frac{1 - \sin \theta}{1 - \sin^2 \theta} = \frac{1 - \sin \theta}{\cos^2 \theta}$
34. $\csc \theta - \sin \theta = \frac{1}{\sin \theta} - \sin \theta = \frac{1 - \sin^2 \theta}{\sin \theta} = \frac{\cos^2 \theta}{\sin \theta} = \cos \theta \cdot \frac{\cos \theta}{\sin \theta} = \cos \theta \cot \theta$
35. $\frac{1 - \sin \theta}{\sec \theta} = \cos \theta (1 - \sin \theta) \cdot \frac{1 + \sin \theta}{1 + \sin \theta} = \frac{\cos \theta (1 - \sin^2 \theta)}{1 + \sin \theta} = \frac{\cos^3 \theta}{1 + \sin \theta}$
36. $\cot \theta - \tan \theta = \frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta} = \frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta} = \frac{1 - 2 \sin^2 \theta}{\sin \theta \cos \theta}$
37. $\frac{\cos(\alpha + \beta)}{\cos \alpha \sin \beta} = \frac{\cos \alpha \cos \beta - \sin \alpha \sin \beta}{\cos \alpha \sin \beta} = \frac{\cos \alpha \cos \beta}{\cos \alpha \sin \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \sin \beta} = \cot \beta - \tan \alpha$
38. $\frac{\cos(\alpha - \beta)}{\cos \alpha \cos \beta} = \frac{\cos \alpha \cos \beta + \sin \alpha \sin \beta}{\cos \alpha \cos \beta} = \frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta} = 1 + \tan \alpha \tan \beta$
39. $(1 + \cos \theta) \left(\tan \frac{\theta}{2} \right) = (1 + \cos \theta) \cdot \frac{\sin \theta}{1 + \cos \theta} = \sin \theta$
40. $2 \cot \theta \cot 2\theta = 2 \left(\frac{\cos \theta}{\sin \theta} \right) \left(\frac{\cos 2\theta}{\sin 2\theta} \right) = \frac{2 \cos \theta (\cos^2 \theta - \sin^2 \theta)}{2 \sin^2 \theta \cos \theta} = \frac{\cos^2 \theta - \sin^2 \theta}{\sin^2 \theta} = \cot^2 \theta - 1$
41. $1 - 8 \sin^2 \theta \cos^2 \theta = 1 - 2(2 \sin \theta \cos \theta)^2 = 1 - 2 \sin^2(2\theta) = \cos(4\theta)$
42. $\frac{\sin(3\theta) \cos \theta - \sin \theta \cos(3\theta)}{\sin(2\theta)} = \frac{\sin(2\theta)}{\sin(2\theta)} = 1$
43. $\frac{\sin(2\theta) + \sin(4\theta)}{\cos(2\theta) + \cos(4\theta)} = \frac{2 \sin(3\theta) \cos(-\theta)}{2 \cos(3\theta) \cos(-\theta)} = \tan(3\theta)$
44. $\frac{\cos(2\theta) - \cos(4\theta)}{\cos(2\theta) + \cos(4\theta)} - \tan \theta \tan(3\theta) = \frac{-2 \sin(3\theta) \sin(-\theta)}{2 \cos(3\theta) \cos(-\theta)} - \tan \theta \tan(3\theta) = \tan(3\theta) \tan \theta - \tan \theta \tan(3\theta) = 0$
45. $\frac{1}{4} (\sqrt{6} - \sqrt{2})$
46. $-2 - \sqrt{3}$
47. $\frac{1}{4} (\sqrt{6} - \sqrt{2})$
48. $\frac{1}{4} (\sqrt{2} - \sqrt{6})$
49. $\frac{1}{2}$
50. $\frac{1}{2}$
51. $\sqrt{2} - 1$
52. $\frac{\sqrt{2 + \sqrt{2}}}{2}$
53. (a) $-\frac{33}{65}$ (b) $-\frac{56}{65}$ (c) $-\frac{63}{65}$ (d) $\frac{33}{56}$ (e) $\frac{24}{25}$ (f) $\frac{119}{169}$ (g) $\frac{5\sqrt{26}}{26}$ (h) $\frac{2\sqrt{5}}{5}$
54. (a) $-\frac{16}{65}$ (b) $-\frac{63}{65}$ (c) $-\frac{56}{65}$ (d) $\frac{16}{63}$ (e) $\frac{24}{25}$ (f) $\frac{119}{169}$ (g) $\frac{\sqrt{26}}{26}$ (h) $-\frac{\sqrt{10}}{10}$
55. (a) $-\frac{63}{65}$ (b) $\frac{16}{65}$ (c) $\frac{33}{65}$ (d) $-\frac{63}{16}$ (e) $\frac{24}{25}$ (f) $-\frac{119}{169}$ (g) $\frac{2\sqrt{13}}{13}$ (h) $-\frac{\sqrt{10}}{10}$
56. (a) $\frac{-\sqrt{3} - 2\sqrt{2}}{6}$ (b) $\frac{1 - 2\sqrt{6}}{6}$ (c) $\frac{-\sqrt{3} + 2\sqrt{2}}{6}$ (d) $\frac{8\sqrt{2} + 9\sqrt{3}}{23}$ (e) $-\frac{\sqrt{3}}{2}$ (f) $-\frac{7}{9}$ (g) $\frac{\sqrt{3}}{3}$ (h) $\frac{\sqrt{3}}{2}$
57. (a) 1 (b) 0 (c) $-\frac{1}{9}$ (d) Not defined (e) $\frac{4\sqrt{5}}{9}$ (f) $-\frac{1}{9}$ (g) $\frac{\sqrt{30}}{6}$ (h) $-\frac{\sqrt{6}\sqrt{3 - \sqrt{5}}}{6}$
58. $\frac{4 + 3\sqrt{3}}{10}$
59. $\frac{33}{65}$
60. $-\frac{48 + 25\sqrt{3}}{39}$
61. $-\frac{\sqrt{2}}{10}$
62. $-\frac{24}{25}$
63. $-\frac{7}{25}$
64. $\left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$
65. $\left\{ \frac{2\pi}{3}, \frac{5\pi}{3} \right\}$
66. $\left\{ \frac{3\pi}{4}, \frac{7\pi}{4} \right\}$
67. $\left\{ 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \right\}$
68. $\left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$
69. $\{0.25, 2.89\}$
70. $\left\{ 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3} \right\}$
71. $\left\{ 0, \frac{\pi}{6}, \frac{5\pi}{6} \right\}$
72. $\left\{ \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6} \right\}$
73. $\left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$
74. $\left\{ \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2} \right\}$
75. $\left\{ \frac{\pi}{2}, \pi \right\}$
76. 0.78
77. -1.11
78. 1.77
79. 1.23
80. 2.90
81. $\{1.11\}$
82. $\{0.87\}$
83. $\{2.22\}$
84. $\left\{ -\frac{\sqrt{3}}{2} \right\}$
85. $\{0\}$
86. $\sin 15^\circ = \sqrt{\frac{1 - \cos 30^\circ}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{3}}{4}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$
- $\sin 15^\circ = \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$
- $\left[\frac{\sqrt{2 - \sqrt{3}}}{2} \right]^2 = \frac{2 - \sqrt{3}}{4} = \frac{4(2 - \sqrt{3})}{4 \cdot 4} = \frac{8 - 4\sqrt{3}}{16} = \frac{6 - 2\sqrt{12} + 2}{16} = \left(\frac{\sqrt{6} - \sqrt{2}}{4} \right)^2$
87. $\cos(2\theta) = 2\cos^2 \theta - 1$

Chapter Test (page 520)

1. $\frac{\pi}{6}$ 2. $-\frac{\pi}{4}$ 3. $\frac{\pi}{5}$ 4. $\frac{7}{3}$ 5. 3 6. $-\frac{4}{3}$ 7. 0.39 8. 0.78 9. 1.25 10. 0.20

11.
$$\frac{\csc \theta + \cot \theta}{\sec \theta + \tan \theta} = \frac{\csc \theta + \cot \theta}{\sec \theta + \tan \theta} \cdot \frac{\csc \theta - \cot \theta}{\csc \theta - \cot \theta} = \frac{\csc^2 \theta - \cot^2 \theta}{(\sec \theta + \tan \theta)(\csc \theta - \cot \theta)} = \frac{1}{(\sec \theta + \tan \theta)(\csc \theta - \cot \theta)}$$

$$= \frac{1}{(\sec \theta + \tan \theta)(\csc \theta - \cot \theta)} \cdot \frac{\sec \theta - \tan \theta}{\sec \theta - \tan \theta} = \frac{\sec \theta - \tan \theta}{(\sec^2 \theta - \tan^2 \theta)(\csc \theta - \cot \theta)} = \frac{\sec \theta - \tan \theta}{\csc \theta - \cot \theta}$$

12.
$$\sin \theta \tan \theta + \cos \theta = \sin \theta \cdot \frac{\sin \theta}{\cos \theta} + \cos \theta = \frac{\sin^2 \theta}{\cos \theta} + \frac{\cos^2 \theta}{\cos \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} = \frac{1}{\cos \theta} = \sec \theta$$

13.
$$\tan \theta + \cot \theta = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{\sin^2 \theta}{\sin \theta \cos \theta} + \frac{\cos^2 \theta}{\sin \theta \cos \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta} = \frac{2}{2 \sin \theta \cos \theta} = \frac{2}{\sin(2\theta)} = 2 \csc(2\theta)$$

14.
$$\frac{\sin(\alpha + \beta)}{\tan \alpha + \tan \beta} = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\frac{\sin \alpha}{\cos \alpha} + \frac{\sin \beta}{\cos \beta}} = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}} = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta}}$$

$$= \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{1} \cdot \frac{\cos \alpha \cos \beta}{\sin \alpha \cos \beta + \cos \alpha \sin \beta} = \cos \alpha \cos \beta$$

15.
$$\sin(3\theta) = \sin(\theta + 2\theta) = \sin \theta \cos(2\theta) + \cos \theta \sin(2\theta) = \sin \theta \cdot (\cos^2 \theta - \sin^2 \theta) + \cos \theta \cdot 2 \sin \theta \cos \theta = \sin \theta \cos^2 \theta - \sin^3 \theta + 2 \sin \theta \cos^2 \theta$$

$$= 3 \sin \theta \cos^2 \theta - \sin^3 \theta = 3 \sin \theta (1 - \sin^2 \theta) - \sin^3 \theta = 3 \sin \theta - 3 \sin^3 \theta - \sin^3 \theta = 3 \sin \theta - 4 \sin^3 \theta$$

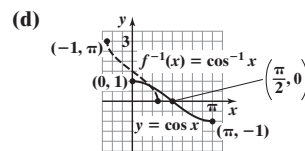
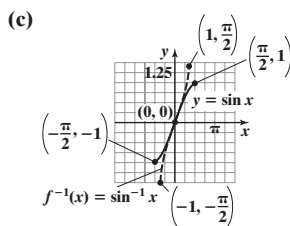
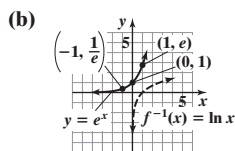
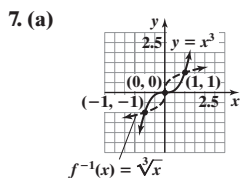
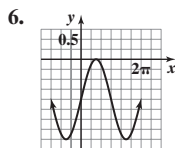
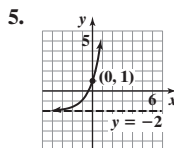
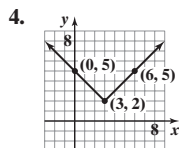
16.
$$\frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = \frac{\frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta}}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}} = \frac{\frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta \cos \theta}}{\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}} = \frac{\sin^2 \theta - \cos^2 \theta}{\sin^2 \theta + \cos^2 \theta} = \frac{(1 - \cos^2 \theta) - \cos^2 \theta}{1} = 1 - 2 \cos^2 \theta$$
 17. $\frac{1}{4}(\sqrt{6} + \sqrt{2})$

18. $2 + \sqrt{3}$ 19. $\frac{\sqrt{5}}{5}$ 20. $\frac{12\sqrt{85}}{49}$ 21. $\frac{2\sqrt{13}(\sqrt{5} - 3)}{39}$ 22. $\frac{2 + \sqrt{3}}{4}$ 23. $\frac{\sqrt{6}}{2}$ 24. $\frac{\sqrt{2}}{2}$ 25. $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$ 26. $\{0, 1.911, \pi, 4.373\}$

27. $\left\{\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}\right\}$ 28. $\{0.285, 3.427\}$ 29. $\{0.253, 2.889\}$

Cumulative Review (page 520)

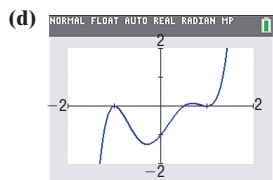
1. $\left\{\frac{-1 - \sqrt{13}}{6}, \frac{-1 + \sqrt{13}}{6}\right\}$ 2. $y + 1 = -1(x - 4)$ or $x + y = 3$; $6\sqrt{2}$; $(1, 2)$ 3. x-axis symmetry; $(0, -3)$, $(0, 3)$, $(3, 0)$



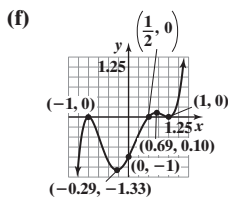
8. (a) $-\frac{2\sqrt{2}}{3}$ (b) $\frac{\sqrt{2}}{4}$ (c) $\frac{4\sqrt{2}}{9}$ (d) $\frac{7}{9}$ (e) $\sqrt{\frac{3+2\sqrt{2}}{6}}$ (f) $-\sqrt{\frac{3-2\sqrt{2}}{6}}$ 9. $\frac{\sqrt{5}}{5}$ 10. (a) $-\frac{2\sqrt{2}}{3}$ (b) $-\frac{2\sqrt{2}}{3}$ (c) $\frac{7}{9}$ (d) $\frac{4\sqrt{2}}{9}$ (e) $\frac{\sqrt{6}}{3}$

11. (a) $f(x) = (2x - 1)(x - 1)^2(x + 1)^2$;
 $\frac{1}{2}$ multiplicity 1; 1 and -1 multiplicity 2

(b) $(0, -1)$; $(\frac{1}{2}, 0)$; $(-1, 0)$; $(1, 0)$ (c) $y = 2x^5$



- (e) Local minimum value -1.33 at $x = -0.29$,
 Local minimum value 0 at $x = 1$
 Local maximum value 0 at $x = -1$,
 Local maximum value 0.10 at $x = 0.69$



- (g) Increasing: $(-\infty, -1]$, $[-0.29, 0.69]$, $[1, \infty)$
 Decreasing: $[-1, -0.29]$, $[0.69, 1]$

12. (a) $\left\{-1, -\frac{1}{2}\right\}$

(b) $\{-1, 1\}$

(c) $(-\infty, -1) \cup \left(-\frac{1}{2}, \infty\right)$

(d) $(-\infty, -1] \cup [1, \infty)$

CHAPTER 8 Applications of Trigonometric Functions

8.1 Assess Your Understanding (page 530)

4. F 5. b 6. angle of elevation 7. T 8. F 9. $\sin \theta = \frac{5}{13}$; $\cos \theta = \frac{12}{13}$; $\tan \theta = \frac{5}{12}$; $\cot \theta = \frac{12}{5}$; $\sec \theta = \frac{13}{12}$; $\csc \theta = \frac{13}{5}$
11. $\sin \theta = \frac{2\sqrt{13}}{13}$; $\cos \theta = \frac{3\sqrt{13}}{13}$; $\tan \theta = \frac{2}{3}$; $\cot \theta = \frac{3}{2}$; $\sec \theta = \frac{\sqrt{13}}{3}$; $\csc \theta = \frac{\sqrt{13}}{2}$
13. $\sin \theta = \frac{\sqrt{3}}{2}$; $\cos \theta = \frac{1}{2}$; $\tan \theta = \sqrt{3}$; $\cot \theta = \frac{\sqrt{3}}{3}$; $\sec \theta = 2$; $\csc \theta = \frac{2\sqrt{3}}{3}$
15. $\sin \theta = \frac{\sqrt{6}}{3}$; $\cos \theta = \frac{\sqrt{3}}{3}$; $\tan \theta = \sqrt{2}$; $\cot \theta = \frac{\sqrt{2}}{2}$; $\sec \theta = \sqrt{3}$; $\csc \theta = \frac{\sqrt{6}}{2}$
17. $\sin \theta = \frac{\sqrt{5}}{5}$; $\cos \theta = \frac{2\sqrt{5}}{5}$; $\tan \theta = \frac{1}{2}$; $\cot \theta = 2$; $\sec \theta = \frac{\sqrt{5}}{2}$; $\csc \theta = \sqrt{5}$
19. 0 21. 1 23. 0 25. 0 27. 1 29. $a \approx 13.74$, $c \approx 14.62$, $A = 70^\circ$ 31. $b \approx 5.03$, $c \approx 7.83$, $A = 50^\circ$ 33. $a \approx 0.71$, $c \approx 4.06$, $B = 80^\circ$
35. $b \approx 10.72$, $c \approx 11.83$, $B = 65^\circ$ 37. $b \approx 3.08$, $a \approx 8.46$, $A = 70^\circ$ 39. $c \approx 5.83$, $A \approx 59.0^\circ$, $B \approx 31.0^\circ$ 41. $b \approx 4.58$, $A \approx 23.6^\circ$, $B \approx 66.4^\circ$
43. 23.6° and 66.4° 45. 4.59 in.; 6.55 in. 47. (a) 5.52 in. or 11.83 in. 49. 70.02 ft 51. 985.91 ft 53. 137.37 m 55. 80.5°
57. (a) 111.96 ft/sec or 76.3 mi/h (b) 82.42 ft/sec or 56.2 mi/h (c) Under 18.8° 59. (a) 2.4898×10^{13} miles (b) 0.000214° 61. 554.52 ft
63. $S76.6^\circ E$ 65. The embankment is 30.5 m high. 67. 3.83 mi 69. 1978.09 ft 71. 60.27 ft 73. The buildings are 7984 ft apart. 75. 69.0°
77. 38.9° 79. The white ball should hit the top cushion 4.125 ft from the upper left corner. 84. Yes 85. $\frac{\sqrt{6} - \sqrt{2}}{4}$ or $\frac{\sqrt{2} - \sqrt{3}}{2}$
86. 0.236, 0.243, 0.248 87. $\left\{ \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$

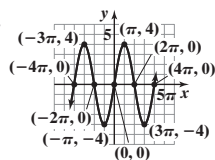
8.2 Assess Your Understanding (page 542)

4. a 5. $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ 6. F 7. F 8. ambiguous case 9. $a \approx 3.23$, $b \approx 3.55$, $A = 40^\circ$ 11. $a \approx 3.25$, $c \approx 4.23$, $B = 45^\circ$
13. $C = 95^\circ$, $c \approx 9.86$, $a \approx 6.36$ 15. $A = 40^\circ$, $a = 2$, $c \approx 3.06$ 17. $C = 120^\circ$, $b \approx 1.06$, $c \approx 2.69$ 19. $A = 100^\circ$, $a \approx 5.24$, $c \approx 0.92$
21. $B = 40^\circ$, $a \approx 5.64$, $b \approx 3.86$ 23. $C = 100^\circ$, $a \approx 1.31$, $b \approx 1.31$ 25. One triangle; $B \approx 30.7^\circ$, $C \approx 99.3^\circ$, $c \approx 3.86$
27. One triangle; $C \approx 36.2^\circ$, $A \approx 43.8^\circ$, $a \approx 3.51$ 29. No triangle 31. Two triangles; $C_1 \approx 30.9^\circ$, $A_1 \approx 129.1^\circ$, $a_1 \approx 9.07$ or $C_2 \approx 149.1^\circ$, $A_2 \approx 10.9^\circ$, $a_2 \approx 2.20$ 33. No triangle 35. Two triangles; $A_1 \approx 57.7^\circ$, $B_1 \approx 97.3^\circ$, $b_1 \approx 2.35$ or $A_2 \approx 122.3^\circ$, $B_2 \approx 32.7^\circ$, $b_2 \approx 1.28$
37. 1490.48 ft 39. 335.16 ft 41. 153.42 ft; 136.59 ft 43. The tree is 39.4 ft high. 45. Adam receives 100.6 more frequent flyer miles.
47. (a) Station Able is about 143.33 mi from the ship; Station Baker is about 135.58 mi from the ship. (b) Approximately 41 min
49. 84.7° ; 183.72 ft 51. 2.64 mi 53. 38.5 in. 55. 449.36 ft 57. 187,600,000 km or 101,440,000 km 59. The diameter is 252 ft.

$$61. \frac{a-b}{c} = \frac{a}{c} - \frac{b}{c} = \frac{\sin A}{\sin C} - \frac{\sin B}{\sin C} = \frac{\sin A - \sin B}{\sin C} = \frac{2 \sin\left(\frac{A-B}{2}\right) \cos\left(\frac{A+B}{2}\right)}{2 \sin\frac{C}{2} \cos\frac{C}{2}} = \frac{\sin\left(\frac{A-B}{2}\right) \cos\left(\frac{\pi}{2} - \frac{C}{2}\right)}{\sin\frac{C}{2} \cos\frac{C}{2}} = \frac{\sin\left(\frac{A-B}{2}\right)}{\cos\frac{C}{2}}$$

$$63. \frac{a-b}{a+b} = \frac{\frac{a-b}{c}}{\frac{a+b}{c}} = \frac{\frac{\sin\left[\frac{1}{2}(A-B)\right]}{\cos\frac{C}{2}}}{\frac{\sin\left[\frac{1}{2}(A+B)\right]}{\sin\frac{C}{2}}} = \frac{\tan\left[\frac{1}{2}(A-B)\right]}{\cot\frac{C}{2}} = \frac{\tan\left[\frac{1}{2}(A-B)\right]}{\tan\left(\frac{\pi}{2} - \frac{C}{2}\right)} = \frac{\tan\left[\frac{1}{2}(A-B)\right]}{\tan\left[\frac{1}{2}(A+B)\right]}$$

69. $\left\{ -3, -\frac{4}{3}, 3 \right\}$ 70. $3\sqrt{5} \approx 6.71$ 71. $-\frac{\sqrt{15}}{7}$ 72.



8.3 Assess Your Understanding (page 549)

3. Cosines 4. a 5. b 6. F 7. F 8. T 9. $b \approx 2.95$, $A \approx 28.7^\circ$, $C \approx 106.3^\circ$ 11. $c \approx 3.75$, $A \approx 32.1^\circ$, $B \approx 52.9^\circ$
13. $A \approx 48.5^\circ$, $B \approx 38.6^\circ$, $C \approx 92.9^\circ$ 15. $A \approx 127.2^\circ$, $B \approx 32.1^\circ$, $C \approx 20.7^\circ$ 17. $c \approx 2.57$, $A \approx 48.6^\circ$, $B \approx 91.4^\circ$
19. $a \approx 2.99$, $B \approx 19.2^\circ$, $C \approx 80.8^\circ$ 21. $b \approx 4.14$, $A \approx 43.0^\circ$, $C \approx 27.0^\circ$ 23. $c \approx 1.69$, $A = 65.0^\circ$, $B = 65.0^\circ$ 25. $A \approx 67.4^\circ$, $B = 90^\circ$, $C \approx 22.6^\circ$
27. $A = 60^\circ$, $B = 60^\circ$, $C = 60^\circ$ 29. $A \approx 33.6^\circ$, $B \approx 62.2^\circ$, $C \approx 84.3^\circ$ 31. $A \approx 97.9^\circ$, $B \approx 52.4^\circ$, $C \approx 29.7^\circ$ 33. $A = 85^\circ$, $a = 14.56$, $c = 14.12$
35. $A = 40.8^\circ$, $B = 60.6^\circ$, $C = 78.6^\circ$ 37. $A = 80^\circ$, $b = 8.74$, $c = 13.80$ 39. Two triangles; $B_1 = 35.4^\circ$, $C_1 = 134.6^\circ$, $c_1 = 12.29$; $B_2 = 144.6^\circ$, $C_2 = 25.4^\circ$, $c_2 = 7.40$ 41. $B = 24.5^\circ$, $C = 95.5^\circ$, $a = 10.44$ 43. 165 yd 45. (a) 26.4° (b) 30.8 h 47. (a) 63.7 ft (b) 66.8 ft (c) 92.8°
49. (a) 492.6 ft (b) 269.3 ft 51. (a) 59.2 mm (b) male 53. (a) $\alpha \approx 9.9^\circ$; $\beta \approx 8.3^\circ$ (b) 21.73 yd (c) 0.36 yd or 13 in. 55. 342.33 ft
57. The footings should be 765 ft apart.

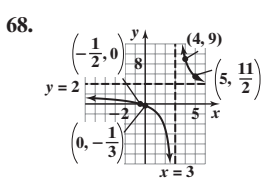
59. Suppose $0 < \theta < \pi$. Then, by the Law of Cosines, $d^2 = r^2 + r^2 - 2r^2 \cos \theta = 4r^2 \left(\frac{1 - \cos \theta}{2} \right) \Rightarrow d = 2r \sqrt{\frac{1 - \cos \theta}{2}} = 2r \sin \frac{\theta}{2}$.

Since, for any angle in $(0, \pi)$, d is strictly less than the length of the arc subtended by θ , that is, $d < r\theta$, then $2r \sin \frac{\theta}{2} < r\theta$, or $2 \sin \frac{\theta}{2} < \theta$.

Since $\cos \frac{\theta}{2} < 1$, then, for $0 < \theta < \pi$, $\sin \theta = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} < 2 \sin \frac{\theta}{2} < \theta$. If $\theta \geq \pi$, then, since $\sin \theta \leq 1$, $\sin \theta < \theta$. Thus $\sin \theta < \theta$ for all $\theta > 0$.

61.
$$\sin \frac{C}{2} = \sqrt{\frac{1 - \cos C}{2}} = \sqrt{\frac{1 - \frac{a^2 + b^2 - c^2}{2ab}}{2}} = \sqrt{\frac{2ab - a^2 - b^2 + c^2}{4ab}} = \sqrt{\frac{c^2 - (a - b)^2}{4ab}} = \sqrt{\frac{(c + a - b)(c + b - a)}{4ab}}$$

$$= \sqrt{\frac{(2s - 2b)(2s - 2a)}{4ab}} = \sqrt{\frac{(s - a)(s - b)}{ab}}$$



69. $\left\{ \frac{\ln 3}{\ln 4 - \ln 3} \right\} \approx \{3.819\}$

70. $\sin \theta = \frac{2\sqrt{6}}{7}$; $\csc \theta = \frac{7\sqrt{6}}{12}$; $\sec \theta = -\frac{7}{5}$; $\cot \theta = -\frac{5\sqrt{6}}{12}$

71. $y = -3 \sin(4x)$

8.4 Assess Your Understanding (page 556)

2. $\frac{1}{2} ab \sin C$ 3. $\sqrt{s(s-a)(s-b)(s-c)}$; $\frac{1}{2}(a+b+c)$ 4. T 5. c 6. c 7. 2.83 9. 2.99 11. 14.98 13. 9.56 15. 3.86 17. 1.48 19. 2.82

21. 30 23. 1.73 25. 19.90 27. $K = \frac{1}{2} ab \sin C = \frac{1}{2} a \sin C \left(\frac{a \sin B}{\sin A} \right) = \frac{a^2 \sin B \sin C}{2 \sin A}$ 29. 0.92 31. 2.27 33. 5.44 35. 9.03 sq ft 37. \$5446.38

39. The area of the lot is about 8376 sq ft. 41. 15.27 in.² 43. The lake area is 1645.14 ft². 45. The ground area is 75174 ft².

47. (a) Area $\triangle OAC = \frac{1}{2} |OC| |AC| = \frac{1}{2} \cdot \frac{|OC|}{1} \cdot \frac{|AC|}{1} = \frac{1}{2} \sin \alpha \cos \alpha$

(e) Area $\triangle OAB = \text{Area } \triangle OAC + \text{Area } \triangle OCB$

(b) Area $\triangle OCB = \frac{1}{2} |BC| |OC| = \frac{1}{2} |OB|^2 \frac{|BC|}{|OB|} \cdot \frac{|OC|}{|OB|} = \frac{1}{2} |OB|^2 \sin \beta \cos \beta$ $\frac{1}{2} |OB| \sin(\alpha + \beta) = \frac{1}{2} \sin \alpha \cos \alpha + \frac{1}{2} |OB|^2 \sin \beta \cos \beta$

(c) Area $\triangle OAB = \frac{1}{2} |BD| |OA| = \frac{1}{2} |OB| \frac{|BD|}{|OB|} = \frac{1}{2} |OB| \sin(\alpha + \beta)$ $\sin(\alpha + \beta) = \frac{1}{|OB|} \sin \alpha \cos \alpha + |OB| \sin \beta \cos \beta$

(d) $\frac{\cos \alpha}{\cos \beta} = \frac{\frac{|OC|}{|OB|}}{\frac{|OC|}{|OB|}} = |OB|$ $\sin(\alpha + \beta) = \frac{\cos \beta}{\cos \alpha} \sin \alpha \cos \alpha + \frac{\cos \alpha}{\cos \beta} \sin \beta \cos \beta$

$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

49. 31,145 ft² 51. (a) The perimeter and area are both 36. (b) The perimeter and area are both 60.

53. $K = \frac{1}{2} ah = \frac{1}{2} ab \sin C \Rightarrow h = b \sin C = \frac{a \sin B \sin C}{\sin A}$

55. $\angle POQ = 180^\circ - \left(\frac{A}{2} + \frac{B}{2} \right) = 180^\circ - \frac{1}{2} (180^\circ - C) = 90^\circ + \frac{C}{2}$, and $\sin \left(90^\circ + \frac{C}{2} \right) = \cos \left(-\frac{C}{2} \right) = \cos \frac{C}{2}$, since cosine is an even function.

Therefore, $r = \frac{c \sin \frac{A}{2} \sin \frac{B}{2}}{\sin \left(90^\circ + \frac{C}{2} \right)} = \frac{c \sin \frac{A}{2} \sin \frac{B}{2}}{\cos \frac{C}{2}}$.

57. $\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \frac{s-a}{r} + \frac{s-b}{r} + \frac{s-c}{r} = \frac{3s - (a+b+c)}{r} = \frac{3s - 2s}{r} = \frac{s}{r}$ 59. $\frac{\sqrt{2+\sqrt{3}}}{4}$ or $\frac{\sqrt{2} + \sqrt{6}}{8}$ square units

63. Maximum value; 17 64. $(-\infty, -3) \cup [-1, 3)$ 65. $\sin t = \frac{\sqrt{2}}{3}$, $\cos t = -\frac{\sqrt{7}}{3}$, $\tan t = -\frac{\sqrt{14}}{7}$, $\csc t = \frac{3\sqrt{2}}{2}$, $\sec t = -\frac{3\sqrt{7}}{7}$, $\cot t = -\frac{\sqrt{14}}{2}$

66. $\csc \theta - \sin \theta = \frac{1}{\sin \theta} - \sin \theta = \frac{1 - \sin^2 \theta}{\sin \theta} = \frac{\cos^2 \theta}{\sin \theta} = \cos \theta \cdot \frac{\cos \theta}{\sin \theta} = \cos \theta \cot \theta$

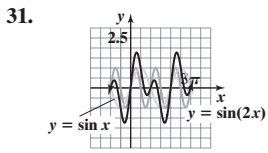
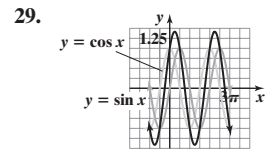
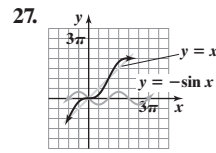
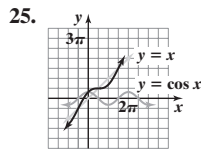
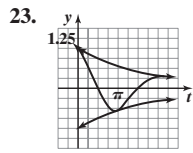
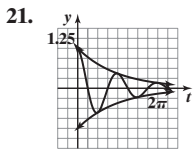
8.5 Assess Your Understanding (page 566)

2. Simple harmonic; amplitude 3. Simple harmonic; damped 4. T 5. $d = -5 \cos(\pi t)$ 7. $d = -6 \cos(2t)$ 9. $d = -5 \sin(\pi t)$

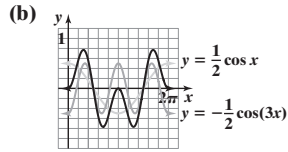
11. $d = -6 \sin(2t)$ 13. (a) Simple harmonic (b) 5 m (c) $\frac{2\pi}{3}$ sec (d) $\frac{3}{2\pi}$ oscillation/sec 15. (a) Simple harmonic (b) 6 m (c) 2 sec

(d) $\frac{1}{2}$ oscillation/sec 17. (a) Simple harmonic (b) 3 m (c) 4π sec (d) $\frac{1}{4\pi}$ oscillation/sec 19. (a) Simple harmonic (b) 2 m

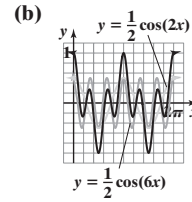
(c) 1 sec (d) 1 oscillation/sec



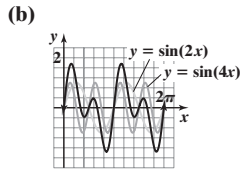
33. (a) $f(x) = \frac{1}{2} [\cos x - \cos(3x)]$



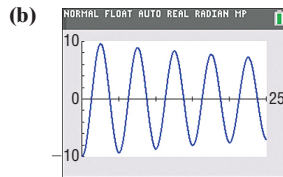
35. (a) $G(x) = \frac{1}{2} [\cos(6x) + \cos(2x)]$



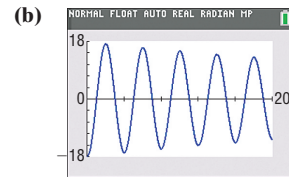
37. (a) $H(x) = \sin(4x) + \sin(2x)$



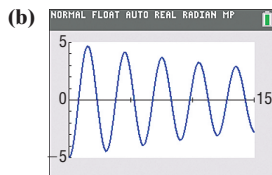
39. (a) $d = -10e^{-0.7t/50} \cos\left(\sqrt{\frac{4\pi^2}{25} - \frac{0.49}{2500}}t\right)$



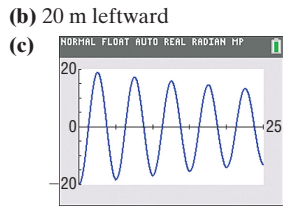
41. (a) $d = -18e^{-0.6t/60} \cos\left(\sqrt{\frac{\pi^2}{4} - \frac{0.36}{3600}}t\right)$



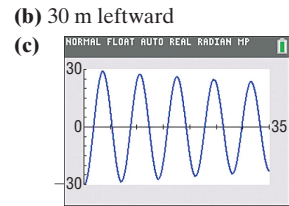
43. (a) $d = -5e^{-0.8t/20} \cos\left(\sqrt{\frac{4\pi^2}{9} - \frac{0.64}{400}}t\right)$



45. (a) The motion is damped. The bob has mass $m = 20$ kg with a damping factor of 0.7 kg/sec.



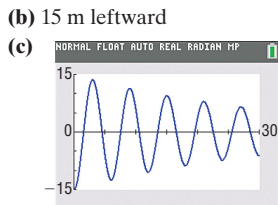
47. (a) The motion is damped. The bob has mass $m = 40$ kg with a damping factor of 0.6 kg/sec.



(d) 18.33 m leftward (e) $d \rightarrow 0$

(d) 28.47 m leftward (e) $d \rightarrow 0$

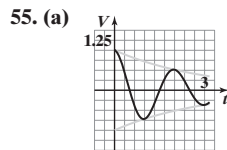
49. (a) The motion is damped. The bob has mass $m = 15$ kg with a damping factor of 0.9 kg/sec.



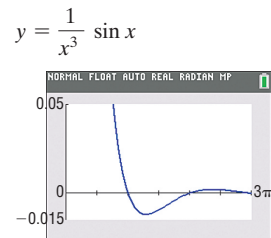
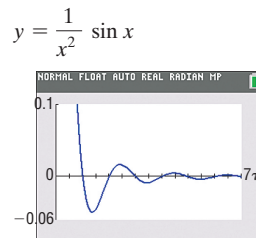
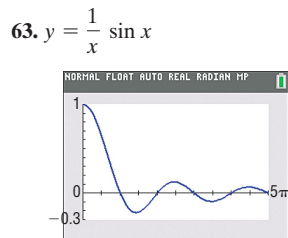
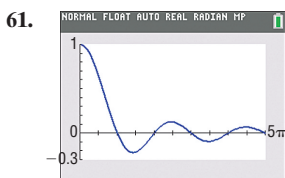
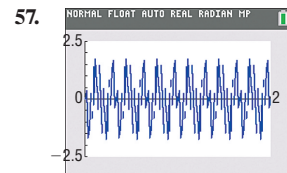
(d) 12.53 m leftward (e) $d \rightarrow 0$

51. $\omega = 1040\pi$; $d = 0.80 \cos(1040\pi t)$

53. $\omega = 880\pi$; $d = 0.01 \sin(880\pi t)$



(b) At $t = 0, t = 2$; at $t = 1, t = 3$
(c) During the approximate intervals $0.35 < t < 0.67, 1.29 < t < 1.75,$ and $2.19 < t \leq 3$



65. $f^{-1}(x) = \frac{4x-3}{x-1}$ 66. $\log_7\left(\frac{xy^3}{x+y}\right)$ 67. $\{4\}$ 68. (a) $\frac{3\sqrt{10}}{10}$ (b) $\frac{\sqrt{10}}{10}$ (c) $\frac{1}{3}$

Review Exercises (page 570)

1. $\sin \theta = \frac{4}{5}$; $\cos \theta = \frac{3}{5}$; $\tan \theta = \frac{4}{3}$; $\cot \theta = \frac{3}{4}$; $\sec \theta = \frac{5}{3}$; $\csc \theta = \frac{5}{4}$ 2. $\sin \theta = \frac{\sqrt{3}}{2}$; $\cos \theta = \frac{1}{2}$; $\tan \theta = \sqrt{3}$; $\cot \theta = \frac{\sqrt{3}}{3}$; $\sec \theta = 2$; $\csc \theta = \frac{2\sqrt{3}}{3}$
3. 0 4. 1 5. 1 6. $A = 70^\circ, b \approx 3.42, a \approx 9.40$ 7. $a \approx 4.58, A \approx 66.4^\circ, B \approx 23.6^\circ$ 8. $C = 100^\circ, b \approx 0.65, c \approx 1.29$
9. $B \approx 56.8^\circ, C \approx 23.2^\circ, b \approx 4.25$ 10. No triangle 11. $b \approx 3.32, A \approx 62.8^\circ, C \approx 17.2^\circ$ 12. $A \approx 36.2^\circ, C \approx 63.8^\circ, c \approx 4.55$

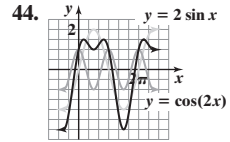
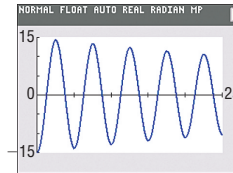
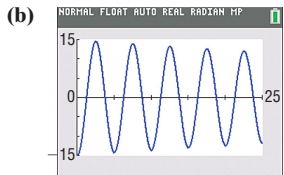
13. No triangle 14. $A \approx 83.3^\circ$, $B \approx 44.0^\circ$, $C \approx 52.6^\circ$ 15. $c \approx 2.32$, $A \approx 16.1^\circ$, $B \approx 123.9^\circ$ 16. $B \approx 36.2^\circ$, $C \approx 63.8^\circ$, $c \approx 4.55$
 17. $A \approx 39.6^\circ$, $B \approx 18.6^\circ$, $C \approx 121.9^\circ$ 18. Two triangles: $B_1 \approx 13.4^\circ$, $C_1 \approx 156.6^\circ$, $c_1 \approx 6.86$ or $B_2 \approx 166.6^\circ$, $C_2 \approx 3.4^\circ$, $c_2 \approx 1.02$
 19. $b \approx 11.52$, $c \approx 10.13$, $C \approx 60^\circ$ 20. $a \approx 5.23$, $B \approx 46.0^\circ$, $C \approx 64.0^\circ$ 21. 1.93 22. 18.79 23. 6 24. 3.80 25. 0.32 26. 1.92 in.^2
 27. 48.2° and 41.8° 28. 23.32 ft 29. 2.15 mi 30. 132.55 ft/min 31. 12.7° 32. 29.97 ft 33. 6.22 mi 34. (a) 131.8 mi (b) 23.1° (c) 0.21 hr
 35. 8798.67 sq ft 36. $S4.0^\circ E$ 37. 76.94 in. 38. 79.69 in. 39. $d = -3 \cos\left(\frac{\pi}{2}t\right)$

40. (a) Simple harmonic (b) 6 ft (c) π s (d) $\frac{1}{\pi}$ oscillation/s
 41. (a) Simple harmonic (b) 2 ft (c) 2 s (d) $\frac{1}{2}$ oscillation/s

42. (a) $d = -15e^{-0.75t/80} \cos\left(\sqrt{\frac{4\pi^2}{25} - \frac{0.5625}{6400}}t\right)$

43. (a) The motion is damped. The bob has mass $m = 20$ kg with a damping factor of 0.6 kg/s.

- (b) 15 m leftward
 (d) 13.92 m leftward
 (e) $d \rightarrow 0$

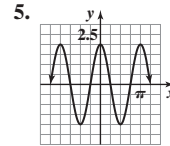
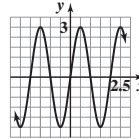
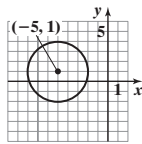


Chapter Test (page 572)

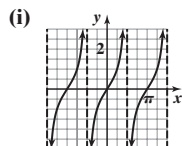
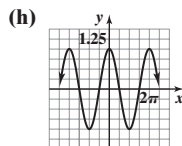
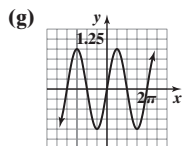
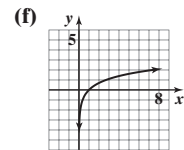
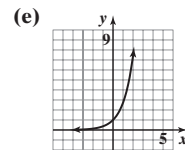
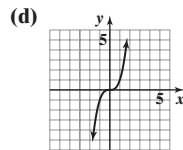
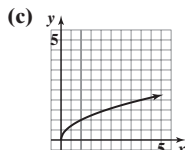
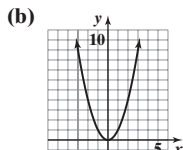
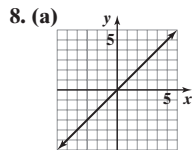
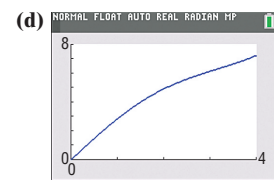
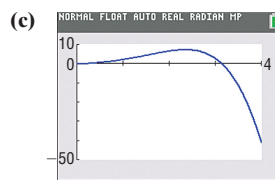
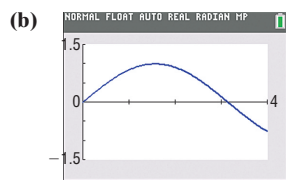
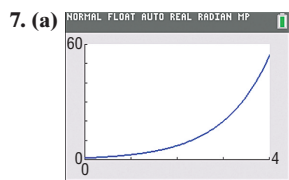
1. $\sin \theta = \frac{\sqrt{5}}{5}$; $\cos \theta = \frac{2\sqrt{5}}{5}$; $\tan \theta = \frac{1}{2}$; $\csc \theta = \sqrt{5}$; $\sec \theta = \frac{\sqrt{5}}{2}$; $\cot \theta = 2$ 2. 0 3. $a = 15.88$, $B \approx 57.5^\circ$, $C \approx 70.5^\circ$
 4. $b \approx 6.85$, $C = 117^\circ$, $c \approx 16.30$ 5. $A \approx 52.4^\circ$, $B \approx 29.7^\circ$, $C \approx 97.9^\circ$ 6. $b \approx 4.72$, $c \approx 1.67$, $B = 105^\circ$ 7. No triangle
 8. $c \approx 7.62$, $A \approx 80.5^\circ$, $B \approx 29.5^\circ$ 9. 15.04 square units 10. 19.81 square units 11. 61.0° 12. 1.3° 13. The area of the shaded region is 9.26 cm^2 .
 14. 54.15 square units 15. Madison will have to swim about 2.23 miles. 16. 12.63 square units 17. The lengths of the sides are 15, 18, and 21.
 18. $d = 5(\sin 42^\circ) \sin\left(\frac{\pi t}{3}\right)$ or $d \approx 3.346 \sin\left(\frac{\pi t}{3}\right)$

Cumulative Review (page 573)

1. $\left\{\frac{1}{3}, 1\right\}$ 2. $(x + 5)^2 + (y - 1)^2 = 9$ 3. $\{x|x \leq -1 \text{ or } x \geq 4\}$ 4.



6. (a) $-\frac{2\sqrt{5}}{5}$ (b) $\frac{\sqrt{5}}{5}$ (c) $-\frac{4}{5}$ (d) $-\frac{3}{5}$ (e) $\sqrt{\frac{5-\sqrt{5}}{10}}$ (f) $-\sqrt{\frac{5+\sqrt{5}}{10}}$



9. Two triangles: $A_1 \approx 59.0^\circ$, $B_1 \approx 81.0^\circ$, $b_1 \approx 23.05$ or $A_2 \approx 121.0^\circ$, $B_2 \approx 19.0^\circ$, $b_2 \approx 7.59$

10. $\left\{-2i, 2i, \frac{1}{3}, 1, 2\right\}$

11. $R(x) = \frac{(2x + 1)(x - 4)}{(x + 5)(x - 3)}$; Domain: $\{x \mid x \neq -5, x \neq 3\}$

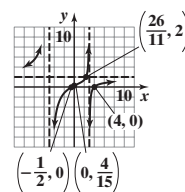
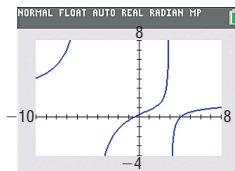
Intercepts: $(-\frac{1}{2}, 0)$, $(4, 0)$, $(0, \frac{4}{15})$

No symmetry

Vertical asymptotes: $x = -5, x = 3$

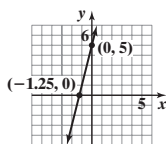
Horizontal asymptote: $y = 2$

Intersects: $(\frac{26}{11}, 2)$

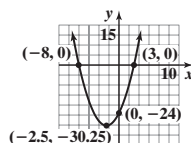


12. $\{2, 26\}$ 13. $\{1\}$ 14. (a) $\{-\frac{5}{4}\}$ (b) $\{2\}$ (c) $\{-\frac{-1 - 3\sqrt{13}}{2}, -\frac{-1 + 3\sqrt{13}}{2}\}$ (d) $\{x \mid x > -\frac{5}{4}\}$ or $(-\frac{5}{4}, \infty)$

(e) $|x| - 8 \leq x \leq 3$ or $[-8, 3]$ (f)



(g)

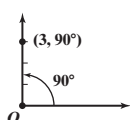


CHAPTER 9 Polar Coordinates; Vectors

9.1 Assess Your Understanding (page 584)

5. pole; polar axis 6. $r \cos \theta$; $r \sin \theta$ 7. b 8. d 9. T 10. F 11. A 13. C 15. B 17. A

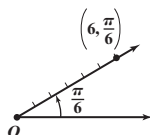
19.



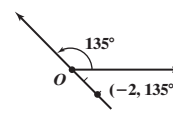
21.



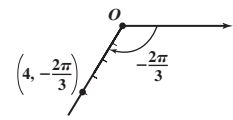
23.



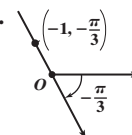
25.



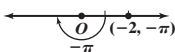
27.



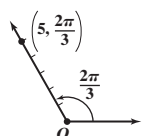
29.



31.

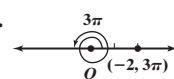


33.



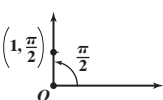
(a) $(5, -\frac{4\pi}{3})$ (b) $(-5, \frac{5\pi}{3})$ (c) $(5, \frac{8\pi}{3})$

35.

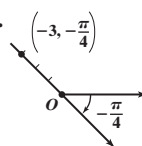


(a) $(2, -2\pi)$
(b) $(-2, \pi)$
(c) $(2, 2\pi)$

37.



39.



41. $(0, 3)$ 43. $(-2, 0)$ 45. $(-3\sqrt{3}, 3)$ 47. $(\sqrt{2}, -\sqrt{2})$ 49. $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$ 51. $(2, 0)$

53. $(-2.57, 7.05)$ 55. $(-4.98, -3.85)$ 57. $(3, 0)$ 59. $(1, \pi)$ 61. $(\sqrt{2}, -\frac{\pi}{4})$

63. $(2, \frac{\pi}{6})$ 65. $(2.47, -1.02)$ 67. $(9.30, 0.47)$ 69. $r^2 = \frac{3}{2}$ or $r = \frac{\sqrt{6}}{2}$

71. $r^2 \cos^2 \theta - 4r \sin \theta = 0$ 73. $r^2 \sin 2\theta = 1$ 75. $r \cos \theta = 4$

77. $x^2 + y^2 - x = 0$ or $(x - \frac{1}{2})^2 + y^2 = \frac{1}{4}$ 79. $(x^2 + y^2)^{3/2} - x = 0$

81. $x^2 + y^2 = 4$ 83. $y^2 = 8(x + 2)$

(a) $(1, -\frac{3\pi}{2})$

(a) $(3, -\frac{5\pi}{4})$

(b) $(-1, \frac{3\pi}{2})$

(b) $(-3, \frac{7\pi}{4})$

(c) $(1, \frac{5\pi}{2})$

(c) $(3, \frac{11\pi}{4})$

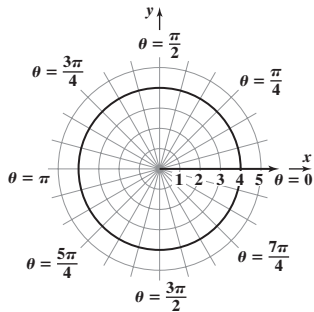
85. (a) $(-10, 36)$ (b) $(2\sqrt{349}, 180^\circ + \tan^{-1}(-\frac{18}{5})) \approx (37.36, 105.5^\circ)$ (c) $(-3, -35)$ (d) $(\sqrt{1234}, 180^\circ + \tan^{-1}(\frac{35}{3})) \approx (35.13, 265.1^\circ)$

90. $\{\frac{19}{15}\}$ 91. 2 or 0 positive real zeros; 1 negative real zero 92. $(-\frac{5}{4}, \frac{9}{2})$ 93. $(0, -11)$

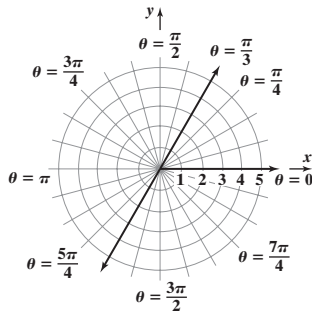
9.2 Assess Your Understanding (page 599)

7. polar equation 8. F 9. $-\theta$ 10. $\pi - \theta$ 11. T 12. $2n; n$ 13. c 14. b

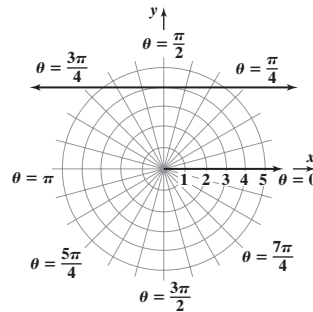
15. $x^2 + y^2 = 16$; circle, radius 4, center at pole



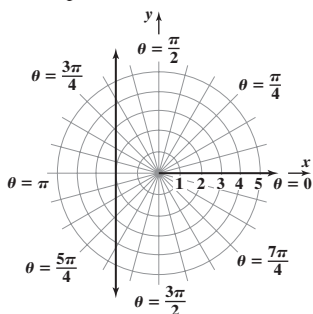
17. $y = \sqrt{3}x$; line through pole, making an angle of $\frac{\pi}{3}$ with polar axis



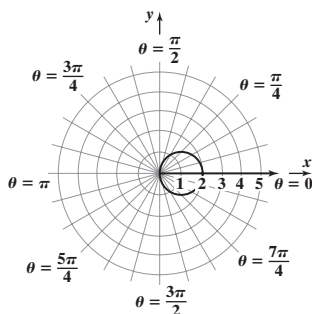
19. $y = 4$; horizontal line 4 units above the pole



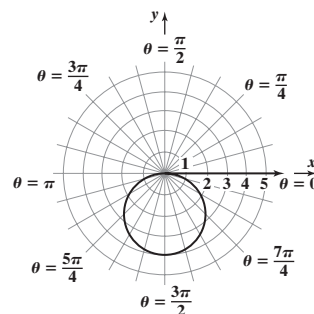
21. $x = -2$; vertical line 2 units to the left of the pole



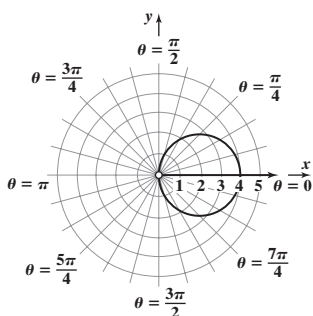
23. $(x - 1)^2 + y^2 = 1$; circle, radius 1, center (1, 0) in rectangular coordinates



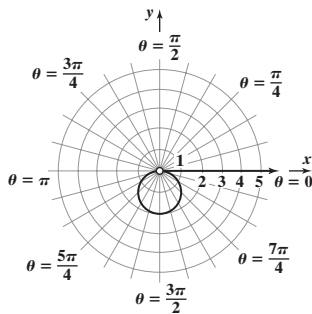
25. $x^2 + (y + 2)^2 = 4$; circle, radius 2, center at (0, -2) in rectangular coordinates



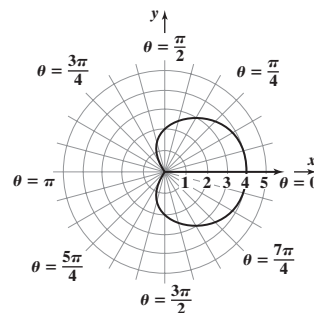
27. $(x - 2)^2 + y^2 = 4, x \neq 0$; circle, radius 2, center at (2, 0) in rectangular coordinates, hole at (0, 0)



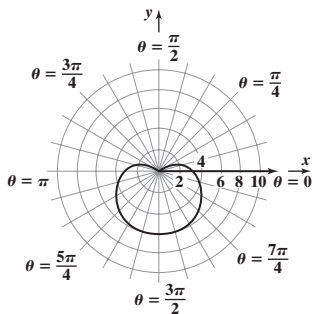
29. $x^2 + (y + 1)^2 = 1, x \neq 0$; circle, radius 1, center at (0, -1) in rectangular coordinates, hole at (0, 0)



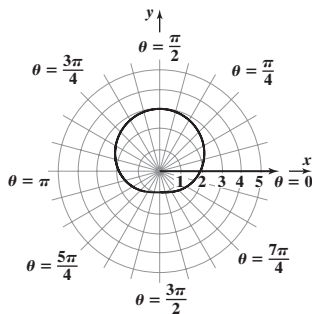
31. E 33. F 35. H 37. D
39. Cardioid



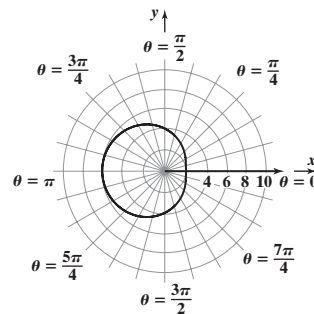
41. Cardioid



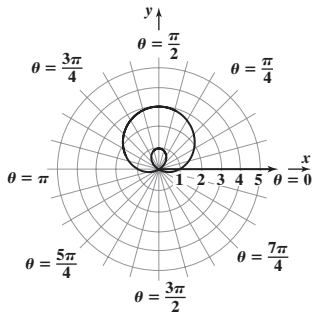
43. Limaçon without inner loop



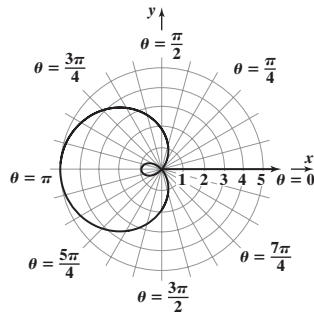
45. Limaçon without inner loop



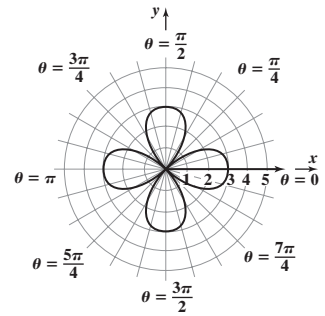
47. Limaçon with inner loop



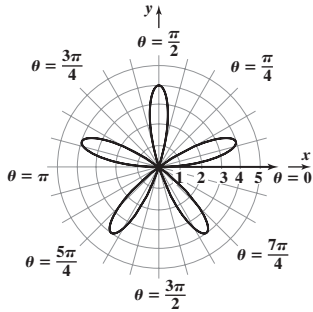
49. Limaçon with inner loop



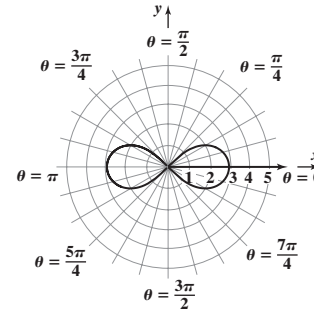
51. Rose



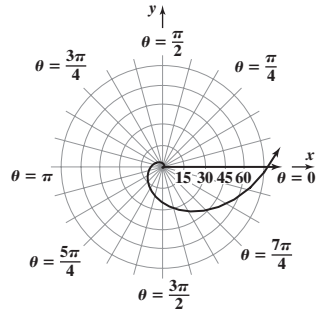
53. Rose



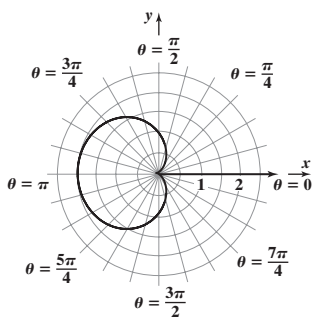
55. Lemniscate



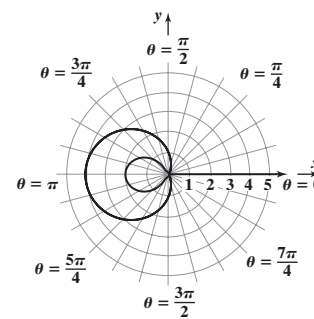
57. Spiral



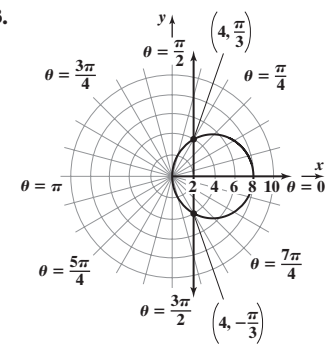
59. Cardioid



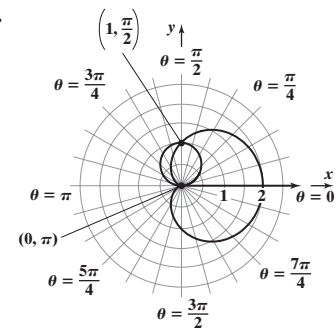
61. Limaçon with inner loop



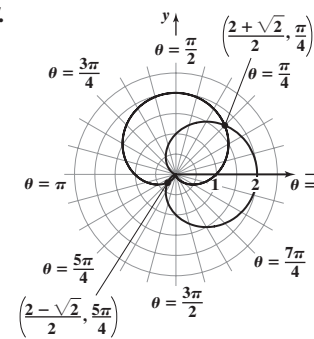
63.



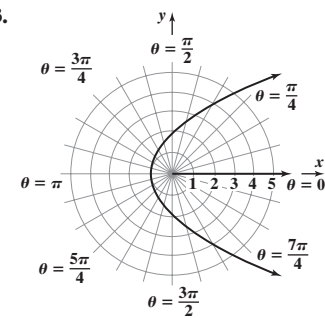
65.



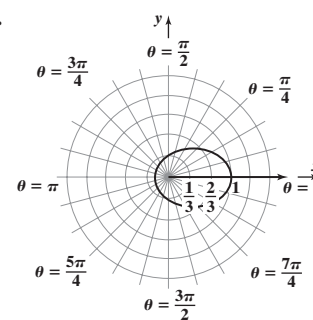
67.



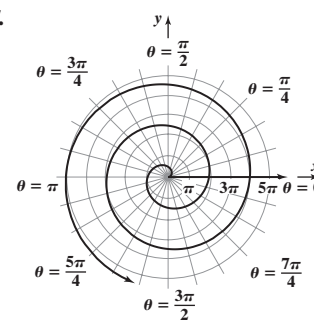
69. $r = 3 + 3 \cos \theta$ 71. $r = 4 + \sin \theta$



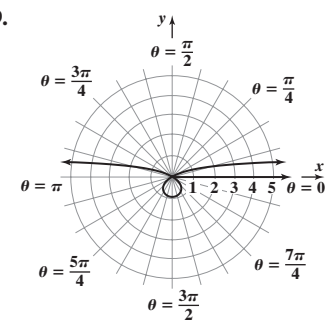
75.

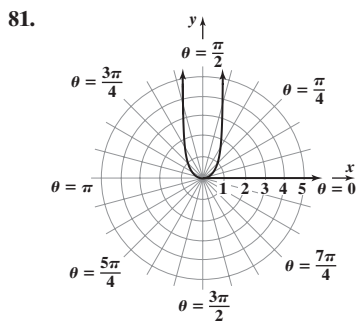


77.



79.





83. $r \sin \theta = a$
 $y = a$

85. $r = 2a \sin \theta$
 $r^2 = 2ar \sin \theta$
 $x^2 + y^2 = 2ay$
 $x^2 + y^2 - 2ay = 0$
 $x^2 + (y - a)^2 = a^2$
Circle, radius a , center at $(0, a)$
in rectangular coordinates

87. $r = 2a \cos \theta$
 $r^2 = 2ar \cos \theta$
 $x^2 + y^2 = 2ax$
 $x^2 - 2ax + y^2 = 0$
 $(x - a)^2 + y^2 = a^2$
Circle, radius a , center at $(a, 0)$
in rectangular coordinates

89. (a) $r^2 = \cos \theta; r^2 = \cos(\pi - \theta)$
 $r^2 = -\cos \theta$
Not equivalent; test fails.
 $(-r)^2 = \cos(-\theta)$
 $r^2 = \cos \theta$
New test works.

(b) $r^2 = \sin \theta; r^2 = \sin(\pi - \theta)$
 $r^2 = \sin \theta$
Test works.
 $(-r)^2 = \sin(-\theta)$
 $r^2 = -\sin \theta$
Not equivalent; new test fails.

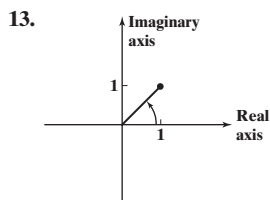
93. $\{x | 3 < x \leq 8\}$, or $(3, 8]$ 94. 420° 95. Amplitude = 2; period = $\frac{2\pi}{5}$ 96. Horizontal asymptote: $y = 0$
Vertical asymptote: $x = 4$

Historical Problems (page 608)

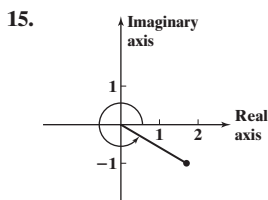
1. (a) $1 + 4i, 1 + i$ (b) $-1, 2 + i$

9.3 Assess Your Understanding (page 608)

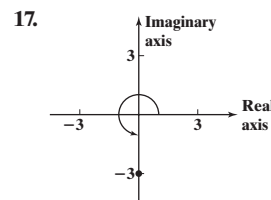
5. real; imaginary 6. magnitude; modulus; argument 7. $r_1 r_2; \theta_1 + \theta_2; \theta_1 + \theta_2$ 8. $r^n; n\theta; n\theta$ 9. three 10. T 11. c 12. a



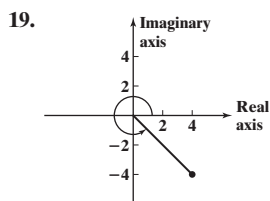
$\sqrt{2} (\cos 45^\circ + i \sin 45^\circ)$



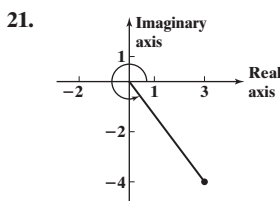
$2 (\cos 330^\circ + i \sin 330^\circ)$



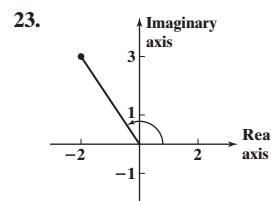
$3 (\cos 270^\circ + i \sin 270^\circ)$



$4\sqrt{2} (\cos 315^\circ + i \sin 315^\circ)$



$5 (\cos 306.9^\circ + i \sin 306.9^\circ)$



$\sqrt{13} (\cos 123.7^\circ + i \sin 123.7^\circ)$

25. $-1 + \sqrt{3}i$ 27. $2\sqrt{2} - 2\sqrt{2}i$ 29. $-3i$ 31. $-0.035 + 0.197i$ 33. $1.970 + 0.347i$

35. $zw = 8 (\cos 60^\circ + i \sin 60^\circ); \frac{z}{w} = \frac{1}{2} (\cos 20^\circ + i \sin 20^\circ)$ 37. $zw = 12 (\cos 40^\circ + i \sin 40^\circ); \frac{z}{w} = \frac{3}{4} (\cos 220^\circ + i \sin 220^\circ)$

39. $zw = 4 (\cos \frac{9\pi}{40} + i \sin \frac{9\pi}{40}); \frac{z}{w} = \cos \frac{\pi}{40} + i \sin \frac{\pi}{40}$ 41. $zw = 4\sqrt{2} (\cos 15^\circ + i \sin 15^\circ); \frac{z}{w} = \sqrt{2} (\cos 75^\circ + i \sin 75^\circ)$

43. $-32 + 32\sqrt{3}i$ 45. $32i$ 47. $\frac{27}{2} + \frac{27\sqrt{3}}{2}i$ 49. $-\frac{25\sqrt{2}}{2} + \frac{25\sqrt{2}}{2}i$ 51. $-4 + 4i$ 53. $-23 + 14.142i$

55. $\sqrt[6]{2} (\cos 15^\circ + i \sin 15^\circ), \sqrt[6]{2} (\cos 135^\circ + i \sin 135^\circ), \sqrt[6]{2} (\cos 255^\circ + i \sin 255^\circ)$

57. $\sqrt[4]{8} (\cos 75^\circ + i \sin 75^\circ), \sqrt[4]{8} (\cos 165^\circ + i \sin 165^\circ), \sqrt[4]{8} (\cos 255^\circ + i \sin 255^\circ), \sqrt[4]{8} (\cos 345^\circ + i \sin 345^\circ)$

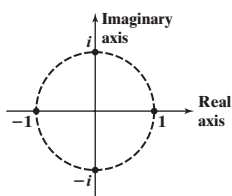
59. $2 (\cos 67.5^\circ + i \sin 67.5^\circ), 2 (\cos 157.5^\circ + i \sin 157.5^\circ), 2 (\cos 247.5^\circ + i \sin 247.5^\circ), 2 (\cos 337.5^\circ + i \sin 337.5^\circ)$

61. $\{\cos 18^\circ + i \sin 18^\circ, \cos 90^\circ + i \sin 90^\circ, \cos 162^\circ + i \sin 162^\circ, \cos 234^\circ + i \sin 234^\circ, \cos 306^\circ + i \sin 306^\circ\}$

63. $1, i, -1, -i$

65. Look at formula (8). $|z_k| = \sqrt[n]{r}$ for all k .

67. Look at formula (8). The z_k are spaced apart by an angle of $\frac{2\pi}{n}$.



69. Assume the theorem is true for $n \geq 1$.

For $n = 0$:
 $z^0 = r^0 [\cos(0 \cdot \theta) + i \sin(0 \cdot \theta)]$
 $1 = 1 \cdot [\cos(0) + i \sin(0)]$
 $1 = 1 \cdot [1 + 0]$
 $1 = 1$ True

For negative integers:

$$z^{-n} = (z^n)^{-1} = (r^n [\cos(n\theta) + i \sin(n\theta)])^{-1} \quad \text{with } n \geq 1$$

$$= \frac{1}{r^n [\cos(n\theta) + i \sin(n\theta)]}$$

$$= \frac{1}{r^n [\cos(n\theta) + i \sin(n\theta)]} \cdot \frac{\cos(n\theta) - i \sin(n\theta)}{\cos(n\theta) - i \sin(n\theta)}$$

$$= \frac{\cos(n\theta) - i \sin(n\theta)}{r^n (\cos^2(n\theta) + \sin^2(n\theta))}$$

$$= \frac{\cos(n\theta) - i \sin(n\theta)}{r^n}$$

$$= r^{-n} [\cos(n\theta) - i \sin(n\theta)]$$

$$= r^{-n} [\cos(-n\theta) + i \sin(-n\theta)]$$

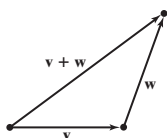
Thus, De Moivre's Theorem is true for all integers.

71. ≈ 40.50 72. $\frac{4}{3}\pi$ 73. $5\sqrt{2}$ 74. Minimum: $f\left(\frac{6}{5}\right) = -\frac{16}{5}$

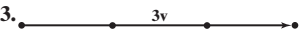
9.4 Assess Your Understanding (page 621)

1. vector 2. 0 3. unit 4. position 5. horizontal; vertical 6. resultant 7. T 8. F 9. a 10. b

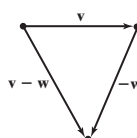
11.



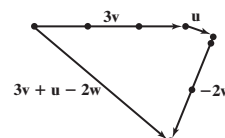
13.



15.

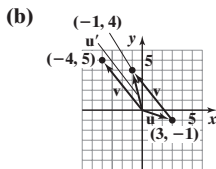


17.

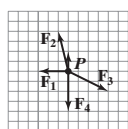


19. T 21. F 23. F 25. T 27. 12 29. $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$ 31. $\mathbf{v} = 2\mathbf{i} + 4\mathbf{j}$ 33. $\mathbf{v} = 8\mathbf{i} - \mathbf{j}$ 35. $\mathbf{v} = -\mathbf{i} + \mathbf{j}$ 37.5 39. $\sqrt{2}$ 41. $\sqrt{13}$ 43. $-\mathbf{j}$
 45. $\sqrt{89}$ 47. $\sqrt{34} - \sqrt{13}$ 49. \mathbf{i} 51. $\frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}$ 53. $\frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}$ 55. $\mathbf{v} = \frac{8\sqrt{5}}{5}\mathbf{i} + \frac{4\sqrt{5}}{5}\mathbf{j}$, or $\mathbf{v} = -\frac{8\sqrt{5}}{5}\mathbf{i} - \frac{4\sqrt{5}}{5}\mathbf{j}$
 57. $\{-2 + \sqrt{21}, -2 - \sqrt{21}\}$ 59. $\mathbf{v} = \frac{5}{2}\mathbf{i} + \frac{5\sqrt{3}}{2}\mathbf{j}$ 61. $\mathbf{v} = -7\mathbf{i} + 7\sqrt{3}\mathbf{j}$ 63. $\mathbf{v} = \frac{25\sqrt{3}}{2}\mathbf{i} - \frac{25}{2}\mathbf{j}$ 65. 45° 67. 150° 69. 333.4° 71. 258.7°
 73. $\mathbf{F} = 20\sqrt{3}\mathbf{i} + 20\mathbf{j}$ 75. $\mathbf{F} = (20\sqrt{3} + 30\sqrt{2})\mathbf{i} + (20 - 30\sqrt{2})\mathbf{j}$
 77. (a) $\mathbf{v}_a = 550\mathbf{j}$; $\mathbf{v}_w = 50\sqrt{2}\mathbf{i} + 50\sqrt{2}\mathbf{j}$
 (b) $\mathbf{v}_g = 50\sqrt{2}\mathbf{i} + (550 + 50\sqrt{2})\mathbf{j}$
 (c) $\|\mathbf{v}_g\| = 624.7$ mph; N 6.5° E
 79. $\mathbf{v} = (250\sqrt{2} - 30)\mathbf{i} + (250\sqrt{2} + 30\sqrt{3})\mathbf{j}$; 518.8 km/h; N 38.6° E
 81. Approximately 4031 lb 83. 8.6° left of direct heading across the river; 1.52 min
 85. (a) N 705° E (b) 12 min 87. Tension in right cable: 1000 lb; tension in left cable: 845.2 lb
 89. Tension in right part: 1088.4 lb; tension in left part: 1089.1 lb 91. $\mu = 0.36$
 93. 13.68 lb 95. The truck must pull with a force of 4635.2 lb.

97. (a) $(-1, 4)$

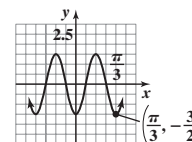


99.



103. $c \approx 4.29$; $A \approx 66.7^\circ$, $B \approx 13.3^\circ$ 104. $-2, 0, 6$ 105. $\sqrt{3}$

106. Amplitude = $\frac{3}{2}$; period = $\frac{\pi}{3}$
 Phase shift = $-\frac{\pi}{2}$



Historical Problem (page 630)

$(a\mathbf{i} + b\mathbf{j}) \cdot (c\mathbf{i} + d\mathbf{j}) = ac + bd$
 Real part $[(a + bi)(c + di)] = \text{real part}[(a - bi)(c + di)] = \text{real part}[ac + adi - bci - bdi^2] = ac + bd$

9.5 Assess Your Understanding (page 630)

2. dot product 3. orthogonal 4. parallel 5. T 6. F 7. d 8. b 9. (a) 0 (b) 90° (c) orthogonal 11. (a) 0 (b) 90° (c) orthogonal
 13. (a) $\sqrt{3} - 1$ (b) 75° (c) neither 15. (a) -50 (b) 180° (c) parallel 17. (a) 0 (b) 90° (c) orthogonal
 19. $\frac{2}{3}$ 21. $\mathbf{v}_1 = \frac{5}{2}\mathbf{i} - \frac{5}{2}\mathbf{j}$, $\mathbf{v}_2 = -\frac{1}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}$ 23. $\mathbf{v}_1 = -\frac{1}{5}\mathbf{i} - \frac{2}{5}\mathbf{j}$, $\mathbf{v}_2 = \frac{6}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}$ 25. $\mathbf{v}_1 = \frac{14}{5}\mathbf{i} + \frac{7}{5}\mathbf{j}$, $\mathbf{v}_2 = \frac{1}{5}\mathbf{i} - \frac{2}{5}\mathbf{j}$ 27. Approximately 1.353
 29. 9 ft-lb

31. (a) $\|\mathbf{I}\| \approx 0.022$; the intensity of the sun's rays is approximately 0.022 W/cm^2 . $\|\mathbf{A}\| = 500$; the area of the solar panel is 500 cm^2 .
 (b) $W = 10$; ten watts of energy is collected. (c) Vectors \mathbf{I} and \mathbf{A} should be parallel with the solar panels facing the sun.
 33. Force required to keep the Sienna from rolling down the hill: 7376 lb ; force perpendicular to the hill: 5248.4 lb
 35. Timmy must exert 85.5 lb . 37. 60° 39. Let $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$. Then $\mathbf{0} \cdot \mathbf{v} = 0a + 0b = 0$.
 41. $\mathbf{v} = \cos \alpha \mathbf{i} + \sin \alpha \mathbf{j}$, $0 \leq \alpha \leq \pi$; $\mathbf{w} = \cos \beta \mathbf{i} + \sin \beta \mathbf{j}$, $0 \leq \beta \leq \pi$. If θ is the angle between \mathbf{v} and \mathbf{w} , then $\mathbf{v} \cdot \mathbf{w} = \cos \theta$, since $\|\mathbf{v}\| = 1$ and $\|\mathbf{w}\| = 1$. Now $\theta = \alpha - \beta$ or $\theta = \beta - \alpha$. Since the cosine function is even,
 $\mathbf{v} \cdot \mathbf{w} = \cos(\alpha - \beta)$. Also, $\mathbf{v} \cdot \mathbf{w} = \cos \alpha \cos \beta + \sin \alpha \sin \beta$. So $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$.
 43. (a) If $\mathbf{u} = a_1\mathbf{i} + b_1\mathbf{j}$ and $\mathbf{v} = a_2\mathbf{i} + b_2\mathbf{j}$, then, since $\|\mathbf{u}\| = \|\mathbf{v}\|$, $a_1^2 + b_1^2 = \|\mathbf{u}\|^2 = \|\mathbf{v}\|^2 = a_2^2 + b_2^2$,
 $(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) = (a_1 + a_2)(a_1 - a_2) + (b_1 + b_2)(b_1 - b_2) = (a_1^2 + b_1^2) - (a_2^2 + b_2^2) = 0$.
 (b) The legs of the angle can be made to correspond to vectors $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$.
 45. $(\|\mathbf{w}\|\mathbf{v} + \|\mathbf{v}\|\mathbf{w}) \cdot (\|\mathbf{w}\|\mathbf{v} - \|\mathbf{v}\|\mathbf{w}) = \|\mathbf{w}\|^2\mathbf{v} \cdot \mathbf{v} - \|\mathbf{w}\|\|\mathbf{v}\|\mathbf{v} \cdot \mathbf{w} + \|\mathbf{v}\|\|\mathbf{w}\|\mathbf{w} \cdot \mathbf{v} - \|\mathbf{v}\|^2\mathbf{w} \cdot \mathbf{w} = \|\mathbf{w}\|^2\mathbf{v} \cdot \mathbf{v} - \|\mathbf{v}\|^2\mathbf{w} \cdot \mathbf{w} = \|\mathbf{w}\|^2\|\mathbf{v}\|^2 - \|\mathbf{v}\|^2\|\mathbf{w}\|^2 = 0$
 47. $\|\mathbf{u} + \mathbf{v}\|^2 - \|\mathbf{u} - \mathbf{v}\|^2 = (\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} + \mathbf{v}) - (\mathbf{u} - \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) = (\mathbf{u} \cdot \mathbf{u} + \mathbf{u} \cdot \mathbf{v} + \mathbf{v} \cdot \mathbf{u} + \mathbf{v} \cdot \mathbf{v}) - (\mathbf{u} \cdot \mathbf{u} - \mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{u} + \mathbf{v} \cdot \mathbf{v}) = 2(\mathbf{u} \cdot \mathbf{v}) + 2(\mathbf{v} \cdot \mathbf{u}) = 4(\mathbf{u} \cdot \mathbf{v})$
 49. $12 \frac{9}{2}$ 50. $\frac{9}{2}$ 51. $(1 - \sin^2\theta)(1 + \tan^2\theta) = (\cos^2\theta)(\sec^2\theta) = \cos^2\theta \cdot \frac{1}{\cos^2\theta} = 1$ 52. $V(x) = x(19 - 2x)(13 - 2x)$, or $V(x) = 4x^3 - 64x^2 + 247x$

9.6 Assess Your Understanding (page 639)

2. xy -plane 3. components 4. 1 5. F 6. T 7. All points of the form $(x, 0, z)$ 9. All points of the form $(x, y, 2)$ 11. All points of the form $(-4, y, z)$
 13. All points of the form $(1, 2, z)$ 15. $\sqrt{21}$ 17. $\sqrt{33}$ 19. $\sqrt{26}$ 21. $(2, 0, 0)$; $(2, 1, 0)$; $(0, 1, 0)$; $(2, 0, 3)$; $(0, 1, 3)$; $(0, 0, 3)$
 23. $(1, 4, 3)$; $(3, 2, 3)$; $(3, 4, 3)$; $(3, 2, 5)$; $(1, 4, 5)$; $(1, 2, 5)$ 25. $(-1, 2, 2)$; $(4, 0, 2)$; $(4, 2, 2)$; $(-1, 2, 5)$; $(4, 0, 5)$; $(-1, 0, 5)$ 27. $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$
 29. $\mathbf{v} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ 31. $\mathbf{v} = 8\mathbf{i} - \mathbf{j}$ 33. 7 35. $\sqrt{3}$ 37. $\sqrt{22}$ 39. $-\mathbf{j} - 2\mathbf{k}$ 41. $\sqrt{105}$ 43. $\sqrt{38} - \sqrt{17}$ 45. \mathbf{i} 47. $\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} - \frac{2}{7}\mathbf{k}$
 49. $\frac{\sqrt{3}}{3}\mathbf{i} + \frac{\sqrt{3}}{3}\mathbf{j} + \frac{\sqrt{3}}{3}\mathbf{k}$ 51. $\mathbf{v} \cdot \mathbf{w} = 0$; $\theta = 90^\circ$ 53. $\mathbf{v} \cdot \mathbf{w} = -2$, $\theta \approx 100.3^\circ$ 55. $\mathbf{v} \cdot \mathbf{w} = 0$; $\theta = 90^\circ$ 57. $\mathbf{v} \cdot \mathbf{w} = 52$; $\theta = 0^\circ$
 59. $\alpha \approx 64.6^\circ$; $\beta \approx 149.0^\circ$; $\gamma \approx 106.6^\circ$; $\mathbf{v} = 7(\cos 64.6^\circ\mathbf{i} + \cos 149.0^\circ\mathbf{j} + \cos 106.6^\circ\mathbf{k})$
 61. $\alpha = \beta = \gamma \approx 54.7^\circ$; $\mathbf{v} = \sqrt{3}(\cos 54.7^\circ\mathbf{i} + \cos 54.7^\circ\mathbf{j} + \cos 54.7^\circ\mathbf{k})$ 63. $\alpha = \beta = 45^\circ$; $\gamma = 90^\circ$; $\mathbf{v} = \sqrt{2}(\cos 45^\circ\mathbf{i} + \cos 45^\circ\mathbf{j} + \cos 90^\circ\mathbf{k})$
 65. $\alpha \approx 60.9^\circ$; $\beta \approx 144.2^\circ$; $\gamma \approx 71.1^\circ$; $\mathbf{v} = \sqrt{38}(\cos 60.9^\circ\mathbf{i} + \cos 144.2^\circ\mathbf{j} + \cos 71.1^\circ\mathbf{k})$ 67. (a) $\mathbf{d} = \mathbf{a} + \mathbf{b} + \mathbf{c} = \langle 7, 1, 5 \rangle$ (b) 8.66 ft
 69. $(x - 3)^2 + (y - 1)^2 + (z - 1)^2 = 1$ 71. Radius = 2, center $(-1, 1, 0)$ 73. Radius = 3, center $(2, -2, -1)$ 75. Radius = $\frac{3\sqrt{2}}{2}$, center $(2, 0, -1)$
 77. 2 newton-meters = 2 joules 79. 9 newton-meters = 9 joules 80. $\left\{x \mid 2 < x \leq \frac{13}{5}\right\}$ or $\left(2, \frac{13}{5}\right]$ 81. $2x^2 + 2x - 5$ 82. $\frac{1}{2}$
 83. $c = 3\sqrt{5} \approx 6.71$; $A \approx 26.6^\circ$; $B \approx 63.4^\circ$

9.7 Assess Your Understanding (page 646)

1. T 2. T 3. T 4. F 5. F 6. T 7. 2 9. 4 11. $-11A + 2B + 5C$ 13. $-6A + 23B - 15C$ 15. (a) $5\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$ (b) $-5\mathbf{i} - 5\mathbf{j} - 5\mathbf{k}$
 (c) $\mathbf{0}$ (d) $\mathbf{0}$ 17. (a) $\mathbf{i} - \mathbf{j} - \mathbf{k}$ (b) $-\mathbf{i} + \mathbf{j} + \mathbf{k}$ (c) $\mathbf{0}$ (d) $\mathbf{0}$ 19. (a) $-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ (b) $\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$ (c) $\mathbf{0}$ (d) $\mathbf{0}$ 21. (a) $3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$
 (b) $-3\mathbf{i} + \mathbf{j} - 4\mathbf{k}$ (c) $\mathbf{0}$ (d) $\mathbf{0}$ 23. $-9\mathbf{i} - 7\mathbf{j} - 3\mathbf{k}$ 25. $9\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$ 27. $\mathbf{0}$ 29. $-27\mathbf{i} - 21\mathbf{j} - 9\mathbf{k}$ 31. $-18\mathbf{i} - 14\mathbf{j} - 6\mathbf{k}$ 33. $\mathbf{0}$ 35. -25
 37. 25 39. $\mathbf{0}$ 41. Any vector of the form $c(-9\mathbf{i} - 7\mathbf{j} - 3\mathbf{k})$, where c is a nonzero scalar 43. Any vector of the form $c(-\mathbf{i} + \mathbf{j} + 5\mathbf{k})$, where c is a

nonzero scalar 45. $\sqrt{166}$ 47. $\sqrt{555}$ 49. $\sqrt{34}$ 51. $\sqrt{998}$ 53. $\frac{11\sqrt{19}}{57}\mathbf{i} + \frac{\sqrt{19}}{57}\mathbf{j} + \frac{7\sqrt{19}}{57}\mathbf{k}$ or $-\frac{11\sqrt{19}}{57}\mathbf{i} - \frac{\sqrt{19}}{57}\mathbf{j} - \frac{7\sqrt{19}}{57}\mathbf{k}$

55. 98 cubic units 57. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = (b_1c_2 - b_2c_1)\mathbf{i} - (a_1c_2 - a_2c_1)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$

$$\|\mathbf{u} \times \mathbf{v}\|^2 = \sqrt{(b_1c_2 - b_2c_1)^2 + (a_1c_2 - a_2c_1)^2 + (a_1b_2 - a_2b_1)^2}$$

$$= b_1^2c_2^2 - 2b_1b_2c_1c_2 + b_2^2c_1^2 + a_1^2c_2^2 - 2a_1a_2c_1c_2 + a_2^2c_1^2 + a_1^2b_2^2 - 2a_1a_2b_1b_2 + a_2^2b_1^2$$

$$\|\mathbf{u}\|^2 = a_1^2 + b_1^2 + c_1^2, \|\mathbf{v}\|^2 = a_2^2 + b_2^2 + c_2^2$$

$$\|\mathbf{u}\|^2\|\mathbf{v}\|^2 = (a_1^2 + b_1^2 + c_1^2)(a_2^2 + b_2^2 + c_2^2) = a_1^2a_2^2 + a_1^2b_2^2 + a_1^2c_2^2 + b_1^2a_2^2 + b_1^2b_2^2 + b_1^2c_2^2 + a_2^2c_1^2 + b_2^2c_1^2 + c_1^2c_2^2$$

$$(\mathbf{u} \cdot \mathbf{v})^2 = (a_1a_2 + b_1b_2 + c_1c_2)^2 = (a_1a_2 + b_1b_2 + c_1c_2)(a_1a_2 + b_1b_2 + c_1c_2)$$

$$= a_1^2a_2^2 + a_1a_2b_1b_2 + a_1a_2c_1c_2 + b_1b_2c_1c_2 + b_1b_2a_1a_2 + b_1^2b_2^2 + b_1b_2c_1c_2 + a_1a_2c_1c_2 + c_1^2c_2^2$$

$$= a_1^2a_2^2 + b_1^2b_2^2 + c_1^2c_2^2 + 2a_1a_2b_1b_2 + 2b_1b_2c_1c_2 + 2a_1a_2c_1c_2$$

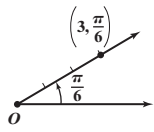
$$\|\mathbf{u}\|^2\|\mathbf{v}\|^2 - (\mathbf{u} \cdot \mathbf{v})^2 = a_1^2b_2^2 + a_1^2c_2^2 + b_1^2a_2^2 + a_2^2c_1^2 + b_2^2c_1^2 + b_1^2c_2^2 - 2a_1a_2b_1b_2 - 2b_1b_2c_1c_2 - 2a_1a_2c_1c_2, \text{ which equals } \|\mathbf{u} \times \mathbf{v}\|^2.$$

59. By Problem 58, since \mathbf{u} and \mathbf{v} are orthogonal, $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\|\|\mathbf{v}\|$. If, in addition, \mathbf{u} and \mathbf{v} are unit vectors, $\|\mathbf{u} \times \mathbf{v}\| = 1 \cdot 1 = 1$.
 61. Assume that $\mathbf{u} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, $\mathbf{v} = d\mathbf{i} + e\mathbf{j} + f\mathbf{k}$, and $\mathbf{w} = h\mathbf{i} + m\mathbf{j} + n\mathbf{k}$. Then $\mathbf{u} \times \mathbf{v} = (bf - ec)\mathbf{i} - (af - dc)\mathbf{j} + (ae - db)\mathbf{k}$,
 $\mathbf{u} \times \mathbf{w} = (bn - mc)\mathbf{i} - (an - lc)\mathbf{j} + (am - lb)\mathbf{k}$, and $\mathbf{v} \times \mathbf{w} = (d + l)\mathbf{i} + (e + m)\mathbf{j} + (f + n)\mathbf{k}$.
 Therefore, $(\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w}) = (bf - ec + bn - mc)\mathbf{i} - (af - dc + an - lc)\mathbf{j} + (ae - db + am - lb)\mathbf{k}$ and $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = [b(f + n) - (e + m)c]\mathbf{i} - [a(f + n) - (d + l)c]\mathbf{j} + [a(e + m) - (d + l)b]\mathbf{k}$
 $= (bf - ec + bn - mc)\mathbf{i} - (af - dc + an - lc)\mathbf{j} + (ae - db + am - lb)\mathbf{k}$, which equals $(\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w})$.

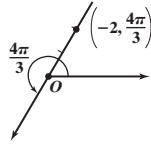
64. $\frac{\pi}{4}$ 65. $(17, 4.22)$, $(-17, 1.08)$ 66. $f^{-1}(x) = \log_7(x - 5) + 1$ 67. $\frac{1}{2}\log_4 x - 3\log_4 z$

Review Exercises (page 649)

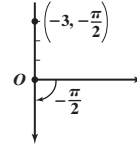
1. $(\frac{3\sqrt{3}}{2}, \frac{3}{2})$



2. $(1, \sqrt{3})$



3. $(0, 3)$



4. $(3\sqrt{2}, \frac{3\pi}{4}), (-3\sqrt{2}, -\frac{\pi}{4})$

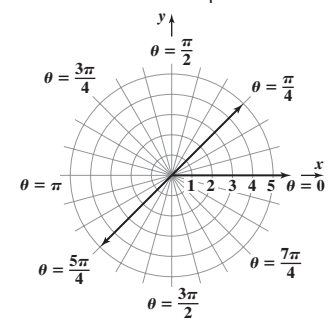
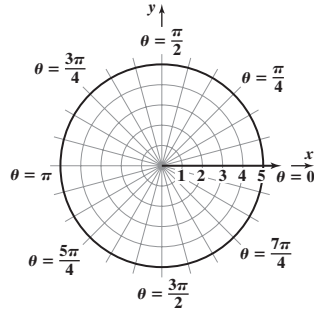
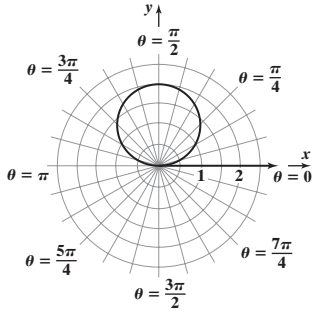
5. $(2, -\frac{\pi}{2}), (-2, \frac{\pi}{2})$

6. $(5, 0.93), (-5, 4.07)$

7. (a) $x^2 + (y - 1)^2 = 1$ (b) circle, radius 1, center $(0, 1)$ in rectangular coordinates

8. (a) $x^2 + y^2 = 25$ (b) circle, radius 5, center at pole

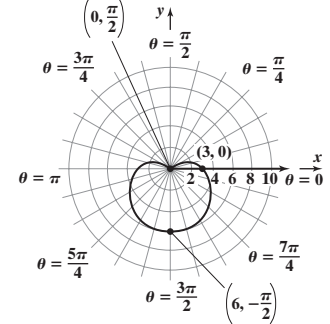
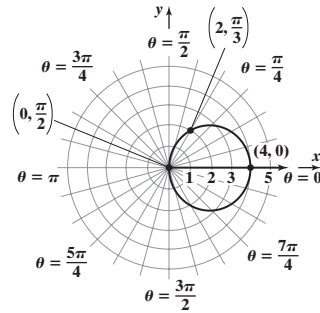
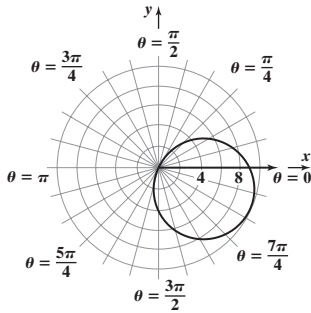
9. (a) $x - y = 0$ (b) line through pole, making an angle of $\frac{\pi}{4}$ with polar axis



10. (a) $(x - 4)^2 + (y + 2)^2 = 25$ (b) circle, radius 5, center $(4, -2)$ in rectangular coordinates

11. Circle; radius 2, center at $(2, 0)$ in rectangular coordinates; symmetric with respect to the polar axis

12. Cardioid; symmetric with respect to the line $\theta = \frac{\pi}{2}$



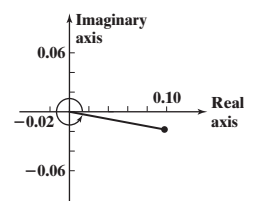
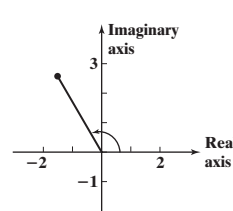
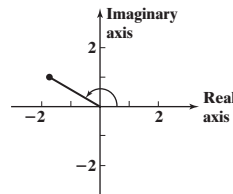
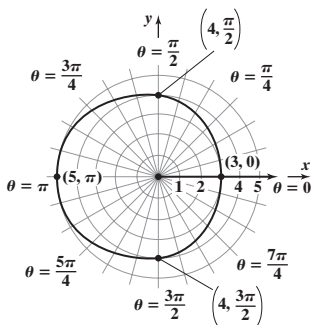
13. Limaçon without inner loop; symmetric with respect to the polar axis

14. $\sqrt{2}(\cos 225^\circ + i \sin 225^\circ)$ 15. $5(\cos 323.1^\circ + i \sin 323.1^\circ)$

16. $-\sqrt{3} + i$

17. $-\frac{3}{2} + \frac{3\sqrt{3}}{2}i$

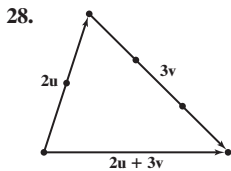
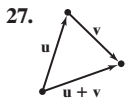
18. $0.10 - 0.02i$



19. $zw = \cos 130^\circ + i \sin 130^\circ$; $\frac{z}{w} = \cos 30^\circ + i \sin 30^\circ$ 20. $zw = 6(\cos 0 + i \sin 0) = 6$; $\frac{z}{w} = \frac{3}{2}(\cos \frac{8\pi}{5} + i \sin \frac{8\pi}{5})$

21. $zw = 5(\cos 5^\circ + i \sin 5^\circ)$; $\frac{z}{w} = 5(\cos 15^\circ + i \sin 15^\circ)$ 22. $\frac{27}{2} + \frac{27\sqrt{3}}{2}i$ 23. $4i$ 24. 64 25. $-527 - 336i$

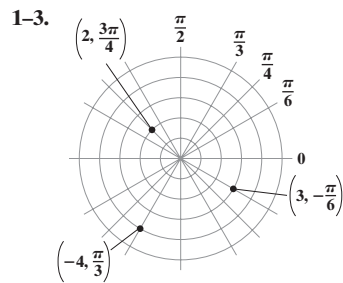
26. $3, 3(\cos 120^\circ + i \sin 120^\circ), 3(\cos 240^\circ + i \sin 240^\circ)$ or $3, -\frac{3}{2} + \frac{3\sqrt{3}}{2}i, -\frac{3}{2} - \frac{3\sqrt{3}}{2}i$



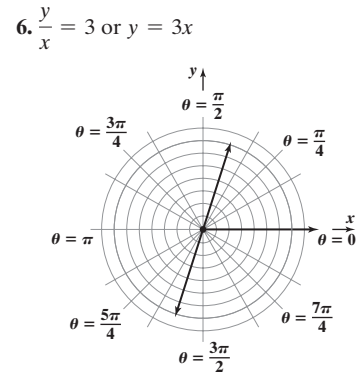
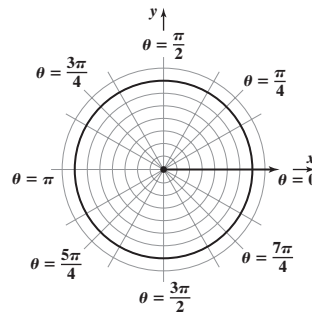
29. $\mathbf{v} = 2\mathbf{i} - 4\mathbf{j}$; $\|\mathbf{v}\| = 2\sqrt{5}$ 30. $\mathbf{v} = -\mathbf{i} + 3\mathbf{j}$; $\|\mathbf{v}\| = \sqrt{10}$ 31. $2\mathbf{i} - 2\mathbf{j}$ 32. $-20\mathbf{i} + 13\mathbf{j}$ 33. $\sqrt{5}$
 34. $\sqrt{5} + 5 \approx 7.24$ 35. $-\frac{2\sqrt{5}}{5}\mathbf{i} + \frac{\sqrt{5}}{5}\mathbf{j}$ 36. $\mathbf{v} = \frac{3}{2}\mathbf{i} + \frac{3\sqrt{3}}{2}\mathbf{j}$ 37. 120° 38. $\sqrt{43} \approx 6.56$
 39. $\mathbf{v} = 3\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$ 40. $21\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$ 41. $\sqrt{38}$ 42. 0 43. $3\mathbf{i} + 9\mathbf{j} + 9\mathbf{k}$ 44. 0
 45. $\frac{\sqrt{19}}{19}\mathbf{i} + \frac{3\sqrt{19}}{19}\mathbf{j} + \frac{3\sqrt{19}}{19}\mathbf{k}$ or $-\frac{\sqrt{19}}{19}\mathbf{i} - \frac{3\sqrt{19}}{19}\mathbf{j} - \frac{3\sqrt{19}}{19}\mathbf{k}$
 46. $\mathbf{v} \cdot \mathbf{w} = -11$; $\theta \approx 169.7^\circ$ 47. $\mathbf{v} \cdot \mathbf{w} = -4$; $\theta \approx 153.4^\circ$ 48. $\mathbf{v} \cdot \mathbf{w} = 1$; $\theta \approx 70.5^\circ$ 49. $\mathbf{v} \cdot \mathbf{w} = 0$; $\theta = 90^\circ$

50. Parallel 51. Neither 52. Orthogonal 53. $\mathbf{v}_1 = \frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}$; $\mathbf{v}_2 = \frac{6}{5}\mathbf{i} + \frac{8}{5}\mathbf{j}$ 54. $\mathbf{v}_1 = \frac{9}{10}(3\mathbf{i} + \mathbf{j})$; $\mathbf{v}_2 = -\frac{7}{10}\mathbf{i} + \frac{21}{10}\mathbf{j}$
 55. $\alpha \approx 56.1^\circ$; $\beta \approx 138^\circ$; $\gamma \approx 68.2^\circ$ 56. $2\sqrt{83}$ 57. $-2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ 58. 0 59. $\sqrt{29} \approx 5.39$ mi/h; 0.4 mi 60. Left cable: 1843.21 lb; right cable: 1630.41 lb 61. 50 ft-lb 62. A force of 6972 lb is needed to keep the van from rolling down the hill. The magnitude of the force perpendicular to the hill is 7969.6 lb.

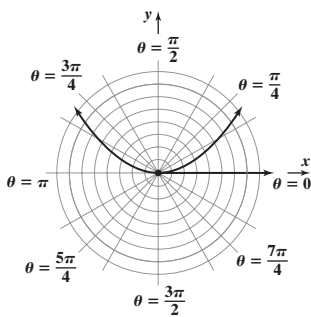
Chapter Test (page 651)



4. $(4, \frac{\pi}{3})$ 5. $x^2 + y^2 = 49$

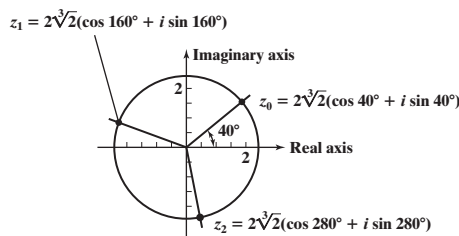


7. $8y = x^2$



8. $r^2 \cos \theta = 5$ is symmetric about the pole, the polar axis, and the line $\theta = \frac{\pi}{2}$.
 9. $r = 5 \sin \theta \cos^2 \theta$ is symmetric about the line $\theta = \frac{\pi}{2}$. The tests for symmetry about the pole and the polar axis fail, so the graph of $r = 5 \sin \theta \cos^2 \theta$ may or may not be symmetric about the pole or the polar axis.

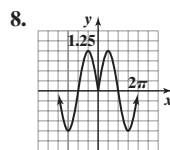
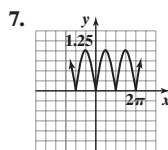
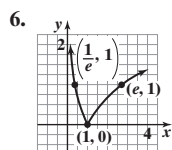
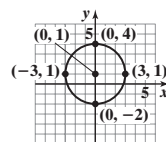
10. $z \cdot w = 6(\cos 107^\circ + i \sin 107^\circ)$ 11. $\frac{w}{z} = \frac{3}{2}(\cos 297^\circ + i \sin 297^\circ)$
 12. $w^5 = 243(\cos 110^\circ + i \sin 110^\circ)$
 13. $z_0 = 2\sqrt[3]{2}(\cos 40^\circ + i \sin 40^\circ)$, $z_1 = 2\sqrt[3]{2}(\cos 160^\circ + i \sin 160^\circ)$, $z_2 = 2\sqrt[3]{2}(\cos 280^\circ + i \sin 280^\circ)$



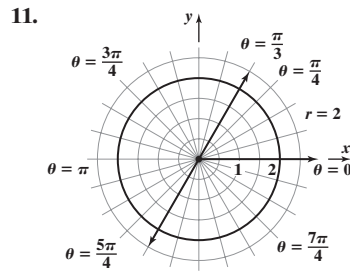
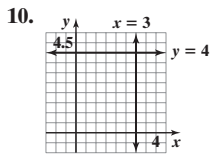
14. $\mathbf{v} = \langle 5\sqrt{2}, -5\sqrt{2} \rangle$ 15. $\|\mathbf{v}\| = 10$ 16. $\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \rangle$ 17. 315° off the positive x -axis 18. $\mathbf{v} = 5\sqrt{2}\mathbf{i} - 5\sqrt{2}\mathbf{j}$
 19. $\mathbf{v}_1 + 2\mathbf{v}_2 - \mathbf{v}_3 = \langle 6, -10 \rangle$ 20. Vectors \mathbf{v}_1 and \mathbf{v}_4 are parallel. 21. Vectors \mathbf{v}_2 and \mathbf{v}_3 are orthogonal. 22. 172.87° 23. $-9\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$
 24. $\alpha \approx 57.7^\circ$, $\beta \approx 143.3^\circ$, $\gamma \approx 74.5^\circ$ 25. $\sqrt{115}$ 26. The cable must be able to endure a tension of approximately 670.82 lb.

Cumulative Review (page 652)

1. $\{-3, 3\}$ 2. $y = \frac{\sqrt{3}}{3}x$ 3. $x^2 + (y - 1)^2 = 9$ 4. $\{x \mid x < \frac{1}{2}\}$ or $(-\infty, \frac{1}{2})$ 5. Symmetry with respect to the y -axis



9. $-\frac{\pi}{6}$



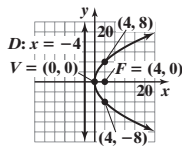
12. Amplitude: 4; period: 2

CHAPTER 10 analytic geometry

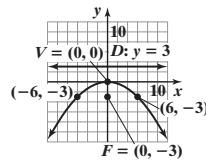
10.2 Assess Your Understanding (page 662)

6. parabola 7. axis of symmetry 8. latus rectum 9. c 10. (3, 2) 11. d 12. c 13. B 15. E 17. H 19. C

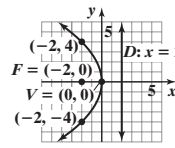
21. $y^2 = 16x$



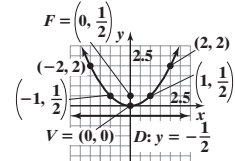
23. $x^2 = -12y$



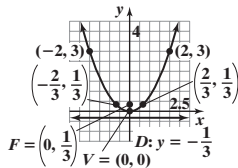
25. $y^2 = -8x$



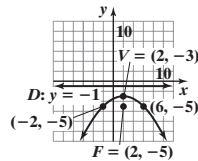
27. $x^2 = 2y$



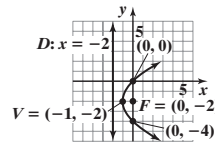
29. $x^2 = \frac{4}{3}y$



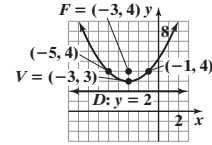
31. $(x - 2)^2 = -8(y + 3)$



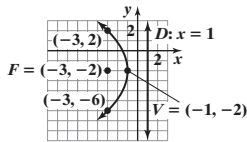
33. $(y + 2)^2 = 4(x + 1)$



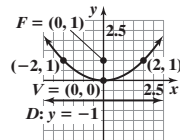
35. $(x + 3)^2 = 4(y - 3)$



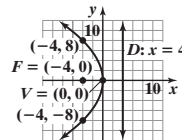
37. $(y + 2)^2 = -8(x + 1)$



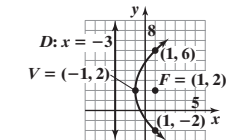
39. Vertex: (0, 0); focus: (0, 1);
directrix: $y = -1$



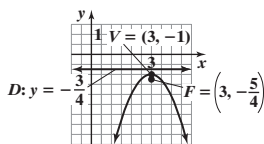
41. Vertex: (0, 0); focus: (-4, 0);
directrix: $x = 4$



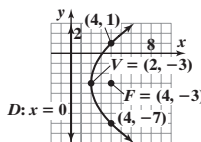
43. Vertex: (-1, 2); focus: (1, 2);
directrix: $x = -3$



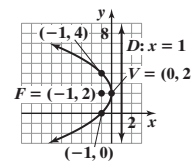
45. Vertex: (3, -1); focus: $(3, -\frac{5}{4})$;
directrix: $y = -\frac{3}{4}$



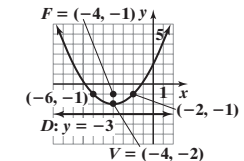
47. Vertex: (2, -3); focus: (4, -3);
directrix: $x = 0$



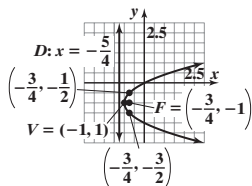
49. Vertex: (0, 2); focus: (-1, 2);
directrix: $x = 1$



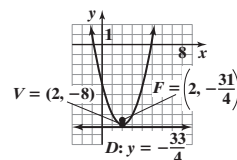
51. Vertex: (-4, -2); focus:
 $(-4, -1)$; directrix: $y = -3$



53. Vertex: (-1, -1); focus:
 $(-\frac{3}{4}, -1)$; directrix: $x = -\frac{5}{4}$



55. Vertex: (2, -8); focus: $(2, -\frac{31}{4})$;
directrix: $y = -\frac{33}{4}$



57. $(y - 1)^2 = x$ 59. $(y - 1)^2 = -(x - 2)$

61. $x^2 = 4(y - 1)$ 63. $y^2 = \frac{1}{2}(x + 2)$

65. 1.5625 ft from the base of the dish, along the axis of symmetry

67. 1 in. from the vertex, along the axis of symmetry

69. 20 ft 71. 0.78125 ft

73. 4.17 ft from the base, along the axis of symmetry

75. 24.31 ft, 18.75 ft, 764 ft

77. (a) $y = -\frac{2}{315}x^2 + 630$

(b) 567 ft; 119.7 ft; 478 ft; 2673 ft; 308 ft; 479.4 ft (c) No

79. $Cy^2 + Dx = 0, C \neq 0, D \neq 0$ This is the equation of a parabola with vertex at $(0, 0)$ and axis of symmetry the x -axis.

$$Cy^2 = -Dx$$

$$y^2 = -\frac{D}{C}x$$

The focus is $(-\frac{D}{4C}, 0)$; the directrix is the line $x = \frac{D}{4C}$. The parabola opens to the right if $-\frac{D}{C} > 0$ and to the left if $-\frac{D}{C} < 0$.

81. $Cy^2 + Dx + Ey + F = 0, C \neq 0$

$$Cy^2 + Ey = -Dx - F$$

$$y^2 + \frac{E}{C}y = -\frac{D}{C}x - \frac{F}{C}$$

$$\left(y + \frac{E}{2C}\right)^2 = -\frac{D}{C}x - \frac{F}{C} + \frac{E^2}{4C^2}$$

$$\left(y + \frac{E}{2C}\right)^2 = -\frac{D}{C}x + \frac{E^2 - 4CF}{4C^2}$$

(a) If $D \neq 0$, then the equation may be written as

$$\left(y + \frac{E}{2C}\right)^2 = -\frac{D}{C}\left(x - \frac{E^2 - 4CF}{4CD}\right)$$

This is the equation of a parabola with vertex at $\left(\frac{E^2 - 4CF}{4CD}, -\frac{E}{2C}\right)$

and axis of symmetry parallel to the x -axis.

(b)–(d) If $D = 0$, the graph of the equation contains no points if

$E^2 - 4CF < 0$, is a single horizontal line if $E^2 - 4CF = 0$, and

is two horizontal lines if $E^2 - 4CF > 0$.

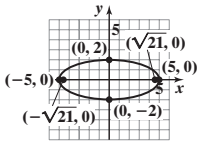
82. $(0, 2), (0, -2), (-36, 0)$; symmetric with respect to the x -axis 83. $\{5\}$

84. $\sin \theta = \frac{5\sqrt{89}}{89}$; $\cos \theta = -\frac{8\sqrt{89}}{89}$; $\csc \theta = \frac{\sqrt{89}}{5}$; $\sec \theta = -\frac{\sqrt{89}}{8}$; $\cot \theta = -\frac{8}{5}$ 85. $-\frac{2\sqrt{10}}{3}$

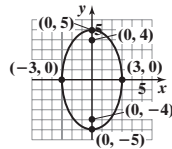
10.3 Assess Your Understanding (page 673)

7. ellipse 8. b 9. $(0, -5); (0, 5)$ 10. 5; 3; x 11. $(-2, -3); (6, -3)$ 12. a 13. C 15. B

17. Vertices: $(-5, 0), (5, 0)$;
foci: $(-\sqrt{21}, 0), (\sqrt{21}, 0)$

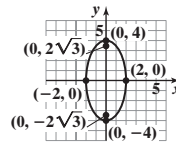


19. Vertices: $(0, -5), (0, 5)$;
foci: $(0, -4), (0, 4)$



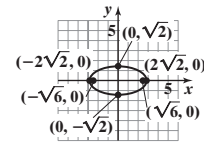
$$21. \frac{x^2}{4} + \frac{y^2}{16} = 1$$

Vertices: $(0, -4), (0, 4)$;
foci: $(0, -2\sqrt{3}), (0, 2\sqrt{3})$



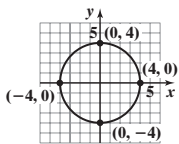
$$23. \frac{x^2}{8} + \frac{y^2}{2} = 1$$

Vertices: $(-2\sqrt{2}, 0), (2\sqrt{2}, 0)$;
foci: $(-\sqrt{6}, 0), (\sqrt{6}, 0)$

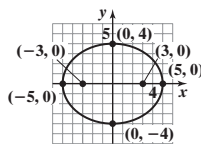


$$25. \frac{x^2}{16} + \frac{y^2}{16} = 1$$

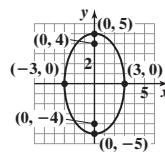
Vertices: $(-4, 0), (4, 0)$;
 $(0, -4), (0, 4)$; focus: $(0, 0)$



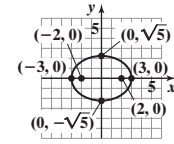
$$27. \frac{x^2}{25} + \frac{y^2}{16} = 1$$



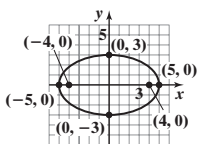
$$29. \frac{x^2}{9} + \frac{y^2}{25} = 1$$



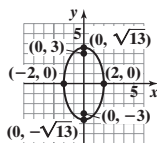
$$31. \frac{x^2}{9} + \frac{y^2}{5} = 1$$



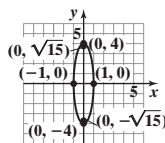
$$33. \frac{x^2}{25} + \frac{y^2}{9} = 1$$



$$35. \frac{x^2}{4} + \frac{y^2}{13} = 1$$



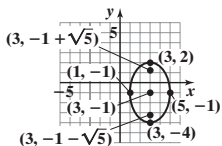
$$37. \frac{x^2}{16} + \frac{y^2}{9} = 1$$



$$39. \frac{(x+1)^2}{4} + (y-1)^2 = 1$$

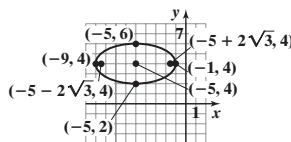
$$41. (x-1)^2 + \frac{y^2}{4} = 1$$

43. Center: $(3, -1)$;
vertices: $(3, -4), (3, 2)$; foci:
 $(3, -1 - \sqrt{5}), (3, -1 + \sqrt{5})$



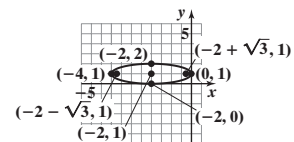
$$45. \frac{(x+5)^2}{16} + \frac{(y-4)^2}{4} = 1$$

Center: $(-5, 4)$;
vertices: $(-9, 4), (-1, 4)$; foci:
 $(-5 - 2\sqrt{3}, 4), (-5 + 2\sqrt{3}, 4)$



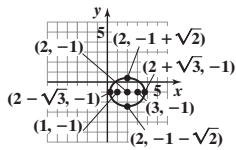
$$47. \frac{(x+2)^2}{4} + (y-1)^2 = 1$$

Center: $(-2, 1)$;
vertices: $(-4, 1), (0, 1)$; foci:
 $(-2 - \sqrt{3}, 1), (-2 + \sqrt{3}, 1)$

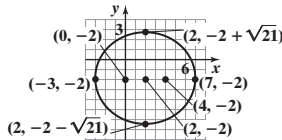


49. $\frac{(x-2)^2}{3} + \frac{(y+1)^2}{2} = 1$

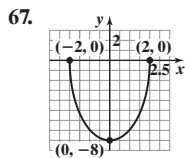
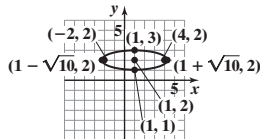
Center: (2, -1); vertices: (2 - √3, -1), (2 + √3, -1); foci: (1, -1), (3, -1)



55. $\frac{(x-2)^2}{25} + \frac{(y+2)^2}{21} = 1$

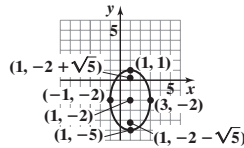


61. $\frac{(x-1)^2}{10} + (y-2)^2 = 1$

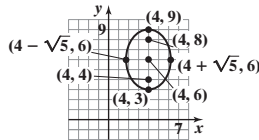


51. $\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$

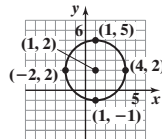
Center: (1, -2); vertices: (1, -5), (1, 1); foci: (1, -2 - √5), (1, -2 + √5)



57. $\frac{(x-4)^2}{5} + \frac{(y-6)^2}{9} = 1$

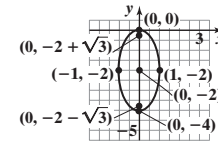


63. $\frac{(x-1)^2}{9} + \frac{(y-2)^2}{9} = 1$

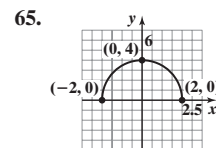
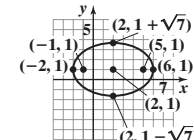


53. $x^2 + \frac{(y+2)^2}{4} = 1$

Center: (0, -2); vertices: (0, -4), (0, 0); foci: (0, -2 - √3), (0, -2 + √3)



59. $\frac{(x-2)^2}{16} + \frac{(y-1)^2}{7} = 1$



69. $\frac{x^2}{100} + \frac{y^2}{36} = 1$

71. 43.3 ft 73. 24.65 ft, 21.65 ft, 13.82 ft 75. 30 ft 77. The elliptical hole will have a major axis of length $2\sqrt{41}$ in. and a minor axis of length 8 in.

79. 91.5 million mi; $\frac{x^2}{(93)^2} + \frac{y^2}{8646.75} = 1$

81. Perihelion: 460.6 million mi; mean distance: 483.8 million mi; $\frac{x^2}{(483.8)^2} + \frac{y^2}{233,524.2} = 1$

83. 35 million mi 85. $5\sqrt{5} - 4$

87. $Ax^2 + Cy^2 + Dx + Ey + F = 0$ $A \neq 0, C \neq 0$

$Ax^2 + Dx + Cy^2 + Ey = -F$

$A\left(x^2 + \frac{D}{A}x\right) + C\left(y^2 + \frac{E}{C}y\right) = -F$

$A\left(x + \frac{D}{2A}\right)^2 + C\left(y + \frac{E}{2C}\right)^2 = -F + \frac{D^2}{4A} + \frac{E^2}{4C}$

(a) If $\frac{D^2}{4A} + \frac{E^2}{4C} - F$ is of the same sign as A (and C), this is the equation of an ellipse with center at $\left(-\frac{D}{2A}, -\frac{E}{2C}\right)$.

(b) If $\frac{D^2}{4A} + \frac{E^2}{4C} - F$, the graph is the single point $\left(-\frac{D}{2A}, -\frac{E}{2C}\right)$.

(c) If $\frac{D^2}{4A} + \frac{E^2}{4C} - F$ is of the sign opposite that of A (and C), the graph contains no points, because in this case, the left side has the sign opposite that of the right side.

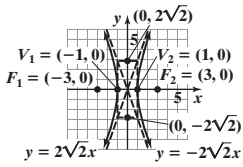
89. Zeros: $5 - 2\sqrt{3}, 5 + 2\sqrt{3}$; x-intercepts: $5 - 2\sqrt{3}, 5 + 2\sqrt{3}$ 90. Domain: $\{x|x \neq 5\}$; Horizontal asymptote: $y = 2$; Vertical asymptote: $x = 5$

91. 6171 ft-lb 92. $b \approx 10.94, c \approx 17.77, B = 38^\circ$

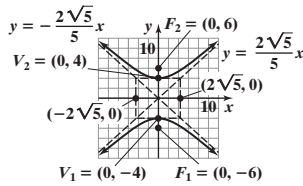
10.4 Assess Your Understanding (page 686)

7. hyperbola 8. transverse axis 9. b 10. (2, 4); (2, -2) 11. (2, 6); (2, -4) 12. c 13. 2; 3; x 14. $y = -\frac{4}{9}x; y = \frac{4}{9}x$ 15. B 17. A

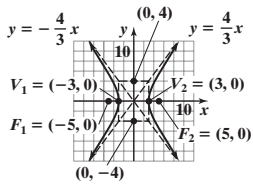
19. $x^2 - \frac{y^2}{8} = 1$



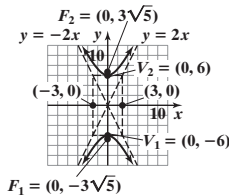
21. $\frac{y^2}{16} - \frac{x^2}{20} = 1$



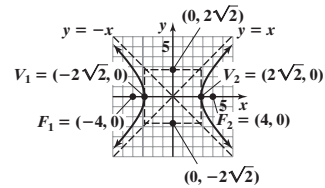
23. $\frac{x^2}{9} - \frac{y^2}{16} = 1$



25. $\frac{y^2}{36} - \frac{x^2}{9} = 1$

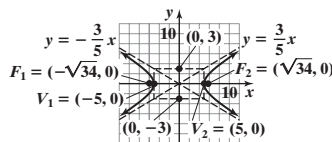


27. $\frac{x^2}{8} - \frac{y^2}{8} = 1$



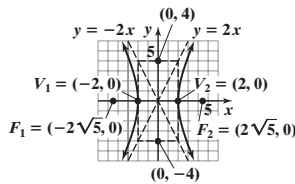
29. $\frac{x^2}{25} - \frac{y^2}{9} = 1$

Center: (0, 0)
 Transverse axis: x-axis
 Vertices: (-5, 0), (5, 0)
 Foci: (-\sqrt{34}, 0), (\sqrt{34}, 0)
 Asymptotes: $y = \pm \frac{3}{5}x$



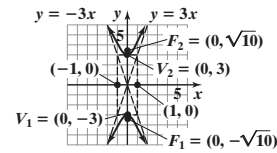
31. $\frac{x^2}{4} - \frac{y^2}{16} = 1$

Center: (0, 0)
 Transverse axis: x-axis
 Vertices: (-2, 0), (2, 0)
 Foci: (-2\sqrt{5}, 0), (2\sqrt{5}, 0)
 Asymptotes: $y = \pm 2x$



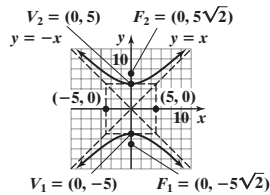
33. $\frac{y^2}{9} - x^2 = 1$

Center: (0, 0)
 Transverse axis: y-axis
 Vertices: (0, -3), (0, 3)
 Foci: (0, -\sqrt{10}), (0, \sqrt{10})
 Asymptotes: $y = \pm 3x$



35. $\frac{y^2}{25} - \frac{x^2}{25} = 1$

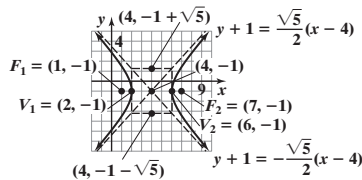
Center: (0, 0)
 Transverse axis: y-axis
 Vertices: (0, -5), (0, 5)
 Foci: (0, -5\sqrt{2}), (0, 5\sqrt{2})
 Asymptotes: $y = \pm x$



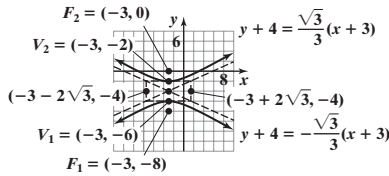
37. $x^2 - y^2 = 1$

39. $\frac{y^2}{36} - \frac{x^2}{9} = 1$

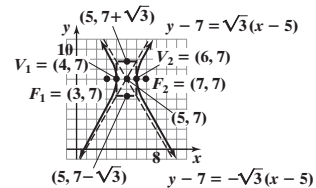
41. $\frac{(x-4)^2}{4} - \frac{(y+1)^2}{5} = 1$



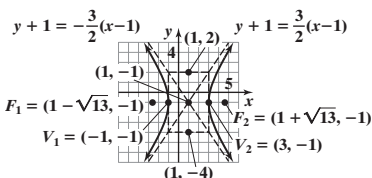
43. $\frac{(y+4)^2}{4} - \frac{(x+3)^2}{12} = 1$



45. $(x-5)^2 - \frac{(y-7)^2}{3} = 1$

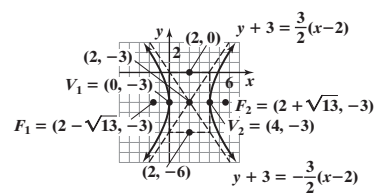


47. $\frac{(x-1)^2}{4} - \frac{(y+1)^2}{9} = 1$



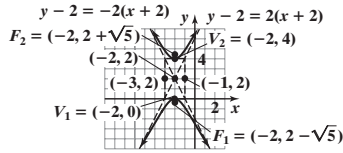
49. $\frac{(x-2)^2}{4} - \frac{(y+3)^2}{9} = 1$

Center: (2, -3)
 Transverse axis: parallel to x-axis
 Vertices: (0, -3), (4, -3)
 Foci: (2 - \sqrt{13}, -3), (2 + \sqrt{13}, -3)
 Asymptotes: $y + 3 = \pm \frac{3}{2}(x - 2)$



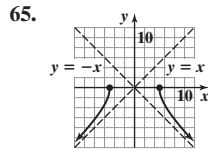
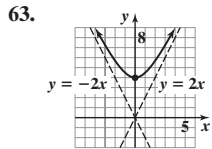
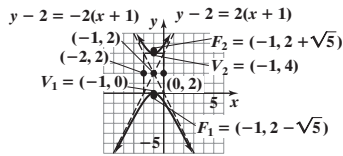
51. $\frac{(y - 2)^2}{4} - (x + 2)^2 = 1$

Center: $(-2, 2)$
 Transverse axis: parallel to y-axis
 Vertices: $(-2, 0), (-2, 4)$
 Foci: $(-2, 2 - \sqrt{5}), (-2, 2 + \sqrt{5})$
 Asymptotes: $y - 2 = \pm 2(x + 2)$



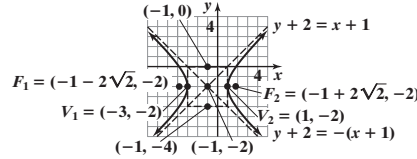
57. $\frac{(y - 2)^2}{4} - (x + 1)^2 = 1$

Center: $(-1, 2)$
 Transverse axis: parallel to y-axis
 Vertices: $(-1, 0), (-1, 4)$
 Foci: $(-1, 2 - \sqrt{5}), (-1, 2 + \sqrt{5})$
 Asymptotes: $y - 2 = \pm 2(x + 1)$



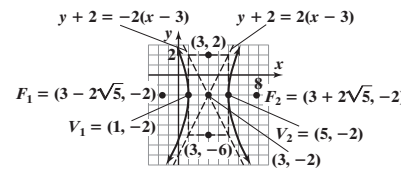
53. $\frac{(x + 1)^2}{4} - \frac{(y + 2)^2}{4} = 1$

Center: $(-1, -2)$
 Transverse axis: parallel to x-axis
 Vertices: $(-3, -2), (1, -2)$
 Foci: $(-1 - 2\sqrt{2}, -2), (-1 + 2\sqrt{2}, -2)$
 Asymptotes: $y + 2 = \pm(x + 1)$



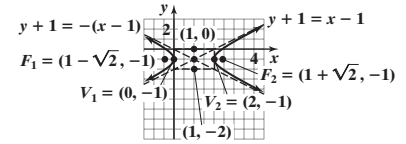
59. $\frac{(x - 3)^2}{4} - \frac{(y + 2)^2}{16} = 1$

Center: $(3, -2)$
 Transverse axis: parallel to x-axis
 Vertices: $(1, -2), (5, -2)$
 Foci: $(3 - 2\sqrt{5}, -2), (3 + 2\sqrt{5}, -2)$
 Asymptotes: $y + 2 = \pm 2(x - 3)$



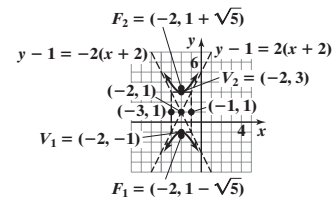
55. $(x - 1)^2 - (y + 1)^2 = 1$

Center: $(1, -1)$
 Transverse axis: parallel to x-axis
 Vertices: $(0, -1), (2, -1)$
 Foci: $(1 - \sqrt{2}, -1), (1 + \sqrt{2}, -1)$
 Asymptotes: $y + 1 = \pm(x - 1)$

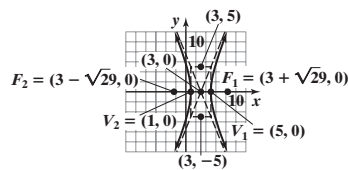


61. $\frac{(y - 1)^2}{4} - (x + 2)^2 = 1$

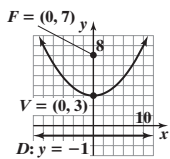
Center: $(-2, 1)$
 Transverse axis: parallel to y-axis
 Vertices: $(-2, -1), (-2, 3)$
 Foci: $(-2, 1 - \sqrt{5}), (-2, 1 + \sqrt{5})$
 Asymptotes: $y - 1 = \pm 2(x + 2)$



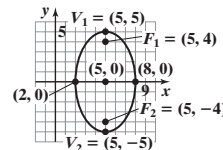
67. Center: $(3, 0)$
 Transverse axis: parallel to x-axis
 Vertices: $(1, 0), (5, 0)$
 Foci: $(3 - \sqrt{29}, 0), (3 + \sqrt{29}, 0)$
 Asymptotes: $y = \pm \frac{5}{2}(x - 3)$



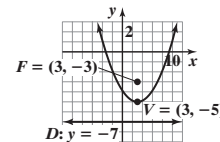
69. Vertex: $(0, 3)$; focus: $(0, 7)$;
 directrix: $y = -1$



71. $\frac{(x - 5)^2}{9} + \frac{y^2}{25} = 1$
 Center: $(5, 0)$; vertices: $(5, 5), (5, -5)$;
 foci: $(5, -4), (5, 4)$

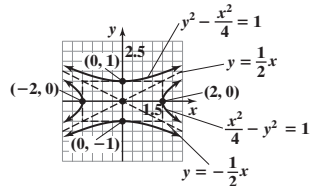


73. $(x - 3)^2 = 8(y + 5)$
 Vertex: $(3, -5)$; focus: $(3, -3)$;
 directrix: $y = -7$



75. The fireworks display is 50,138 ft north of the person at point A. 77. The tower is 592.4 ft tall. 79. (a) $y = \pm x$ (b) $\frac{x^2}{100} - \frac{y^2}{100} = 1, x \geq 0$

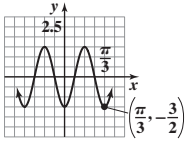
81. If the eccentricity is close to 1, the "opening" of the hyperbola is very small. As e increases, the opening gets bigger.
 83. $\frac{x^2}{4} - y^2 = 1$; asymptotes: $y = \pm \frac{1}{2}x$
 $y^2 - \frac{x^2}{4} = 1$; asymptotes: $y = \pm \frac{1}{2}x$



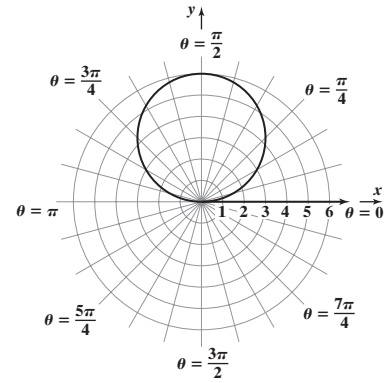
85. $Ax^2 + Cy^2 + F = 0$
 $Ax^2 + Cy^2 = -F$

If A and C are opposite in sign and $F \neq 0$, this equation may be written as $\frac{x^2}{(-F/A)} + \frac{y^2}{(-F/C)} = 1$, where $-F/A$ and $-F/C$ are opposite in sign. This is the equation of a hyperbola with center $(0, 0)$. The transverse axis is the x -axis if $-F/A > 0$; the transverse axis is the y -axis if $-F/C < 0$.

87. Amplitude = $\frac{3}{2}$; Period = $\frac{\pi}{3}$; Phase shift = $-\frac{\pi}{2}$



90. $x^2 + (y - 3)^2 = 9$; circle, radius 3, center at (0, 3) in rectangular coordinates



88. $c \approx 13.16$, $A \approx 31.6^\circ$, $B = 48.4^\circ$ 89. $(6, -6\sqrt{3})$

10.5 Assess Your Understanding (page 696)

5. $\cot(2\theta) = \frac{A-C}{B}$ 6. d 7. $B^2 - 4AC < 0$ 8. c 9. T 10. F 11. Parabola 13. Ellipse 15. Hyperbola

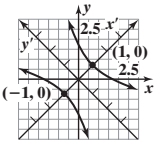
17. Hyperbola 19. Circle 21. $x = \frac{\sqrt{2}}{2}(x' - y')$, $y = \frac{\sqrt{2}}{2}(x' + y')$ 23. $x = \frac{\sqrt{2}}{2}(x' - y')$, $y = \frac{\sqrt{2}}{2}(x' + y')$

25. $x = \frac{1}{2}(x' - \sqrt{3}y')$, $y = \frac{1}{2}(\sqrt{3}x' + y')$ 27. $x = \frac{\sqrt{5}}{5}(x' - 2y')$, $y = \frac{\sqrt{5}}{5}(2x' + y')$ 29. $x = \frac{\sqrt{13}}{13}(3x' - 2y')$, $y = \frac{\sqrt{13}}{13}(2x' + 3y')$

31. $\theta = 45^\circ$ (see Problem 21)

$$x'^2 - \frac{y'^2}{3} = 1$$

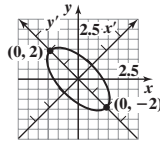
Hyperbola
Center at origin
Transverse axis is the x' -axis.
Vertices at $(\pm 1, 0)$



33. $\theta = 45^\circ$ (see Problem 23)

$$x'^2 + \frac{y'^2}{4} = 1$$

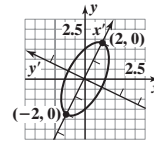
Ellipse
Center at (0, 0)
Major axis is the y' -axis.
Vertices at $(0, \pm 2)$



35. $\theta = 60^\circ$ (see Problem 25)

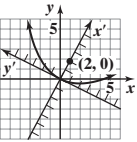
$$\frac{x'^2}{4} + y'^2 = 1$$

Ellipse
Center at (0, 0)
Major axis is the x' -axis.
Vertices at $(\pm 2, 0)$



37. $\theta \approx 63^\circ$ (see Problem 27)

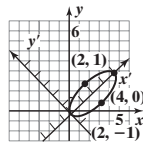
$y'^2 = 8x'$
Parabola
Vertex at (0, 0)
Focus at (2, 0)



39. $\theta \approx 34^\circ$ (see Problem 29)

$$\frac{(x' - 2)^2}{4} + y'^2 = 1$$

Ellipse
Center at (2, 0)
Major axis is the x' -axis.
Vertices at (4, 0) and (0, 0)

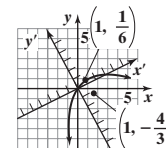


41. $\cot(2\theta) = \frac{7}{24}$

$$\theta = \sin^{-1}\left(\frac{3}{5}\right) \approx 37^\circ$$

$$(x' - 1)^2 = -6\left(y' - \frac{1}{6}\right)$$

Parabola
Vertex at $\left(1, \frac{1}{6}\right)$
Focus at $\left(1, -\frac{4}{3}\right)$



43. Hyperbola 45. Hyperbola 47. Parabola 49. Ellipse 51. Ellipse

53. Refer to equation (6): $A' = A \cos^2 \theta + B \sin \theta \cos \theta + C \sin^2 \theta$

$$B' = B(\cos^2 \theta - \sin^2 \theta) + 2(C - A)(\sin \theta \cos \theta)$$

$$C' = A \sin^2 \theta - B \sin \theta \cos \theta + C \cos^2 \theta$$

$$D' = D \cos \theta + E \sin \theta$$

$$E' = -D \sin \theta + E \cos \theta$$

$$F' = F$$

55. Use Problem 53 to find $B'^2 - 4A'C'$. After much cancellation, $B'^2 - 4A'C' = B^2 - 4AC$.

57. The distance between P_1 and P_2 in the $x'y'$ -plane equals $\sqrt{(x_2' - x_1')^2 + (y_2' - y_1')^2}$.

Assuming that $x' = x \cos \theta - y \sin \theta$ and $y' = x \sin \theta + y \cos \theta$, then

$$(x_2' - x_1')^2 = (x_2 \cos \theta - y_2 \sin \theta - x_1 \cos \theta + y_1 \sin \theta)^2$$

$$= \cos^2 \theta (x_2 - x_1)^2 - 2 \sin \theta \cos \theta (x_2 - x_1)(y_2 - y_1) + \sin^2 \theta (y_2 - y_1)^2, \text{ and}$$

$$(y_2' - y_1')^2 = (x_2 \sin \theta + y_2 \cos \theta - x_1 \sin \theta - y_1 \cos \theta)^2 = \sin^2 \theta (x_2 - x_1)^2 + 2 \sin \theta \cos \theta (x_2 - x_1)(y_2 - y_1) + \cos^2 \theta (y_2 - y_1)^2.$$

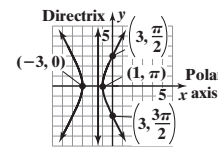
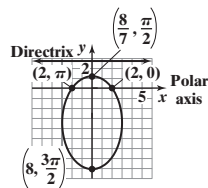
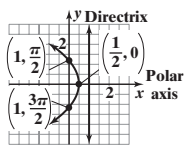
Therefore, $(x_2' - x_1')^2 + (y_2' - y_1')^2 = \cos^2 \theta (x_2 - x_1)^2 + \sin^2 \theta (x_2 - x_1)^2 + \sin^2 \theta (y_2 - y_1)^2 + \cos^2 \theta (y_2 - y_1)^2$

$$= (x_2 - x_1)^2 (\cos^2 \theta + \sin^2 \theta) + (y_2 - y_1)^2 (\sin^2 \theta + \cos^2 \theta) = (x_2 - x_1)^2 + (y_2 - y_1)^2.$$

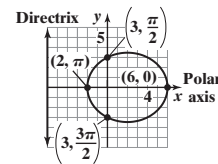
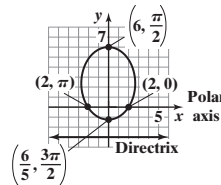
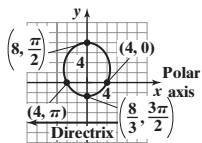
61. $A \approx 39.4^\circ, B \approx 54.7^\circ, C \approx 85.9^\circ$ 62. 38.5 63. $r^2 \cos \theta \sin \theta = 1$ 64. $\sqrt{29}(\cos 291.8^\circ + i \sin 291.8^\circ)$

10.6 Assess Your Understanding (page 703)

3. conic; focus; directrix 4. 1; <1; >1 5. T 6. T 7. Parabola; directrix is perpendicular to the polar axis, 1 unit to the right of the pole.
 9. Hyperbola; directrix is parallel to the polar axis, $\frac{4}{3}$ units below the pole. 11. Ellipse; directrix is perpendicular to the polar axis, $\frac{3}{2}$ units to the left of the pole.
 13. Parabola; directrix is perpendicular to the polar axis, 1 unit to the right of the pole; vertex is at $(\frac{1}{2}, 0)$.
 15. Ellipse; directrix is parallel to the polar axis, $\frac{8}{3}$ units above the pole; vertices are at $(\frac{8}{7}, \frac{\pi}{2})$ and $(8, \frac{3\pi}{2})$.
 17. Hyperbola; directrix is perpendicular to the polar axis, $\frac{3}{2}$ units to the left of the pole; vertices are at $(-3, 0)$ and $(1, \pi)$.



19. Ellipse; directrix is parallel to the polar axis, 8 units below the pole; vertices are at $(8, \frac{\pi}{2})$ and $(8, \frac{3\pi}{2})$.
 21. Ellipse; directrix is parallel to the polar axis, 3 units below the pole; vertices are at $(6, \frac{\pi}{2})$ and $(\frac{6}{5}, \frac{3\pi}{2})$.
 23. Ellipse; directrix is perpendicular to the polar axis, 6 units to the left of the pole; vertices are at $(6, 0)$ and $(2, \pi)$.



25. $y^2 + 2x - 1 = 0$ 27. $16x^2 + 7y^2 + 48y - 64 = 0$ 29. $3x^2 - y^2 + 12x + 9 = 0$ 31. $4x^2 + 3y^2 - 16y - 64 = 0$

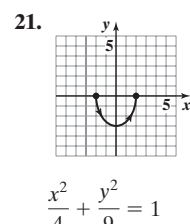
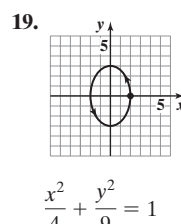
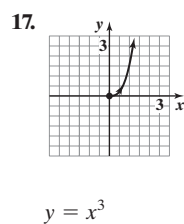
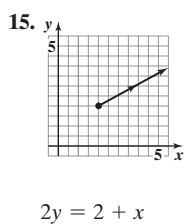
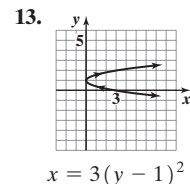
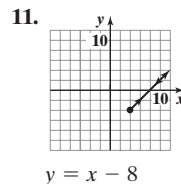
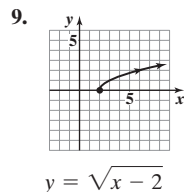
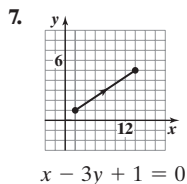
33. $9x^2 + 5y^2 - 24y - 36 = 0$ 35. $3x^2 + 4y^2 - 12x - 36 = 0$ 37. $r = \frac{1}{1 + \sin \theta}$ 39. $r = \frac{12}{5 - 4 \cos \theta}$ 41. $r = \frac{12}{1 - 6 \sin \theta}$

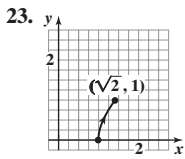
43. Use $d(D, P) = p - r \cos \theta$ in the derivation of equation (a) in Table 5. 45. Use $d(D, P) = p + r \sin \theta$ in the derivation of equation (a) in Table 5.

47. 27.81 48. Amplitude = 4; Period = 10π 49. $\{\frac{\pi}{3}, \pi, \frac{5\pi}{3}\}$ 50. 26

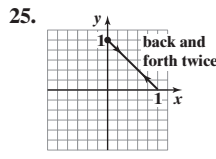
10.7 Assess Your Understanding (page 714)

2. plane curve; parameter 3. b 4. a 5. F 6. T





$x^2 - y^2 = 1$



$x + y = 1$

27. $x = t$ or $x = \frac{t+1}{4}$

$y = 4t - 1$ $y = t$

29. $x = t$ or $x = t^3$
 $y = t^2 + 1$ $y = t^6 + 1$

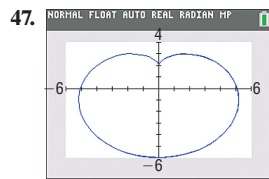
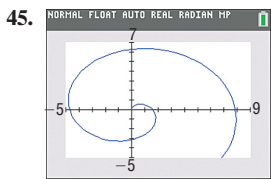
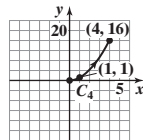
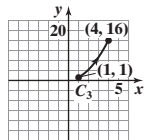
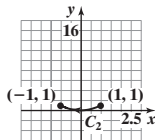
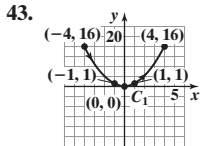
31. $x = t$ or $x = \sqrt[3]{t}$
 $y = t^3$ $y = t$

33. $x = t$ or $x = t^3$
 $y = t^{2/3}, t \geq 0$ $y = t^2, t \geq 0$

35. $x = t + 2, y = t, 0 \leq t \leq 5$ 37. $x = 3 \cos t, y = 2 \sin t, 0 \leq t \leq 2\pi$

39. $x = 2 \cos(\pi t), y = -3 \sin(\pi t), 0 \leq t \leq 2$

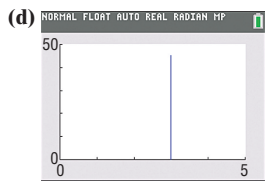
41. $x = 2 \sin(2\pi t), y = 3 \cos(2\pi t), 0 \leq t \leq 1$



49. (a) $x = 3$
 $y = -16t^2 + 50t + 6$

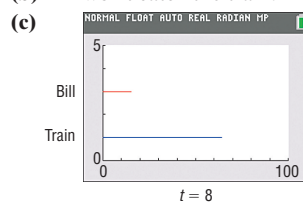
(b) 3.24 s

(c) 1.56 s; 45.06 ft



51. (a) Train: $x_1 = t^2, y_1 = 1$;
 Bill: $x_2 = 5(t - 5), y_2 = 3$

(b) Bill won't catch the train.

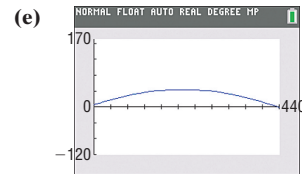


53. (a) $x = (145 \cos 20^\circ)t$
 $y = -16t^2 + (145 \sin 20^\circ)t + 5$

(b) 3.20 s

(c) 435.65 ft

(d) 1.55 s; 43.43 ft

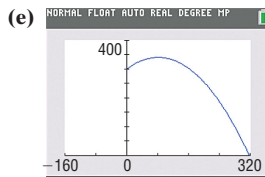


55. (a) $x = (40 \cos 45^\circ)t$
 $y = -4.9t^2 + (40 \sin 45^\circ)t + 300$

(b) 11.23 s

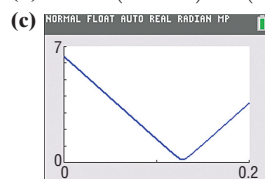
(c) 317.52 m

(d) 2.89 s; 340.82 m

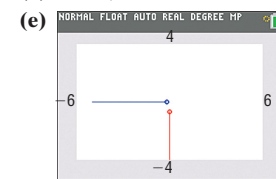


57. (a) Camry: $x = 40t - 5, y = 0$; Chevy Impala: $x = 0, y = 30t - 4$

(b) $d = \sqrt{(40t - 5)^2 + (30t - 4)^2}$

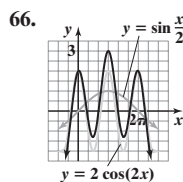
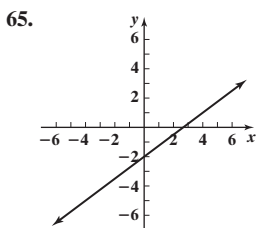


(d) 0.2 mi; 768 min



59. (a) $x = \frac{\sqrt{2}}{2}v_0t, y = -16t^2 + \frac{\sqrt{2}}{2}v_0t + 3$ (b) Maximum height is 139.1 ft. (c) The ball is 272.25 ft from home plate.

(d) Yes, the ball will clear the wall by about 99.5 ft. 61. The orientation is from (x_1, y_1) to (x_2, y_2) .



67. Approximately 2733 miles 68. (a) Simple harmonic (b) 2 m (c) $\frac{\pi}{2}$ s (d) $\frac{2}{\pi}$ oscillations/s

Review Exercises (page 719)

1. Parabola; vertex: $(0, 0)$; focus: $(-4, 0)$; directrix: $x = 4$ 2. Hyperbola; center: $(0, 0)$; vertices: $(5, 0)$ and $(-5, 0)$; foci: $(\sqrt{26}, 0)$ and $(-\sqrt{26}, 0)$; asymptotes: $y = \frac{1}{5}x$ and $y = -\frac{1}{5}x$ 3. Ellipse; center: $(0, 0)$; vertices: $(0, 5)$ and $(0, -5)$; foci: $(0, 3)$ and $(0, -3)$

4. $x^2 = -4(y - 1)$: Parabola; vertex: $(0, 1)$; focus: $(0, 0)$; directrix: $y = 2$ 5. $\frac{x^2}{2} - \frac{y^2}{8} = 1$: Hyperbola; center: $(0, 0)$; vertices: $(\sqrt{2}, 0)$ and $(-\sqrt{2}, 0)$; foci: $(\sqrt{10}, 0)$ and $(-\sqrt{10}, 0)$; asymptotes: $y = 2x$ and $y = -2x$ 6. $(x - 2)^2 = 2(y + 2)$: Parabola; vertex: $(2, -2)$; focus: $(2, -\frac{3}{2})$; directrix: $y = -\frac{5}{2}$

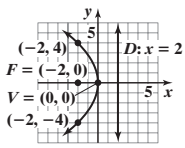
7. $\frac{(y - 2)^2}{4} - (x - 1)^2 = 1$: Hyperbola; center: $(1, 2)$; vertices: $(1, 4)$ and $(1, 0)$; foci: $(1, 2 + \sqrt{5})$ and $(1, 2 - \sqrt{5})$; asymptotes: $y - 2 = \pm 2(x - 1)$

8. $\frac{(x - 2)^2}{9} + \frac{(y - 1)^2}{4} = 1$: Ellipse; center: $(2, 1)$; vertices: $(5, 1)$ and $(-1, 1)$; foci: $(2 + \sqrt{5}, 1)$ and $(2 - \sqrt{5}, 1)$

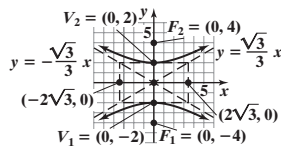
9. $(x - 2)^2 = -4(y + 1)$: Parabola; vertex: $(2, -1)$; focus: $(2, -2)$; directrix: $y = 0$

10. $\frac{(x - 1)^2}{4} + \frac{(y + 1)^2}{9} = 1$: Ellipse; center: $(1, -1)$; vertices: $(1, 2)$ and $(1, -4)$; foci: $(1, -1 + \sqrt{5})$ and $(1, -1 - \sqrt{5})$

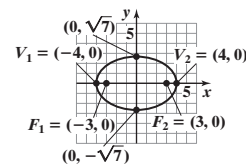
11. $y^2 = -8x$



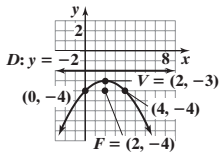
12. $\frac{y^2}{4} - \frac{x^2}{12} = 1$



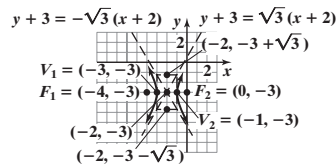
13. $\frac{x^2}{16} + \frac{y^2}{7} = 1$



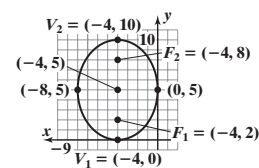
14. $(x - 2)^2 = -4(y + 3)$



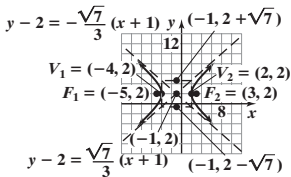
15. $(x + 2)^2 - \frac{(y + 3)^2}{3} = 1$



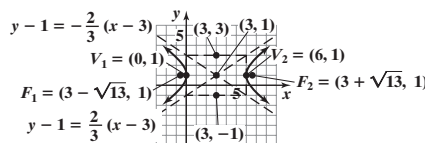
16. $\frac{(x + 4)^2}{16} + \frac{(y - 5)^2}{25} = 1$



17. $\frac{(x + 1)^2}{9} - \frac{(y - 2)^2}{7} = 1$



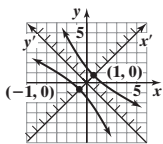
18. $\frac{(x - 3)^2}{9} - \frac{(y - 1)^2}{4} = 1$



19. Parabola 20. Ellipse
21. Parabola 22. Hyperbola
23. Ellipse

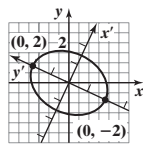
24. $x'^2 - \frac{y'^2}{9} = 1$

Hyperbola
Center at the origin
Transverse axis the x' -axis
Vertices at $(\pm 1, 0)$



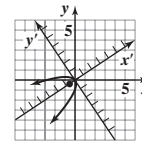
25. $\frac{x'^2}{2} + \frac{y'^2}{4} = 1$

Ellipse
Center at origin
Major axis the y' -axis
Vertices at $(0, \pm 2)$

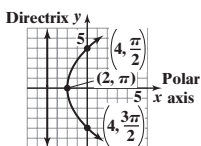


26. $y'^2 = -\frac{4\sqrt{13}}{13}x'$

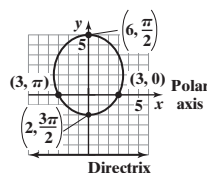
Parabola
Vertex at the origin
Focus on the x' -axis at $(-\frac{\sqrt{13}}{13}, 0)$



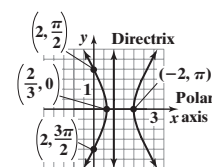
27. Parabola; directrix is perpendicular to the polar axis, 4 units to the left of the pole; vertex is $(2, \pi)$.



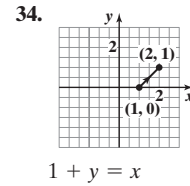
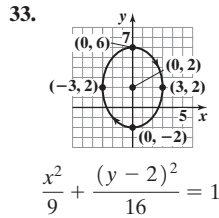
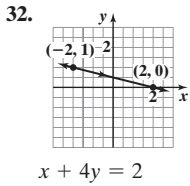
28. Ellipse; directrix is parallel to the polar axis, 6 units below the pole; vertices are $(6, \frac{\pi}{2})$ and $(2, \frac{3\pi}{2})$.



29. Hyperbola; directrix is perpendicular to the polar axis, 1 unit to the right of the pole; vertices are $(\frac{2}{3}, 0)$ and $(-2, \pi)$.



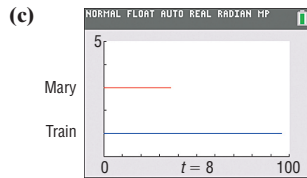
30. $y^2 - 8x - 16 = 0$ 31. $3x^2 - y^2 - 8x + 4 = 0$



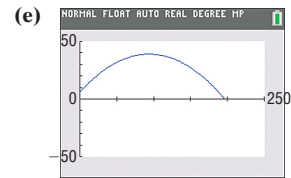
35. $x = t, y = -2t + 4, -\infty < t < \infty$ 36. $x = 4 \cos\left(\frac{\pi}{2}t\right), y = 3 \sin\left(\frac{\pi}{2}t\right), 0 \leq t \leq 4$ 37. $\frac{x^2}{5} - \frac{y^2}{4} = 1$ 38. The ellipse $\frac{x^2}{16} + \frac{y^2}{7} = 1$
 $x = \frac{t - 4}{-2}, y = t, -\infty < t < \infty$

39. $\frac{1}{4}$ ft or 3 in. 40. 19.72 ft, 18.86 ft, 14.91 ft 41. 450 ft

42. (a) Train: $x_1 = \frac{3}{2}t^2, y_1 = 1$
 Mary: $x_2 = 6(t - 2), y_2 = 3$
 (b) Mary won't catch the train.

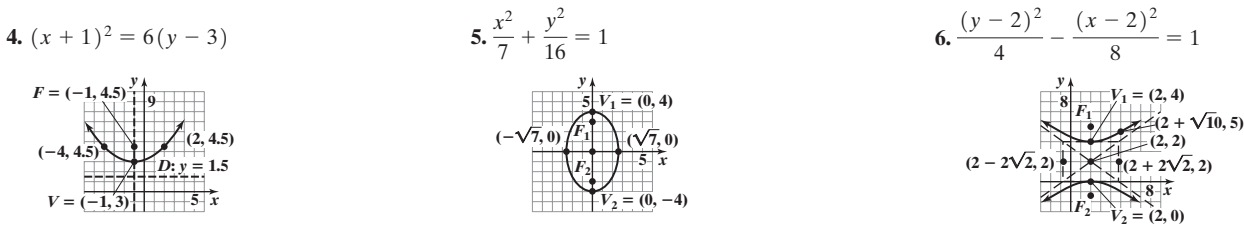


43. (a) $x = (80 \cos 35^\circ)t$
 $y = -16t^2 + (80 \sin 35^\circ)t + 6$
 (b) 2.9932 s
 (c) 1.4339 s; 38.9 ft
 (d) 196.15 ft

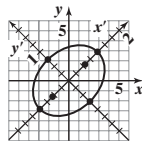


Chapter Test (page 720)

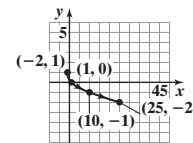
- Hyperbola; center: $(-1, 0)$; vertices: $(-3, 0)$ and $(1, 0)$; foci: $(-1 - \sqrt{13}, 0)$ and $(-1 + \sqrt{13}, 0)$; asymptotes: $y = -\frac{3}{2}(x + 1)$ and $y = \frac{3}{2}(x + 1)$
- Parabola; vertex: $(1, -\frac{1}{2})$; focus: $(1, \frac{3}{2})$; directrix: $y = -\frac{5}{2}$
- Ellipse; center: $(-1, 1)$; foci: $(-1 - \sqrt{3}, 1)$ and $(-1 + \sqrt{3}, 1)$; vertices: $(-4, 1)$ and $(2, 1)$



7. Hyperbola 8. Ellipse 9. Parabola
 10. $x'^2 + 2y'^2 = 1$. This is the equation of an ellipse with center at $(0, 0)$ in the $x'y'$ -plane. The vertices are at $(-1, 0)$ and $(1, 0)$ in the $x'y'$ -plane.



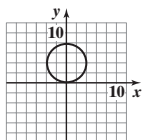
11. Hyperbola: $(x + 2)^2 - \frac{y^2}{3} = 1$ 12. $y = 1 - \sqrt{\frac{x + 2}{3}}$



13. The microphone should be located $\frac{2}{3}$ ft from the base of the reflector, along its axis of symmetry.

Cumulative Review (page 721)

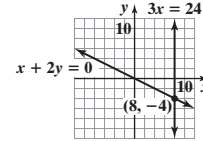
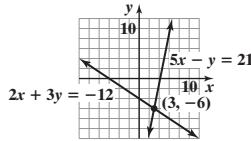
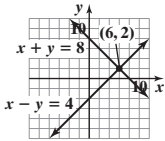
1. $-6x + 5 - 3h$ 2. $\left\{-5, -\frac{1}{3}, 2\right\}$ 3. $\{x | -3 \leq x \leq 2\}$ or $[-3, 2]$ 4. (a) Domain: $(-\infty, \infty)$; range: $(2, \infty)$
 (b) $y = \log_3(x - 2)$; domain: $(2, \infty)$; range: $(-\infty, \infty)$ 5. (a) $\{18\}$ (b) $(2, 18]$
 6. (a) $y = 2x - 2$ (b) $(x - 2)^2 + y^2 = 4$ (c) $\frac{x^2}{9} + \frac{y^2}{4} = 1$ (d) $y = 2(x - 1)^2$ (e) $y^2 - \frac{x^2}{3} = 1$ (f) $y = 4^x$
 7. $\theta = \frac{\pi}{12} \pm \pi k, k$ is any integer; $\theta = \frac{5\pi}{12} \pm \pi k, k$ is any integer 8. $\theta = \frac{\pi}{6}$
 9. $r = 8 \sin \theta$ 10. $\left\{x \mid x \neq \frac{3\pi}{4} \pm \pi k, k \text{ is an integer}\right\}$ 11. $\{22.5^\circ\}$ 12. $y = \frac{x^2}{5} + 5$



CHAPTER 11 Systems of Equations and Inequalities

11.1 Assess Your Understanding (page 734)

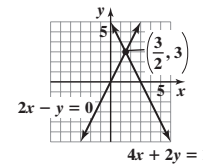
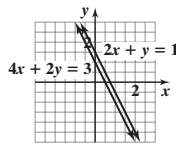
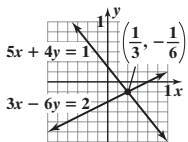
3. inconsistent 4. consistent; independent 5. (3, -2) 6. consistent; dependent 7. b 8. a 9. $\begin{cases} 2(2) - (-1) = 5 \\ 5(2) + 2(-1) = 8 \end{cases}$
11. $\begin{cases} 3(2) - 4\left(\frac{1}{2}\right) = 4 \\ \frac{1}{2}(2) - 3\left(\frac{1}{2}\right) = -\frac{1}{2} \end{cases}$ 13. $\begin{cases} 4 - 1 = 3 \\ \frac{1}{2}(4) + 1 = 3 \end{cases}$ 15. $\begin{cases} 3(1) + 3(-1) + 2(2) = 4 \\ 1 - (-1) - 2 = 0 \\ 2(-1) - 3(2) = -8 \end{cases}$ 17. $\begin{cases} 3(2) + 3(-2) + 2(2) = 4 \\ 2 - 3(-2) + 2 = 10 \\ 5(2) - 2(-2) - 3(2) = 8 \end{cases}$
19. $x = 6, y = 2; (6, 2)$ 21. $x = 3, y = -6; (3, -6)$ 23. $x = 8, y = -4; (8, -4)$



25. $x = \frac{1}{3}, y = -\frac{1}{6}; \left(\frac{1}{3}, -\frac{1}{6}\right)$

27. Inconsistent

29. $x = \frac{3}{2}, y = 3; \left(\frac{3}{2}, 3\right)$



31. $\{(x, y) | x = 4 - 2y, y \text{ is any real number}\}$, or $\{(x, y) | y = \frac{4-x}{2}, x \text{ is any real number}\}$ 33. $x = 1, y = 1; (1, 1)$ 35. $x = \frac{3}{2}, y = 1; \left(\frac{3}{2}, 1\right)$

37. $x = 4, y = 3; (4, 3)$ 39. $x = \frac{4}{3}, y = \frac{1}{5}; \left(\frac{4}{3}, \frac{1}{5}\right)$ 41. $x = \frac{1}{5}, y = \frac{1}{3}; \left(\frac{1}{5}, \frac{1}{3}\right)$ 43. $x = 8, y = 2, z = 0; (8, 2, 0)$

45. $x = 2, y = -1, z = 1; (2, -1, 1)$ 47. Inconsistent 49. $\{(x, y, z) | x = 5z - 2, y = 4z - 3; z \text{ is any real number}\}$ 51. Inconsistent

53. $x = 1, y = 3, z = -2; (1, 3, -2)$ 55. $x = -3, y = \frac{1}{2}, z = 1; \left(-3, \frac{1}{2}, 1\right)$ 57. Length 30 ft; width 15 ft

59. 23 commercial launches and 69 noncommercial launches 61. 22.5 lb 63. Smartphone: \$325; tablet: \$640

65. Average wind speed 25 mph; average airspeed 175 mph 67. 80 \$25 sets and 120 \$45 sets 69. \$9.96

71. Mix 50 mg of first compound with 75 mg of second. 73. $a = \frac{4}{3}, b = -\frac{5}{3}, c = 1$ 75. $Y = 9000, r = 0.06$ 77. $I_1 = \frac{10}{71}, I_2 = \frac{65}{71}, I_3 = \frac{55}{71}$

79. 100 orchestra, 210 main, and 190 balcony seats 81. 1.5 chicken, 1 corn, 2 milk

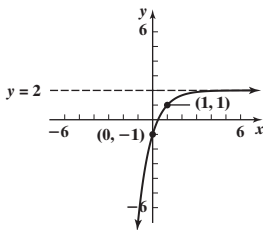
83. If $x =$ price of hamburgers, $y =$ price of fries,

and $z =$ price of colas, then $x = 5.5 - z, y = \frac{41}{30} + \frac{1}{3}z, \$1.20 \leq z \leq \$1.80$.
There is not sufficient information:

x	\$4.26	\$4.02	\$3.72
y	\$1.78	\$1.86	\$1.96
z	\$1.24	\$1.48	\$1.78

85. It will take Beth 30 hr, Bill 24 hr, and Edie 40 hr.

89.



90. $-2i, 2i, -6, 6$

91. $\frac{\pi}{9}$ 92. $2(\cos 150^\circ + i \sin 150^\circ)$

11.2 Assess Your Understanding (page 750)

1. matrix 2. augmented 3. third; fifth 4. T 5. b 6. c 7. $\begin{bmatrix} 1 & -5 & 5 \\ 4 & 3 & 6 \end{bmatrix}$ 9. $\begin{bmatrix} 2 & 3 & 6 \\ 4 & -6 & -2 \end{bmatrix}$ 11. $\begin{bmatrix} 0.01 & -0.03 & 0.06 \\ 0.13 & 0.10 & 0.20 \end{bmatrix}$

13. $\begin{bmatrix} 1 & -1 & 1 & 10 \\ 3 & 3 & 0 & 5 \\ 1 & 1 & 2 & 2 \end{bmatrix}$ 15. $\begin{bmatrix} 1 & 1 & -1 & 2 \\ 3 & -2 & 0 & 2 \\ 5 & 3 & -1 & 1 \end{bmatrix}$ 17. $\begin{bmatrix} 1 & -1 & -1 & 10 \\ 2 & 1 & 2 & -1 \\ -3 & 4 & 0 & 5 \\ 4 & -5 & 1 & 0 \end{bmatrix}$ 19. $\begin{cases} x - 3y = -2 & (1) \\ 2x - 5y = 5 & (2) \end{cases}; \begin{bmatrix} 1 & -3 & -2 \\ 0 & 1 & 9 \end{bmatrix}$

21. $\begin{cases} x - 3y + 4z = 3 & (1) \\ 3x - 5y + 6z = 6 & (2) \\ -5x + 3y + 4z = 6 & (3) \end{cases}; \begin{bmatrix} 1 & -3 & 4 & 3 \\ 0 & 4 & -6 & -3 \\ 0 & -12 & 24 & 21 \end{bmatrix}$ 23. $\begin{cases} x - 3y + 2z = -6 & (1) \\ 2x - 5y + 3z = -4 & (2) \\ -3x - 6y + 4z = 6 & (3) \end{cases}; \begin{bmatrix} 1 & -3 & 2 & -6 \\ 0 & 1 & -1 & 8 \\ 0 & -15 & 10 & -12 \end{bmatrix}$

25.
$$\begin{cases} 5x - 3y + z = -2 & (1) \\ 2x - 5y + 6z = -2 & (2) \\ -4x + y + 4z = 6 & (3) \end{cases} \left[\begin{array}{ccc|c} 1 & 7 & -11 & 2 \\ 2 & -5 & 6 & -2 \\ 0 & -9 & 16 & 2 \end{array} \right]$$

27.
$$\begin{cases} x = 5 \\ y = -1 \end{cases}$$

 Consistent; $x = 5, y = -1$ or $(5, -1)$

29.
$$\begin{cases} x = 1 \\ y = 2 \\ 0 = 3 \end{cases}$$

 Inconsistent

31.
$$\begin{cases} x + 2z = -1 \\ y - 4z = -2 \\ 0 = 0 \end{cases}$$

Consistent:

$$\begin{cases} x = -1 - 2z \\ y = -2 + 4z \\ z \text{ is any real number or} \\ \{(x, y, z) \mid x = -1 - 2z, \\ y = -2 + 4z, \\ z \text{ is any real number}\} \end{cases}$$

33.
$$\begin{cases} x_1 = 1 \\ x_2 + x_4 = 2 \\ x_3 + 2x_4 = 3 \end{cases}$$

Consistent:

$$\begin{cases} x_1 = 1, x_2 = 2 - x_4 \\ x_3 = 3 - 2x_4 \\ x_4 \text{ is any real number or} \\ \{(x_1, x_2, x_3, x_4) \mid x_1 = 1, \\ x_2 = 2 - x_4, x_3 = 3 - 2x_4, \\ x_4 \text{ is any real number}\} \end{cases}$$

35.
$$\begin{cases} x_1 + 4x_4 = 2 \\ x_2 + x_3 + 3x_4 = 3 \\ 0 = 0 \end{cases}$$

Consistent:

$$\begin{cases} x_1 = 2 - 4x_4 \\ x_2 = 3 - x_3 - 3x_4 \\ x_3, x_4 \text{ are any real numbers or} \\ \{(x_1, x_2, x_3, x_4) \mid x_1 = 2 - 4x_4, \\ x_2 = 3 - x_3 - 3x_4, x_3, x_4 \text{ are} \\ \text{any real numbers}\} \end{cases}$$

37.
$$\begin{cases} x_1 + x_4 = -2 \\ x_2 + 2x_4 = 2 \\ x_3 - x_4 = 0 \\ 0 = 0 \end{cases}$$

Consistent:

$$\begin{cases} x_1 = -2 - x_4 \\ x_2 = 2 - 2x_4 \\ x_3 = x_4 \\ x_4 \text{ is any real number or} \\ \{(x_1, x_2, x_3, x_4) \mid x_1 = -2 - x_4, \\ x_2 = 2 - 2x_4, x_3 = x_4, x_4 \text{ is any} \\ \text{real number}\} \end{cases}$$

39. $x = 6, y = 2; (6, 2)$ 41. $x = \frac{1}{2}, y = \frac{3}{4}; (\frac{1}{2}, \frac{3}{4})$ 43. $x = 4 - 2y, y$ is any real number; $\{(x, y) \mid x = 4 - 2y, y \text{ is any real number}\}$

45. $x = \frac{3}{2}, y = 1; (\frac{3}{2}, 1)$ 47. $x = \frac{4}{3}, y = \frac{1}{5}; (\frac{4}{3}, \frac{1}{5})$ 49. $x = 8, y = 2, z = 0; (8, 2, 0)$ 51. $x = 2, y = -1, z = 1; (2, -1, 1)$ 53. Inconsistent

55. $x = 5z - 2, y = 4z - 3$, where z is any real number; $\{(x, y, z) \mid x = 5z - 2, y = 4z - 3, z \text{ is any real number}\}$ 57. Inconsistent

59. $x = 1, y = 3, z = -2; (1, 3, -2)$ 61. $x = -3, y = \frac{1}{2}, z = 1; (-3, \frac{1}{2}, 1)$ 63. $x = \frac{1}{3}, y = \frac{2}{3}, z = 1; (\frac{1}{3}, \frac{2}{3}, 1)$

65. $x = 1, y = 2, z = 0, w = 1; (1, 2, 0, 1)$ 67. $y = 0, z = 1 - x$, x is any real number; $\{(x, y, z) \mid y = 0, z = 1 - x, x \text{ is any real number}\}$

69. $x = 2, y = z - 3$, z is any real number; $\{(x, y, z) \mid x = 2, y = z - 3, z \text{ is any real number}\}$ 71. $x = \frac{13}{9}, y = \frac{7}{18}, z = \frac{19}{18}; (\frac{13}{9}, \frac{7}{18}, \frac{19}{18})$

73. $x = \frac{7}{5} - \frac{3}{5}z - \frac{2}{5}w, y = -\frac{8}{5} + \frac{7}{5}z + \frac{13}{5}w$, where z and w are any real numbers; $\{(x, y, z, w) \mid x = \frac{7}{5} - \frac{3}{5}z - \frac{2}{5}w, y = -\frac{8}{5} + \frac{7}{5}z + \frac{13}{5}w, z \text{ and } w \text{ are any real numbers}\}$ 75. $y = -2x^2 + x + 3$ 77. $f(x) = 3x^3 - 4x^2 + 5$ 79. 1.5 salmon steak, 2 baked eggs, 1 acorn squash

81. \$4000 in Treasury bills, \$4000 in Treasury bonds, \$2000 in corporate bonds 83. 8 Deltas, 5 Betas, 10 Sigmas 85. $I_1 = \frac{44}{23}, I_2 = 2, I_3 = \frac{16}{23}, I_4 = \frac{28}{23}$

87. (a)

	Amount Invested At		
	7%	9%	11%
0	10,000	10,000	
1000	8000	11,000	
2000	6000	12,000	
3000	4000	13,000	
4000	2000	14,000	
5000	0	15,000	

(b)

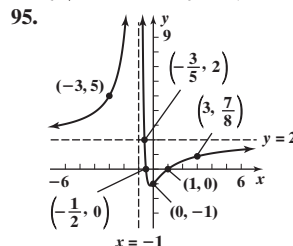
	Amount Invested At		
	7%	9%	11%
12,500	12,500	0	
14,500	8500	2000	
16,500	4500	4000	
18,750	0	6250	

(c) All the money invested at 7% provides \$2100, more than what is required.

89.

First Supplement	Second Supplement	Third Supplement
50 mg	75 mg	0 mg
36 mg	76 mg	8 mg
22 mg	77 mg	16 mg
8 mg	78 mg	24 mg

94. $\{x \mid -1 < x < 6\}$ or $(-1, 6)$



96. $\{x \mid x \text{ is any real number}\}$ or $(-\infty, \infty)$ 97. 2.42

11.3 Assess Your Understanding (page 763)

1. $ad - bc$ 2. $\begin{vmatrix} 5 & 3 \\ -3 & -4 \end{vmatrix}$ 3. F 4. F 5. F 6. a 7. 22 9. -2 11. 10 13. -26 15. $x = 6, y = 2; (6, 2)$ 17. $x = 3, y = 2; (3, 2)$

19. $x = 8, y = -4; (8, -4)$ 21. $x = 4, y = -2; (4, -2)$ 23. Not applicable 25. $x = \frac{1}{2}, y = \frac{3}{4}; (\frac{1}{2}, \frac{3}{4})$ 27. $x = \frac{1}{10}, y = \frac{2}{5}; (\frac{1}{10}, \frac{2}{5})$

29. $x = \frac{3}{2}, y = 1; (\frac{3}{2}, 1)$ 31. $x = \frac{4}{3}, y = \frac{1}{5}; (\frac{4}{3}, \frac{1}{5})$ 33. $x = 1, y = 3, z = -2; (1, 3, -2)$ 35. $x = -3, y = \frac{1}{2}, z = 1; (-3, \frac{1}{2}, 1)$

37. Not applicable 39. $x = 0, y = 0, z = 0; (0, 0, 0)$ 41. Not applicable 43. -4 45. 12 47. 8 49. 8 51. -5 53. $\frac{13}{11}$ 55. 0 or -9

57. $(y_1 - y_2)x - (x_1 - x_2)y + (x_1y_2 - x_2y_1) = 0$
 $(y_1 - y_2)x + (x_2 - x_1)y = x_2y_1 - x_1y_2$
 $(x_2 - x_1)y - (x_2 - x_1)y_1 = (y_2 - y_1)x + x_2y_1 - x_1y_2 - (x_2 - x_1)y_1$
 $(x_2 - x_1)(y - y_1) = (y_2 - y_1)x - (y_2 - y_1)x_1$
 $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

59. The triangle has an area of 5 square units. 61. 50.5 square units 63. $(x - 3)^2 + (y + 2)^2 = 25$

65. If $a = 0$, we have

$by = s$
 $cx + dy = t$
 Thus, $y = \frac{s}{b}$ and

$x = \frac{t - dy}{c} = \frac{tb - ds}{bc}$

Using Cramer's Rule, we get

$x = \frac{sd - tb}{-bc} = \frac{tb - sd}{bc}$

$y = \frac{-sc}{-bc} = \frac{s}{b}$

If $b = 0$, we have

$ax = s$
 $cx + dy = t$
 Since $D = ad \neq 0$, then
 $a \neq 0$ and $d \neq 0$.

Thus, $x = \frac{s}{a}$ and

$y = \frac{t - cx}{d} = \frac{ta - cs}{ad}$

Using Cramer's Rule, we get

$x = \frac{sd}{ad} = \frac{s}{a}$

$y = \frac{ta - cs}{ad}$

If $c = 0$, we have

$ax + by = s$
 $dy = t$
 Since $D = ad \neq 0$, then
 $a \neq 0$ and $d \neq 0$.

Thus, $y = \frac{t}{d}$ and

$x = \frac{s - by}{a} = \frac{sd - bt}{ad}$

Using Cramer's Rule, we get

$x = \frac{sd - bt}{ad}$

$y = \frac{at}{ad} = \frac{t}{d}$

If $d = 0$, we have

$ax + by = s$
 $cx = t$
 Since $D = -bc \neq 0$, then
 $b \neq 0$ and $c \neq 0$.

Thus, $x = \frac{t}{c}$ and

$y = \frac{s - ax}{b} = \frac{sc - at}{bc}$

Using Cramer's Rule, we get

$x = \frac{-tb}{-bc} = \frac{t}{c}$

$y = \frac{at - sc}{-bc} = \frac{sc - at}{bc}$

67. $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ ka_{21} & ka_{22} & ka_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -ka_{21}(a_{12}a_{33} - a_{32}a_{13}) + ka_{22}(a_{11}a_{33} - a_{31}a_{13}) - ka_{23}(a_{11}a_{32} - a_{31}a_{12})$

$= k[-a_{21}(a_{12}a_{33} - a_{32}a_{13}) + a_{22}(a_{11}a_{33} - a_{31}a_{13}) - a_{23}(a_{11}a_{32} - a_{31}a_{12})] = k \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

69. $\begin{vmatrix} a_{11} + ka_{21} & a_{12} + ka_{22} & a_{13} + ka_{23} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (a_{11} + ka_{21})(a_{22}a_{33} - a_{32}a_{23}) - (a_{12} + ka_{22})(a_{21}a_{33} - a_{31}a_{23}) + (a_{13} + ka_{23})(a_{21}a_{32} - a_{31}a_{22})$

$= a_{11}a_{22}a_{33} - a_{11}a_{32}a_{23} + ka_{21}a_{22}a_{33} - ka_{21}a_{32}a_{23} - a_{12}a_{21}a_{33} + a_{12}a_{31}a_{23}$
 $- ka_{22}a_{21}a_{33} + ka_{22}a_{31}a_{23} + a_{13}a_{21}a_{32} - a_{13}a_{31}a_{22} + ka_{23}a_{21}a_{32} - ka_{23}a_{31}a_{22}$
 $= a_{11}a_{22}a_{33} - a_{11}a_{32}a_{23} - a_{12}a_{21}a_{33} + a_{12}a_{31}a_{23} + a_{13}a_{21}a_{32} - a_{13}a_{31}a_{22}$
 $= a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$

$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

70. $\mathbf{v} = 9\mathbf{i} - 4\mathbf{j}; \sqrt{97}$ 71. $\pm \frac{1}{2}, \pm \frac{5}{2}, \pm 1, \pm 2, \pm 5, \pm 10$

72.  73. 0

Historical Problems (page 778)

1. (a) $2 - 5i \leftrightarrow \begin{bmatrix} 2 & -5 \\ 5 & 2 \end{bmatrix}, 1 + 3i \leftrightarrow \begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -5 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 17 & 1 \\ -1 & 17 \end{bmatrix}$ (c) $17 + i$ (d) $17 + i$

2. $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & 0 \\ 0 & b^2 + a^2 \end{bmatrix}$; the product is a real number.

3. (a) $x = k(ar + bs) + l(cr + ds) = r(ka + lc) + s(kb + ld)$
 $y = m(ar + bs) + n(cr + ds) = r(ma + nc) + s(mb + nd)$ (b) $A = \begin{bmatrix} ka + lc & kb + ld \\ ma + nc & mb + nd \end{bmatrix}$

11.4 Assess Your Understanding (page 778)

1. square 2. T 3. F 4. inverse 5. T 6. $A^{-1}B$ 7. a 8. d 9. $\begin{bmatrix} 4 & 4 & -5 \\ -1 & 5 & 4 \end{bmatrix}$ 11. $\begin{bmatrix} 0 & 12 & -20 \\ 4 & 8 & 24 \end{bmatrix}$ 13. $\begin{bmatrix} -8 & 7 & -15 \\ 7 & 0 & 22 \end{bmatrix}$

15. $\begin{bmatrix} 28 & -9 \\ 4 & 23 \end{bmatrix}$ 17. Not defined 19. $\begin{bmatrix} 1 & 14 & -14 \\ 2 & 22 & -18 \\ 3 & 0 & 28 \end{bmatrix}$ 21. $\begin{bmatrix} 15 & 21 & -16 \\ 22 & 34 & -22 \\ -11 & 7 & 22 \end{bmatrix}$ 23. $\begin{bmatrix} 25 & -9 \\ 4 & 20 \end{bmatrix}$ 25. $\begin{bmatrix} -13 & 7 & -12 \\ -18 & 10 & -14 \\ 17 & -7 & 34 \end{bmatrix}$ 27. $\begin{bmatrix} -2 & 4 & 2 & 8 \\ 2 & 1 & 4 & 6 \end{bmatrix}$

29. $\begin{bmatrix} 5 & 14 \\ 9 & 16 \end{bmatrix}$ 31. Not defined 33. $\begin{bmatrix} 9 & 2 \\ 34 & 13 \\ 47 & 20 \end{bmatrix}$ 35. $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ 37. $\begin{bmatrix} 1 & -5 \\ -1 & 3 \end{bmatrix}$ 39. $\begin{bmatrix} 1 & -\frac{1}{a} \\ -1 & \frac{2}{a} \end{bmatrix}$ 41. $\begin{bmatrix} 3 & -3 & 1 \\ -2 & 2 & -1 \\ -4 & 5 & -2 \end{bmatrix}$ 43. $\begin{bmatrix} -\frac{5}{7} & \frac{1}{7} & \frac{3}{7} \\ \frac{9}{7} & \frac{1}{7} & -\frac{4}{7} \\ \frac{3}{7} & -\frac{2}{7} & \frac{1}{7} \end{bmatrix}$

45. $x = 3, y = 2; (3, 2)$ 47. $x = -5, y = 10; (-5, 10)$ 49. $x = 2, y = -1; (2, -1)$ 51. $x = \frac{1}{2}, y = 2; (\frac{1}{2}, 2)$ 53. $x = -2, y = 1; (-2, 1)$

55. $x = \frac{2}{a}, y = \frac{3}{a}; (\frac{2}{a}, \frac{3}{a})$ 57. $x = -2, y = 3, z = 5; (-2, 3, 5)$ 59. $x = \frac{1}{2}, y = -\frac{1}{2}, z = 1; (\frac{1}{2}, -\frac{1}{2}, 1)$

61. $x = -\frac{34}{7}, y = \frac{85}{7}, z = \frac{12}{7}; (-\frac{34}{7}, \frac{85}{7}, \frac{12}{7})$ 63. $x = \frac{1}{3}, y = 1, z = \frac{2}{3}; (\frac{1}{3}, 1, \frac{2}{3})$ 65. $\begin{bmatrix} 4 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{4} & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{4} & 0 \\ 0 & 0 & -\frac{1}{2} & 1 \end{bmatrix}$

67. $\begin{bmatrix} 15 & 3 & 1 & 0 \\ 10 & 2 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{1}{5} & \frac{1}{15} & 0 \\ 10 & 2 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{1}{5} & \frac{1}{15} & 0 \\ 0 & 0 & -\frac{2}{3} & 1 \end{bmatrix}$

69. $\begin{bmatrix} -3 & 1 & -1 & 1 & 0 & 0 \\ 1 & -4 & -7 & 0 & 1 & 0 \\ 1 & 2 & 5 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 5 & 0 & 0 & 1 \\ 1 & -4 & -7 & 0 & 1 & 0 \\ -3 & 1 & -1 & 1 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 5 & 0 & 0 & 1 \\ 0 & -6 & -12 & 0 & 1 & -1 \\ 0 & 7 & 14 & 1 & 0 & 3 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 1 & 2 & 5 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & -\frac{1}{6} & \frac{1}{6} \\ 0 & 1 & 2 & \frac{1}{7} & 0 & \frac{3}{7} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 5 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & -\frac{1}{6} & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{7} & \frac{1}{6} & \frac{11}{42} \end{bmatrix}$

71. $\begin{bmatrix} 0.01 & 0.05 & -0.01 \\ 0.01 & -0.02 & 0.01 \\ -0.02 & 0.01 & 0.03 \end{bmatrix}$ 73. $\begin{bmatrix} 0.02 & -0.04 & -0.01 & 0.01 \\ -0.02 & 0.05 & 0.03 & -0.03 \\ 0.02 & 0.01 & -0.04 & 0.00 \\ -0.02 & 0.06 & 0.07 & 0.06 \end{bmatrix}$

75. $x = 4.57, y = -6.44, z = -24.07; (4.57, -6.44, -24.07)$ 77. $x = -1.19, y = 2.46, z = 8.27; (-1.19, 2.46, 8.27)$

79. $x = -5, y = 7; (-5, 7)$ 81. $x = -4, y = 2, z = \frac{5}{2}; (-4, 2, \frac{5}{2})$

83. Inconsistent; \emptyset 85. $x = -\frac{1}{5}z + \frac{1}{5}, y = \frac{1}{5}z - \frac{6}{5}$, where z is any real number; $\{(x, y, z) \mid x = -\frac{1}{5}z + \frac{1}{5}, y = \frac{1}{5}z - \frac{6}{5}, z \text{ is any real number}\}$

87. (a) $A = \begin{bmatrix} 6 & 9 \\ 3 & 12 \end{bmatrix}; B = \begin{bmatrix} 128.00 \\ 341.60 \end{bmatrix}$ (b) $AB = \begin{bmatrix} 3842.40 \\ 4483.20 \end{bmatrix}$; Nikki's total tuition is \$3842.40, and Joe's total tuition is \$4483.20.

89. (a) $\begin{bmatrix} 500 & 350 & 400 \\ 700 & 500 & 850 \end{bmatrix}; \begin{bmatrix} 500 & 700 \\ 350 & 500 \\ 400 & 850 \end{bmatrix}$ (b) $\begin{bmatrix} 15 \\ 8 \\ 3 \end{bmatrix}$ (c) $\begin{bmatrix} 11,500 \\ 17,050 \end{bmatrix}$ (d) $[0.10 \ 0.05]$ (e) \$2002.50

91. (a) $K^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$ (b) $M = \begin{bmatrix} 13 & 1 & 20 \\ 8 & 9 & 19 \\ 6 & 21 & 14 \end{bmatrix}$ (c) Math is fun.

93. If $D = ad - bc \neq 0$, then $a \neq 0$ and $d \neq 0$, or $b \neq 0$ and $c \neq 0$. Assuming the former,

$$\left[\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 1 & \frac{b}{a} & \frac{1}{a} & 0 \\ c & d & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 1 & \frac{b}{a} & \frac{1}{a} & 0 \\ 0 & \frac{D}{a} & -\frac{c}{a} & 1 \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 1 & \frac{b}{a} & \frac{1}{a} & 0 \\ 0 & 1 & -\frac{c}{D} & \frac{a}{D} \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & \frac{d}{D} & -\frac{b}{D} \\ 0 & 1 & -\frac{c}{D} & \frac{a}{D} \end{array} \right]$$

$$R_1 = \frac{1}{a}r_1 \quad R_2 = -cr_1 + r_2 \quad R_2 = \frac{a}{D}r_2 \quad R_1 = -\frac{b}{a}r_2 + r_1$$

95. (a) $B_3 = A + A^2 + A^3 = \begin{bmatrix} 2 & 4 & 5 & 2 & 3 \\ 5 & 3 & 2 & 5 & 4 \\ 4 & 2 & 2 & 4 & 2 \\ 2 & 2 & 3 & 2 & 3 \\ 1 & 3 & 2 & 1 & 2 \end{bmatrix}$; Yes, all pages can reach every other page within 3 clicks. (b) Page 3

97. (a) $(3 - 2\sqrt{3}, 3\sqrt{3} + 2)$ (b) $R^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$; This is the rotation matrix needed to get the translated coordinates back to the original coordinates.

102. $x^6 - 4x^5 - 3x^4 + 18x^3$ 103. $\frac{\sqrt{u^2 - 1}}{|u|}$ 104. $\{0, 3\}$ 105. $\mathbf{v} \cdot \mathbf{w} = -5; \theta = 180^\circ$

11.5 Assess Your Understanding (page 788)

5. Proper 7. Improper; $1 + \frac{9}{x^2 - 4}$ 9. Improper; $5x + \frac{22x - 1}{x^2 - 4}$ 11. Improper; $1 + \frac{-2(x - 6)}{(x + 4)(x - 3)}$ 13. $\frac{-4}{x} + \frac{4}{x - 1}$ 15. $\frac{1}{x} + \frac{-x}{x^2 + 1}$

17. $\frac{-1}{x - 1} + \frac{2}{x - 2}$ 19. $\frac{1}{4} + \frac{3}{4} + \frac{1}{2}$ 21. $\frac{1}{12} + \frac{-1}{12}(x + 4)$ 23. $\frac{1}{4} + \frac{1}{4} + \frac{-1}{4} + \frac{1}{4}$

25. $\frac{-5}{x + 2} + \frac{5}{x + 1} + \frac{-4}{(x + 1)^2}$ 27. $\frac{1}{4} + \frac{1}{x^2} + \frac{-1}{4}(x + 4)$ 29. $\frac{2}{3} + \frac{1}{3}(x + 1)$ 31. $\frac{2}{7} + \frac{1}{7}$ 33. $\frac{3}{4} + \frac{1}{4}$

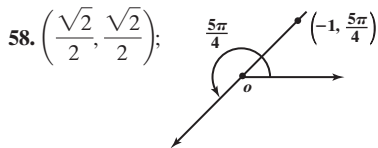
35. $\frac{1}{x^2 + 4} + \frac{2x - 1}{(x^2 + 4)^2}$ 37. $\frac{-1}{x} + \frac{2}{x - 3} + \frac{-1}{x + 1}$ 39. $\frac{4}{x - 2} + \frac{-3}{x - 1} + \frac{-1}{(x - 1)^2}$ 41. $\frac{x}{(x^2 + 16)^2} + \frac{-16x}{(x^2 + 16)^3}$

43. $\frac{-8}{2x + 1} + \frac{4}{x - 3}$ 45. $\frac{-2}{9} + \frac{-1}{3} + \frac{1}{6} + \frac{1}{18}$ 47. $x - 2 + \frac{10x - 11}{x^2 + 3x - 4}; \frac{51}{x + 4} + \frac{-1}{x - 1}; x - 2 + \frac{51}{x + 4} - \frac{1}{x - 1}$

49. $x - \frac{x}{x^2 + 1}$ 51. $x^2 - 4x + 7 + \frac{-11x - 32}{x^2 + 4x + 4}; \frac{-11}{x + 2} + \frac{-10}{(x + 2)^2}; x^2 - 4x + 7 - \frac{11}{x + 2} - \frac{10}{(x + 2)^2}$

53. $x + 1 + \frac{2x^3 + x^2 - x + 1}{x^4 - 2x^2 + 1}; \frac{1}{x + 1} + \frac{1}{(x + 1)^2} + \frac{1}{x - 1} + \frac{3}{(x - 1)^2}; x + 1 + \frac{1}{x + 1} + \frac{1}{(x + 1)^2} + \frac{1}{x - 1} + \frac{3}{(x - 1)^2}$

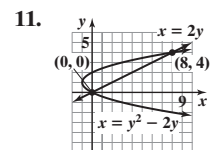
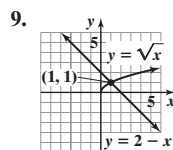
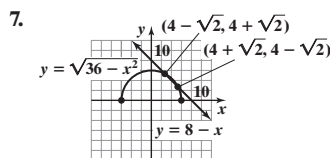
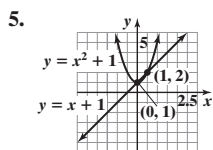
55. 3.85 years 56. -2 57. 1

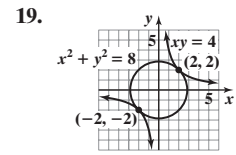
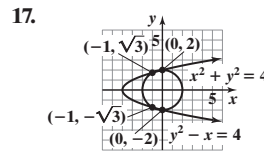
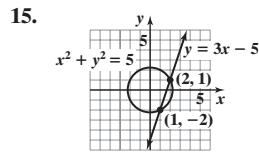
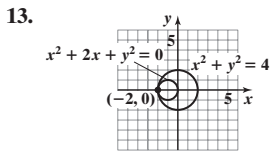


Historical Problem (page 796)

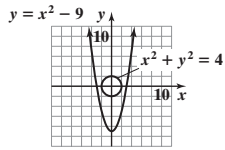
$x = 6$ units, $y = 8$ units

11.6 Assess Your Understanding (page 796)

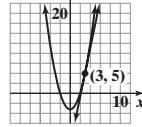




21. No points of intersection



23. $y = x^2 - 4$ $y = 6x - 13$



25. $x = 1, y = 4; x = -1, y = -4; x = 2\sqrt{2}, y = \sqrt{2}; x = -2\sqrt{2}, y = -\sqrt{2}$ or $(1, 4), (-1, -4), (2\sqrt{2}, \sqrt{2}), (-2\sqrt{2}, -\sqrt{2})$

27. $x = 0, y = 1; x = -\frac{2}{3}, y = -\frac{1}{3}$ or $(0, 1), (-\frac{2}{3}, -\frac{1}{3})$ 29. $x = 0, y = -1; x = \frac{5}{2}, y = -\frac{7}{2}$ or $(0, -1), (\frac{5}{2}, -\frac{7}{2})$

31. $x = 2, y = \frac{1}{3}; x = \frac{1}{2}, y = \frac{4}{3}$ or $(2, \frac{1}{3}), (\frac{1}{2}, \frac{4}{3})$ 33. $x = 3, y = 2; x = 3, y = -2; x = -3, y = 2; x = -3, y = -2$ or $(3, 2), (3, -2), (-3, 2), (-3, -2)$

35. $x = \frac{1}{2}, y = \frac{3}{2}; x = \frac{1}{2}, y = -\frac{3}{2}; x = -\frac{1}{2}, y = \frac{3}{2}; x = -\frac{1}{2}, y = -\frac{3}{2}$ or $(\frac{1}{2}, \frac{3}{2}), (\frac{1}{2}, -\frac{3}{2}), (-\frac{1}{2}, \frac{3}{2}), (-\frac{1}{2}, -\frac{3}{2})$

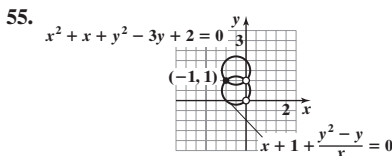
37. $x = \sqrt{2}, y = 2\sqrt{2}; x = -\sqrt{2}, y = -2\sqrt{2}$ or $(\sqrt{2}, 2\sqrt{2}), (-\sqrt{2}, -2\sqrt{2})$ 39. No real solution exists.

41. $x = \frac{8}{3}, y = \frac{2\sqrt{10}}{3}; x = -\frac{8}{3}, y = \frac{2\sqrt{10}}{3}; x = \frac{8}{3}, y = -\frac{2\sqrt{10}}{3}; x = -\frac{8}{3}, y = -\frac{2\sqrt{10}}{3}$ or $(\frac{8}{3}, \frac{2\sqrt{10}}{3}), (-\frac{8}{3}, \frac{2\sqrt{10}}{3}), (\frac{8}{3}, -\frac{2\sqrt{10}}{3}), (-\frac{8}{3}, -\frac{2\sqrt{10}}{3})$

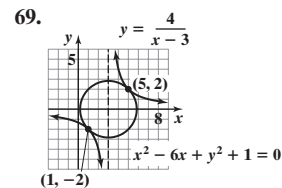
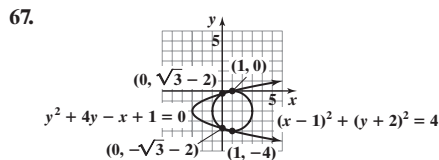
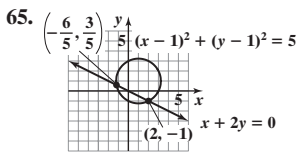
43. $x = 1, y = \frac{1}{2}; x = -1, y = \frac{1}{2}; x = 1, y = -\frac{1}{2}; x = -1, y = -\frac{1}{2}$ or $(1, \frac{1}{2}), (-1, \frac{1}{2}), (1, -\frac{1}{2}), (-1, -\frac{1}{2})$ 45. No real solution exists.

47. $x = \sqrt{3}, y = \sqrt{3}; x = -\sqrt{3}, y = -\sqrt{3}; x = 2, y = 1; x = -2, y = -1$ or $(\sqrt{3}, \sqrt{3}), (-\sqrt{3}, -\sqrt{3}), (2, 1), (-2, -1)$

49. $x = 0, y = -2; x = 0, y = 1; x = 2, y = -1$ or $(0, -2), (0, 1), (2, -1)$ 51. $x = 2, y = 8$ or $(2, 8)$ 53. $x = 81, y = 3$ or $(81, 3)$



57. $x = 0.48, y = 0.62$ 59. $x = -1.65, y = -0.89$
61. $x = 0.58, y = 1.86; x = 1.81, y = 1.05; x = 0.58, y = -1.86; x = 1.81, y = -1.05$
63. $x = 2.35, y = 0.85$



71. 3 and 1; -1 and -3 73. 2 and 2; -2 and -2 75. $\frac{1}{2}$ and $\frac{1}{3}$ 77. 5 79. 5 in. by 3 in. 81. 2 cm and 4 cm 83. Tortoise: 7 m/h, hare: $7\frac{1}{2}$ m/h

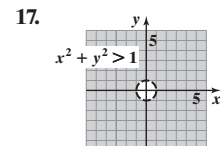
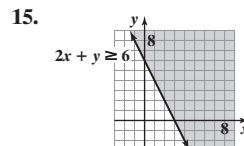
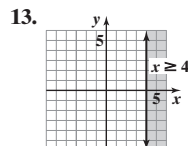
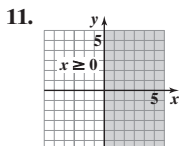
85. 12 cm by 18 cm 87. $x = 60$ ft; $y = 30$ ft 89. $l = \frac{P + \sqrt{P^2 - 16A}}{4}; w = \frac{P - \sqrt{P^2 - 16A}}{4}$ 91. $y = 4x - 4$ 93. $y = 2x + 1$

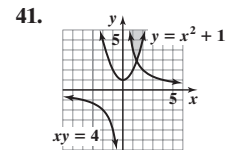
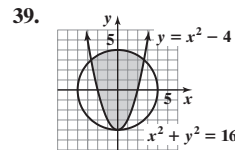
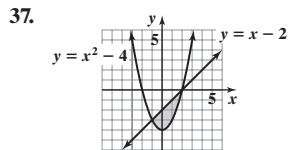
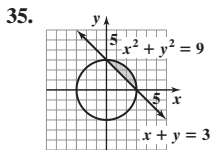
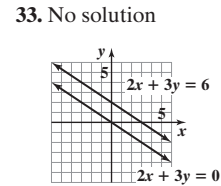
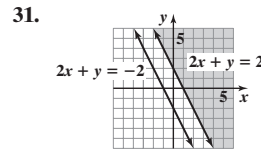
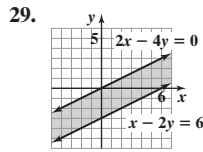
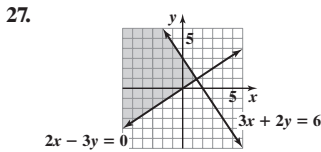
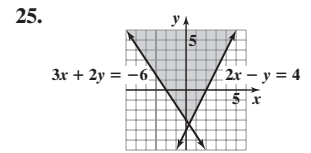
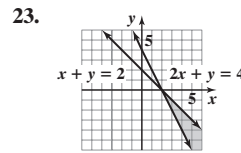
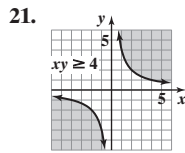
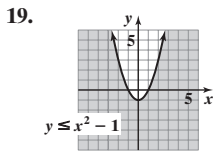
95. $y = -\frac{1}{3}x + \frac{7}{3}$ 97. $y = 2x - 3$ 99. $r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}; r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ 101. (a) 4.274 ft by 4.274 ft or 0.093 ft by 0.093 ft

102. $\frac{-3 - \sqrt{65}}{7}, \frac{-3 + \sqrt{65}}{7}$ 103. $y = -\frac{2}{5}x - 3$ 104. $\sin \theta = -\frac{7}{25}; \cos \theta = -\frac{24}{25}; \tan \theta = \frac{7}{24}; \csc \theta = -\frac{25}{7}; \sec \theta = -\frac{25}{24}$ 105. $\approx 15.8^\circ$

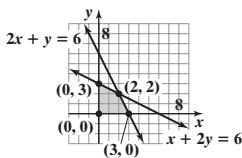
11.7 Assess Your Understanding (page 806)

7. dashes; solid 8. half-planes 9. F 10. unbounded

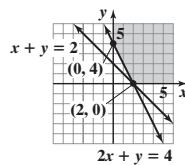




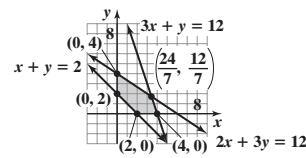
43. Bounded



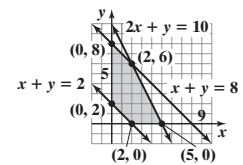
45. Unbounded



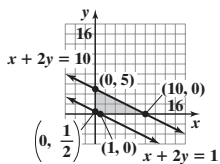
47. Bounded



49. Bounded



51. Bounded



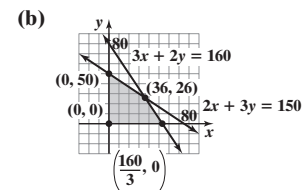
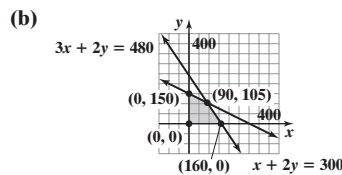
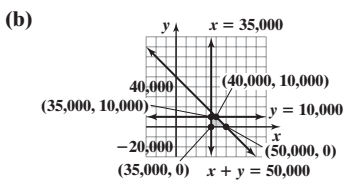
$$53. \begin{cases} x \leq 4 \\ x + y \leq 6 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

$$55. \begin{cases} x \leq 20 \\ y \geq 15 \\ x + y \leq 50 \\ x - y \leq 0 \\ x \geq 0 \end{cases}$$

57. (a)
$$\begin{cases} x + y \leq 50,000 \\ x \geq 35,000 \\ y \leq 10,000 \\ y \geq 0 \end{cases}$$

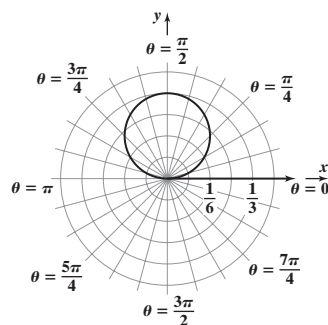
59. (a)
$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x + 2y \leq 300 \\ 3x + 2y \leq 480 \end{cases}$$

61. (a)
$$\begin{cases} 3x + 2y \leq 160 \\ 2x + 3y \leq 150 \\ x \geq 0 \\ y \geq 0 \end{cases}$$



62. 52

63. $x^2 + \left(y - \frac{1}{6}\right)^2 = \frac{1}{36}$;
circle with radius $\frac{1}{6}$ and center $\left(0, \frac{1}{6}\right)$;



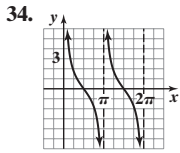
64. $f(-1) = -5$;
 $f(2) = 28$

65. $\left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

11.8 Assess Your Understanding (page 814)

1. objective function 2. T 3. Maximum value is 11; minimum value is 3. 5. Maximum value is 65; minimum value is 4.
7. Maximum value is 67; minimum value is 20. 9. The maximum value of z is 12, and it occurs at the point $(6, 0)$.
11. The minimum value of z is 4, and it occurs at the point $(2, 0)$. 13. The maximum value of z is 20, and it occurs at the point $(0, 4)$.
15. The minimum value of z is 8, and it occurs at the point $(0, 2)$. 17. The maximum value of z is 50, and it occurs at the point $(10, 0)$.
19. Produce 8 downhill and 24 cross-country; \$1760; \$1920 which is the profit when producing 16 downhill and 16 cross-country.

21. Rent 15 rectangular tables and 16 round tables for a minimum cost of \$1252. 23. (a) \$10,000 in a junk bond and \$10,000 in Treasury bills (b) \$12,000 in a junk bond and \$8000 in Treasury bills 25. 100 lb of ground beef should be mixed with 50 lb of pork.
 27. Manufacture 10 racing skates and 15 figure skates. 29. Order 2 metal samples and 4 plastic samples; \$34
 31. (a) Configure with 10 first-class seats and 120 coach seats. (b) Configure with 15 first-class seats and 120 coach seats.
 33. $\{-10\}$



Domain: $\{x | x \neq k\pi, k \text{ is an integer}\}$; range: $(-\infty, \infty)$

35. 89.1 years 36. $y = 3x + 7$

Review Exercises (page 818)

1. $x = 2, y = -1$ or $(2, -1)$ 2. $x = 2, y = \frac{1}{2}$ or $(2, \frac{1}{2})$ 3. $x = 2, y = -1$ or $(2, -1)$ 4. $x = \frac{11}{5}, y = -\frac{3}{5}$ or $(\frac{11}{5}, -\frac{3}{5})$ 5. Inconsistent
 6. $x = 2, y = 3$ or $(2, 3)$ 7. $y = -\frac{2}{5}x + 2$, where x is any real number, or $\{(x, y) | y = -\frac{2}{5}x + 2, x \text{ is any real number}\}$
 8. $x = -1, y = 2, z = -3$ or $(-1, 2, -3)$
 9. $x = \frac{7}{4}z + \frac{39}{4}, y = \frac{9}{8}z + \frac{69}{8}$, where z is any real number, or $\{(x, y, z) | x = \frac{7}{4}z + \frac{39}{4}, y = \frac{9}{8}z + \frac{69}{8}, z \text{ is any real number}\}$ 10. Inconsistent
 11. $\begin{cases} 3x + 2y = 8 \\ x + 4y = -1 \end{cases}$ 12. $\begin{cases} x + 2y + 5z = -2 \\ 5x - 3z = 8 \\ 2x - y = 0 \end{cases}$ 13. $\begin{bmatrix} 4 & -4 \\ 3 & 9 \\ 4 & 4 \end{bmatrix}$ 14. $\begin{bmatrix} 6 & 0 \\ 12 & 24 \\ -6 & 12 \end{bmatrix}$ 15. $\begin{bmatrix} 4 & -3 & 0 \\ 12 & -2 & -8 \\ -2 & 5 & -4 \end{bmatrix}$ 16. $\begin{bmatrix} 9 & -31 \\ -6 & -3 \end{bmatrix}$

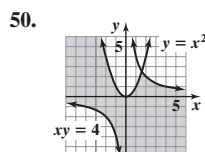
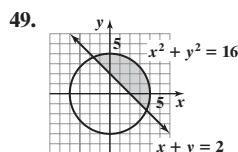
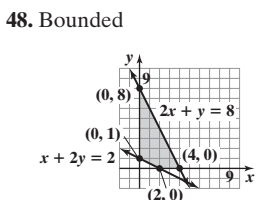
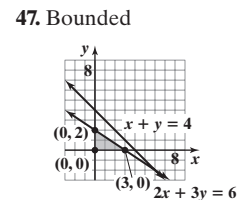
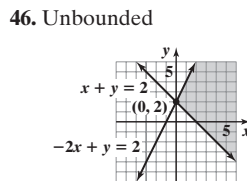
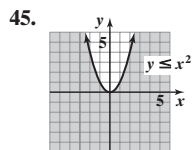
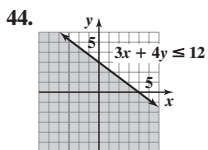
17. $\begin{bmatrix} \frac{1}{2} & -1 \\ -1 & \frac{2}{3} \end{bmatrix}$ 18. $\begin{bmatrix} -\frac{5}{7} & \frac{9}{7} & \frac{3}{7} \\ \frac{1}{7} & \frac{1}{7} & -\frac{2}{7} \\ \frac{3}{7} & -\frac{4}{7} & \frac{1}{7} \end{bmatrix}$ 19. Singular 20. $x = \frac{2}{5}, y = \frac{1}{10}$ or $(\frac{2}{5}, \frac{1}{10})$ 21. $x = 9, y = \frac{13}{3}, z = \frac{13}{3}$ or $(9, \frac{13}{3}, \frac{13}{3})$

22. Inconsistent 23. $x = -\frac{1}{2}, y = -\frac{2}{3}, z = -\frac{3}{4}$ or $(-\frac{1}{2}, -\frac{2}{3}, -\frac{3}{4})$
 24. $z = -1, x = y + 1$, where y is any real number, or $\{(x, y, z) | x = y + 1, z = -1, y \text{ is any real number}\}$
 25. $x = 4, y = 2, z = 3, t = -2$ or $(4, 2, 3, -2)$ 26. 5 27. 108 28. -100 29. $x = 2, y = -1$ or $(2, -1)$ 30. $x = 2, y = 3$ or $(2, 3)$

31. $x = -1, y = 2, z = -3$ or $(-1, 2, -3)$ 32. 16 33. -8 34. $\frac{-\frac{3}{2}}{x} + \frac{\frac{3}{2}}{x-4}$ 35. $\frac{-3}{x-1} + \frac{3}{x} + \frac{4}{x^2}$ 36. $\frac{-\frac{1}{10}}{x+1} + \frac{\frac{1}{10}x + \frac{9}{10}}{x^2 + 9}$

37. $\frac{x}{x^2 + 4} + \frac{-4x}{(x^2 + 4)^2}$ 38. $\frac{\frac{1}{2}}{x^2 + 1} + \frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}}{x+1}$ 39. $x = -\frac{2}{5}, y = -\frac{11}{5}$; $x = -2, y = 1$ or $(-\frac{2}{5}, -\frac{11}{5}), (-2, 1)$

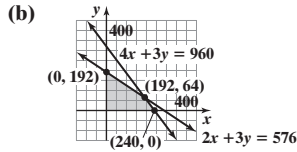
40. $x = 2\sqrt{2}, y = \sqrt{2}; x = -2\sqrt{2}, y = -\sqrt{2}$ or $(2\sqrt{2}, \sqrt{2}), (-2\sqrt{2}, -\sqrt{2})$
 41. $x = 0, y = 0; x = -3, y = 3; x = 3, y = 3$ or $(0, 0), (-3, 3), (3, 3)$
 42. $x = \sqrt{2}, y = -\sqrt{2}; x = -\sqrt{2}, y = \sqrt{2}; x = \frac{4}{3}\sqrt{2}, y = -\frac{2}{3}\sqrt{2}; x = -\frac{4}{3}\sqrt{2}, y = \frac{2}{3}\sqrt{2}$ or $(\sqrt{2}, -\sqrt{2}), (-\sqrt{2}, \sqrt{2}), (\frac{4}{3}\sqrt{2}, -\frac{2}{3}\sqrt{2}), (-\frac{4}{3}\sqrt{2}, \frac{2}{3}\sqrt{2})$ 43. $x = 1, y = -1$ or $(1, -1)$



51. The maximum value is 32 when $x = 0$ and $y = 8$. 52. The minimum value is 3 when $x = 1$ and $y = 0$. 53. 10 54. A is any real number, $A \neq 10$.
 55. $y = -\frac{1}{3}x^2 - \frac{2}{3}x + 1$ 56. Mix 70 lb of \$6.00 coffee and 30 lb of \$9.00 coffee. 57. Buy 1 small, 5 medium, and 2 large.

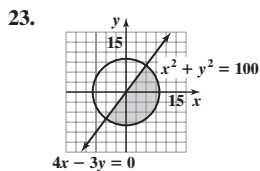
58. (a)
$$\begin{cases} x \geq 0 \\ y \geq 0 \\ 4x + 3y \leq 960 \\ 2x + 3y \leq 576 \end{cases}$$

59. Speedboat: 36.67 km/hr; Aguarico River: 3.33 km/hr
 60. Bruce: 4 hr; Bryce: 2 hr; Marty: 8 hr
 61. Produce 35 gasoline engines and 15 diesel engines; the factory is producing an excess of 15 gasoline engines and 0 diesel engines.



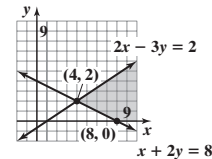
Chapter Test (page 820)

1. $x = 3, y = -1$ or $(3, -1)$ 2. Inconsistent 3. $x = -z + \frac{18}{7}, y = z - \frac{17}{7}$, where z is any real number, or
 $\left\{ (x, y, z) \mid x = -z + \frac{18}{7}, y = z - \frac{17}{7}, z \text{ is any real number} \right\}$ 4. $x = \frac{1}{3}, y = -2, z = 0$ or $(\frac{1}{3}, -2, 0)$ 5. $\begin{bmatrix} 4 & -5 & 1 & 0 \\ -2 & -1 & 0 & -25 \\ 1 & 5 & -5 & 10 \end{bmatrix}$
 6. $\begin{cases} 3x + 2y + 4z = -6 \\ 1x + 0y + 8z = 2 \\ -2x + 1y + 3z = -11 \end{cases}$ or $\begin{cases} 3x + 2y + 4z = -6 \\ x + 8z = 2 \\ -2x + y + 3z = -11 \end{cases}$ 7. $\begin{bmatrix} 6 & 4 \\ 1 & -11 \\ 5 & 12 \end{bmatrix}$ 8. $\begin{bmatrix} -11 & -19 \\ -3 & 5 \\ 6 & -22 \end{bmatrix}$ 9. $\begin{bmatrix} 4 & 10 & 26 \\ 1 & -11 & 2 \\ -1 & 26 & 3 \end{bmatrix}$ 10. $\begin{bmatrix} 16 & 17 \\ 3 & -10 \end{bmatrix}$
 11. $\begin{bmatrix} 2 & -1 \\ -\frac{5}{2} & \frac{3}{2} \end{bmatrix}$ 12. $\begin{bmatrix} 3 & 3 & -4 \\ -2 & -2 & 3 \\ -4 & -5 & 7 \end{bmatrix}$ 13. $x = \frac{1}{2}, y = 3$ or $(\frac{1}{2}, 3)$ 14. $x = -\frac{1}{4}y + 7$, where y is any real number, or
 $\left\{ (x, y) \mid x = -\frac{1}{4}y + 7, y \text{ is any real number} \right\}$ 15. $x = 1, y = -2, z = 0$ or $(1, -2, 0)$ 16. Inconsistent 17. -29 18. -12
 19. $x = -2, y = -5$ or $(-2, -5)$ 20. $x = 1, y = -1, z = 4$ or $(1, -1, 4)$ 21. $(1, -3)$ and $(1, 3)$ 22. $(3, 4)$ and $(1, 2)$



24. $\frac{3}{x+3} + \frac{-2}{(x+3)^2}$
 25. $\frac{-\frac{1}{3}}{x} + \frac{\frac{1}{3}x}{(x^2+3)} + \frac{5x}{(x^2+3)^2}$

26. Unbounded



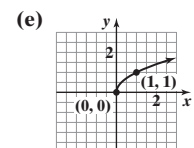
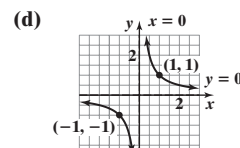
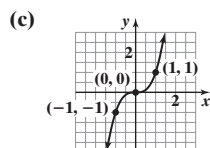
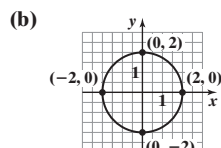
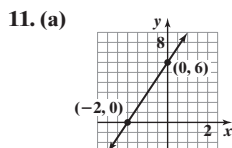
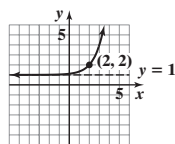
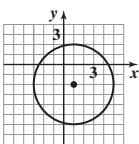
27. The maximum value of z is 64, and it occurs at the point $(0, 8)$. 28. Flare jeans cost \$24.50, camisoles cost \$8.50, and T-shirts cost \$6.00.

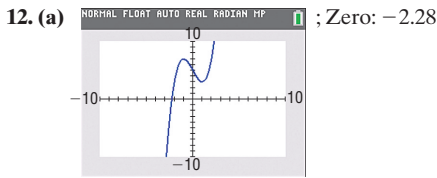
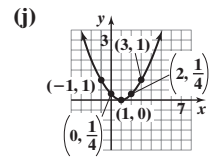
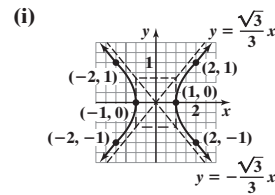
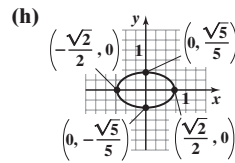
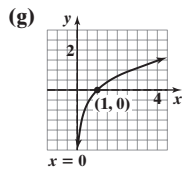
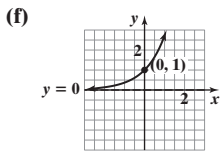
Cumulative Review (page 821)

1. $\{0, \frac{1}{2}\}$ 2. $\{5\}$ 3. $\{-1, -\frac{1}{2}, 3\}$ 4. $\{-2\}$ 5. $\{\frac{5}{2}\}$ 6. $\{\frac{1}{\ln 3}\}$ 7. Odd; symmetric with respect to the origin
 8. Center: $(1, -2)$; radius = 4

9. Domain: all real numbers
 Range: $\{y \mid y > 1\}$
 Horizontal asymptote: $y = 1$

10. $f^{-1}(x) = \frac{5}{x} - 2$
 Domain of f : $\{x \mid x \neq -2\}$
 Range of f : $\{y \mid y \neq 0\}$
 Domain of f^{-1} : $\{x \mid x \neq 0\}$
 Range of f^{-1} : $\{y \mid y \neq -2\}$





- (b) Local maximum of 7 at $x = -1$;
local minimum of 3 at $x = 1$
(c) $(-\infty, -1]$, $[1, \infty)$

CHAPTER 12 Sequences; Induction; the Binomial Theorem

12.1 Assess Your Understanding (page 833)

5. sequence 6. True 7. $n(n-1) \cdots 3 \cdot 2 \cdot 1$ 8. b 9. summation 10. b 11. 3,628,800 13. 504 15. 1260 17. $s_1 = 1, s_2 = 2, s_3 = 3, s_4 = 4, s_5 = 5$

19. $a_1 = \frac{1}{3}, a_2 = \frac{1}{2}, a_3 = \frac{3}{5}, a_4 = \frac{2}{3}, a_5 = \frac{5}{7}$ 21. $c_1 = 1, c_2 = -4, c_3 = 9, c_4 = -16, c_5 = 25$ 23. $s_1 = \frac{1}{2}, s_2 = \frac{2}{5}, s_3 = \frac{2}{7}, s_4 = \frac{8}{41}, s_5 = \frac{8}{61}$

25. $t_1 = -\frac{1}{6}, t_2 = \frac{1}{12}, t_3 = -\frac{1}{20}, t_4 = \frac{1}{30}, t_5 = -\frac{1}{42}$ 27. $b_1 = \frac{1}{e}, b_2 = \frac{2}{e^2}, b_3 = \frac{3}{e^3}, b_4 = \frac{4}{e^4}, b_5 = \frac{5}{e^5}$ 29. $a_n = \frac{n}{n+1}$ 31. $a_n = \frac{1}{2^{n-1}}$

33. $a_n = (-1)^{n+1}$ 35. $a_n = (-1)^{n+1}n$ 37. $a_1 = 2, a_2 = 5, a_3 = 8, a_4 = 11, a_5 = 14$ 39. $a_1 = -2, a_2 = 0, a_3 = 3, a_4 = 7, a_5 = 12$

41. $a_1 = 5, a_2 = 10, a_3 = 20, a_4 = 40, a_5 = 80$ 43. $a_1 = 3, a_2 = \frac{3}{2}, a_3 = \frac{1}{2}, a_4 = \frac{1}{8}, a_5 = \frac{1}{40}$ 45. $a_1 = 1, a_2 = 2, a_3 = 2, a_4 = 4, a_5 = 8$

47. $a_1 = A, a_2 = A + d, a_3 = A + 2d, a_4 = A + 3d, a_5 = A + 4d$

49. $a_1 = \sqrt{2}, a_2 = \sqrt{2 + \sqrt{2}}, a_3 = \sqrt{2 + \sqrt{2 + \sqrt{2}}}$ 51. $3 + 4 + \cdots + (n+2)$ 53. $\frac{1}{2} + 2 + \frac{9}{2} + \cdots + \frac{n^2}{2}$ 55. $1 + \frac{1}{3} + \frac{1}{9} + \cdots + \frac{1}{3^n}$
 $a_4 = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}}}$ 57. $\frac{1}{3} + \frac{1}{9} + \cdots + \frac{1}{3^n}$ 59. $\ln 2 - \ln 3 + \ln 4 - \cdots + (-1)^n \ln n$ 61. $\sum_{k=1}^{20} k$
 $a_5 = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2}}}}}$

63. $\sum_{k=1}^{13} \frac{k}{k+1}$ 65. $\sum_{k=0}^6 (-1)^k \left(\frac{1}{3}\right)^k$ 67. $\sum_{k=1}^n \frac{3^k}{k}$ 69. $\sum_{k=0}^n (a + kd)$ or $\sum_{k=1}^{n+1} [a + (k-1)d]$ 71. 200 73. 820 75. 1110 77. 1560 79. 3570

81. 44,000 83. (a) 2162 (b) After 26 months 85. (a) $A_0 = 1500, A_n = (1.0125)A_{n-1} + 750$ (b) After 99 quarters (c) \$213,073.11

87. (a) \$2930 (b) 14 payments have been made. (c) 36 payments; \$3584.62 (d) \$584.62

89. (a) $a_0 = 150,000, a_n = (1.005)a_{n-1} - 899.33$

(b) \$149,850.67 (c)

(d) After 58 payments or 4 years and 10 months later
(e) After 359 payments of \$899.33, plus last payment of \$895.10
(f) \$173,754.57

(g) (a) $a_0 = 150,000, a_n = (1.005)a_{n-1} - 999.33$

(b) \$149,750.67 (c)

(d) After 37 payments or 3 years and 1 month later
(e) After 278 payments of \$999.33, plus last payment of \$353.69 $(1.005) = \$355.46$
(f) \$128,169.20

91. 21 pairs 93. Fibonacci sequence 95. (a) 3.630170833 (b) 3.669060828 (c) 3.669296668 (d) 12

97. (a) $a_1 = 0.4; a_2 = 0.7; a_3 = 1; a_4 = 1.6; a_5 = 2.8; a_6 = 5.2; a_7 = 10; a_8 = 19.6$

(b) Except for term 5, which has no match, Bode's formula provides excellent approximations for the mean distances of the planets from the sun.

(c) The mean distance of Ceres from the sun is approximated by $a_5 = 2.8$, and that of Uranus is $a_8 = 19.6$.

(d) $a_9 = 38.8; a_{10} = 77.2$

(e) Pluto's distance is approximated by a_9 , but no term approximates Neptune's mean distance from the sun.

(f) According to Bode's Law, the mean orbital distance of Eris will be 154 AU from the sun.

99. $a_0 = 2; a_5 = 2.236067977; 2.236067977$ 101. $a_0 = 4; a_5 = 4.582575695; 4.582575695$ 103. 1, 3, 6, 10, 15, 21, 28

105. $u_n = 1 + 2 + 3 + \dots + n = \sum_{k=1}^n k = \frac{n(n+1)}{2}$, and from Problem 104, $u_{n-1} = \frac{(n+1)(n+2)}{2}$.

Thus, $u_{n+1} + u_n = \frac{(n+1)(n+2)}{2} + \frac{n(n+1)}{2} = \frac{(n+1)[(n+2) + n]}{2} = (n+1)^2$.

108. \$2654.39 109. $\sqrt{2}(\cos 225^\circ + i \sin 225^\circ)$ 110. 0 111. $(y-4)^2 = 16(x+3)$

12.2 Assess Your Understanding (page 842)

1. arithmetic 2. F 3. 17 4. T 5. d 6. c 7. $s_n - s_{n-1} = (n+4) - [(n-1) + 4] = n+4 - (n+3) = n+4 - n - 3 = 1$, a constant;
 $d = 1; s_1 = 5, s_2 = 6, s_3 = 7, s_4 = 8$

9. $a_n - a_{n-1} = (2n-5) - [2(n-1) - 5] = 2n-5 - (2n-2-5) = 2n-5 - (2n-7) = 2n-5 - 2n + 7 = 2$, a constant;
 $d = 2; a_1 = -3, a_2 = -1, a_3 = 1, a_4 = 3$

11. $c_n - c_{n-1} = (6-2n) - [6-2(n-1)] = 6-2n - (6-2n+2) = 6-2n - (8-2n) = 6-2n - 8 + 2n = -2$, a constant;
 $d = -2; c_1 = 4, c_2 = 2, c_3 = 0, c_4 = -2$

13. $t_n - t_{n-1} = \left(\frac{1}{2} - \frac{1}{3}n\right) - \left[\frac{1}{2} - \frac{1}{3}(n-1)\right] = \frac{1}{2} - \frac{1}{3}n - \left(\frac{1}{2} - \frac{1}{3}n + \frac{1}{3}\right) = \frac{1}{2} - \frac{1}{3}n - \left(\frac{5}{6} - \frac{1}{3}n\right) = \frac{1}{2} - \frac{1}{3}n - \frac{5}{6} + \frac{1}{3}n = -\frac{1}{3}$, a constant;
 $d = -\frac{1}{3}; t_1 = \frac{1}{6}, t_2 = -\frac{1}{6}, t_3 = -\frac{1}{2}, t_4 = -\frac{5}{6}$

15. $s_n - s_{n-1} = \ln 3^n - \ln 3^{n-1} = n \ln 3 - (n-1) \ln 3 = n \ln 3 - (n \ln 3 - \ln 3) = n \ln 3 - n \ln 3 + \ln 3 = \ln 3$, a constant;
 $d = \ln 3; s_1 = \ln 3, s_2 = 2 \ln 3, s_3 = 3 \ln 3, s_4 = 4 \ln 3$

17. $a_n = 3n - 1; a_{51} = 152$ 19. $a_n = 8 - 3n; a_{51} = -145$ 21. $a_n = \frac{1}{2}(n-1); a_{51} = 25$ 23. $a_n = \sqrt{2}n; a_{51} = 51\sqrt{2}$ 25. 200 27. -266 29. $\frac{83}{2}$

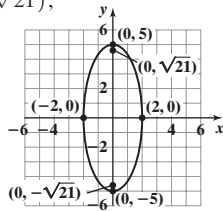
31. $a_1 = -13; d = 3; a_n = a_{n-1} + 3; a_n = -16 + 3n$ 33. $a_1 = -53; d = 6; a_n = a_{n-1} + 6; a_n = -59 + 6n$

35. $a_1 = 28; d = -2; a_n = a_{n-1} - 2; a_n = 30 - 2n$ 37. $a_1 = 25; d = -2; a_n = a_{n-1} - 2; a_n = 27 - 2n$ 39. n^2 41. $\frac{n}{2}(9+5n)$ 43. 1260 45. 324

47. 30,919 49. 10,036 51. 6080 53. -1925 55. 15,960 57. $-\frac{3}{2}$ 59. 24 terms 61. 1185 seats 63. 210 beige and 190 blue

65. $\{T_n\} = \{-5.5n + 67\}; T_5 = 39.5^\circ\text{F}$ 67. The amphitheater has 1647 seats. 69. 8 yr 72. 16.42% 73. $\mathbf{v} = 4\mathbf{i} - 6\mathbf{j}$

74. Ellipse: Center: (0, 0); Vertices: (0, -5), (0, 5); Foci: (0, $-\sqrt{21}$), (0, $\sqrt{21}$); 75. $\begin{bmatrix} \frac{1}{2} & 0 \\ \frac{3}{2} & -1 \end{bmatrix}$



Historical Problems (page 850)

1. $1\frac{2}{3}$ loaves, $10\frac{5}{6}$ loaves, 20 loaves, $29\frac{1}{6}$ loaves, $38\frac{1}{3}$ loaves 2. (a) 1 person (b) 2401 kittens (c) 2800

12.3 Assess Your Understanding (page 850)

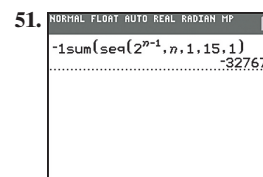
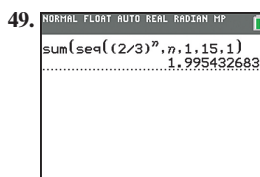
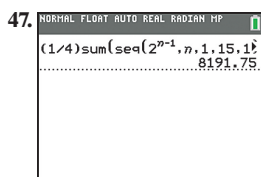
1. $a_n = a_1 \cdot r^{n-1}$ 2. $\frac{2}{3}$ 3. c 4. $\frac{a}{1-r}$ 5. b 6. T 7. F 8. T 9. $r = 3; s_1 = 3, s_2 = 9, s_3 = 27, s_4 = 81$

11. $r = \frac{1}{2}; a_1 = -\frac{3}{2}, a_2 = -\frac{3}{4}, a_3 = -\frac{3}{8}, a_4 = -\frac{3}{16}$ 13. $r = 2; c_1 = \frac{1}{4}, c_2 = \frac{1}{2}, c_3 = 1, c_4 = 2$ 15. $r = 2^{1/3}; e_1 = 2^{1/3}, e_2 = 2^{2/3}, e_3 = 2, e_4 = 2^{4/3}$

17. $r = \frac{3}{2}; t_1 = \frac{1}{2}, t_2 = \frac{3}{4}, t_3 = \frac{9}{8}, t_4 = \frac{27}{16}$ 19. $a_5 = 162; a_n = 2 \cdot 3^{n-1}$ 21. $a_5 = 5; a_n = 5 \cdot (-1)^{n-1}$ 23. $a_5 = 0; a_n = 0$

25. $a_5 = 4\sqrt{2}; a_n = (\sqrt{2})^n$ 27. $a_7 = \frac{1}{64}$ 29. $a_9 = 1$ 31. $a_8 = 0.00000004$ 33. $a_n = 7 \cdot 2^{n-1}$ 35. $a_n = -3 \cdot \left(-\frac{1}{3}\right)^{n-1} = \left(-\frac{1}{3}\right)^{n-2}$

37. $a_n = -(-3)^{n-1}$ 39. $a_n = \frac{7}{15}(15)^{n-1} = 7 \cdot 15^{n-2}$ 41. $-\frac{1}{4}(1-2^n)$ 43. $2\left[1 - \left(\frac{2}{3}\right)^n\right]$ 45. $1 - 2^n$



53. Converges; $\frac{3}{2}$ 55. Converges; 16 57. Converges; $\frac{8}{5}$ 59. Diverges 61. Converges; $\frac{20}{3}$ 63. Diverges 65. Converges; $\frac{18}{5}$ 67. Converges; 6

69. Arithmetic; $d = 1$; 1375 71. Neither 73. Arithmetic; $d = -\frac{2}{3}$; -700 75. Neither 77. Geometric; $r = \frac{2}{3}$; $2\left[1 - \left(\frac{2}{3}\right)^{50}\right]$

79. Geometric; $r = -2$; $-\frac{1}{3}[1 - (-2)^{50}]$ 81. Geometric; $r = 3^{1/2}$; $-\frac{\sqrt{3}}{2}(1 + \sqrt{3})(1 - 3^{25})$

83. -4 85. \$47,271.37 87. (a) 0.775 ft (b) 8th (c) 15.88 ft (d) 20 ft 89. \$349,496.41 91. \$96,885.98 93. \$305.10 95. 1.845×10^{19} 97. 10
99. \$72.67 per share 101. December 20, 2015; \$9999.92 103. \$5633.36 105. Option B results in more money (\$524,287 versus \$500,500).

107. Total pay: \$41,943.03; pay on day 22: \$20,971.52 112. 2.121 113. $\frac{8}{17}\mathbf{i} - \frac{15}{17}\mathbf{j}$ 114. $\frac{x^2}{4} - \frac{y^2}{12} = 1$ 115. 54

12.4 Assess Your Understanding (page 857)

1. (I) $n = 1$: $2(1) = 2$ and $1(1 + 1) = 2$

(II) If $2 + 4 + 6 + \dots + 2k = k(k + 1)$, then $2 + 4 + 6 + \dots + 2k + 2(k + 1) = (2 + 4 + 6 + \dots + 2k) + 2(k + 1)$
 $= k(k + 1) + 2(k + 1) = k^2 + 3k + 2 = (k + 1)(k + 2) = (k + 1)[(k + 1) + 1]$.

3. (I) $n = 1$: $1 + 2 = 3$ and $\frac{1}{2}(1)(1 + 5) = \frac{1}{2}(6) = 3$

(II) If $3 + 4 + 5 + \dots + (k + 2) = \frac{1}{2}k(k + 5)$, then $3 + 4 + 5 + \dots + (k + 2) + [(k + 1) + 2]$
 $= [3 + 4 + 5 + \dots + (k + 2)] + (k + 3) = \frac{1}{2}k(k + 5) + k + 3 = \frac{1}{2}(k^2 + 7k + 6) = \frac{1}{2}(k + 1)(k + 6) = \frac{1}{2}(k + 1)[(k + 1) + 5]$.

5. (I) $n = 1$: $3(1) - 1 = 2$ and $\frac{1}{2}(1)[3(1) + 1] = \frac{1}{2}(4) = 2$

(II) If $2 + 5 + 8 + \dots + (3k - 1) = \frac{1}{2}k(3k + 1)$, then $2 + 5 + 8 + \dots + (3k - 1) + [3(k + 1) - 1]$
 $= [2 + 5 + 8 + \dots + (3k - 1)] + (3k + 2) = \frac{1}{2}k(3k + 1) + (3k + 2) = \frac{1}{2}(3k^2 + 7k + 4) = \frac{1}{2}(k + 1)(3k + 4)$
 $= \frac{1}{2}(k + 1)[3(k + 1) + 1]$.

7. (I) $n = 1$: $2^{1-1} = 1$ and $2^1 - 1 = 1$

(II) If $1 + 2 + 2^2 + \dots + 2^{k-1} = 2^k - 1$, then $1 + 2 + 2^2 + \dots + 2^{k-1} + 2^{(k+1)-1} = (1 + 2 + 2^2 + \dots + 2^{k-1}) + 2^k$
 $= 2^k - 1 + 2^k = 2(2^k) - 1 = 2^{k+1} - 1$.

9. (I) $n = 1$: $4^{1-1} = 1$ and $\frac{1}{3}(4^1 - 1) = \frac{1}{3}(3) = 1$

(II) If $1 + 4 + 4^2 + \dots + 4^{k-1} = \frac{1}{3}(4^k - 1)$, then $1 + 4 + 4^2 + \dots + 4^{k-1} + 4^{(k+1)-1} = (1 + 4 + 4^2 + \dots + 4^{k-1}) + 4^k$
 $= \frac{1}{3}(4^k - 1) + 4^k = \frac{1}{3}[4^k - 1 + 3(4^k)] = \frac{1}{3}[4(4^k) - 1] = \frac{1}{3}(4^{k+1} - 1)$.

11. (I) $n = 1$: $\frac{1}{1 \cdot 2} = \frac{1}{2}$ and $\frac{1}{1 + 1} = \frac{1}{2}$

(II) If $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{k(k + 1)} = \frac{k}{k + 1}$, then $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{k(k + 1)} + \frac{1}{(k + 1)[(k + 1) + 1]}$
 $= \left[\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{k(k + 1)} \right] + \frac{1}{(k + 1)(k + 2)} = \frac{k}{k + 1} + \frac{1}{(k + 1)(k + 2)} = \frac{k(k + 2) + 1}{(k + 1)(k + 2)}$
 $= \frac{k^2 + 2k + 1}{(k + 1)(k + 2)} = \frac{(k + 1)^2}{(k + 1)(k + 2)} = \frac{k + 1}{k + 2} = \frac{k + 1}{(k + 1) + 1}$.

13. (I) $n = 1$: $1^2 = 1$ and $\frac{1}{6} \cdot 1 \cdot 2 \cdot 3 = 1$

(II) If $1^2 + 2^2 + 3^2 + \dots + k^2 = \frac{1}{6}k(k + 1)(2k + 1)$, then $1^2 + 2^2 + 3^2 + \dots + k^2 + (k + 1)^2$
 $= (1^2 + 2^2 + 3^2 + \dots + k^2) + (k + 1)^2 = \frac{1}{6}k(k + 1)(2k + 1) + (k + 1)^2 = \frac{1}{6}(2k^3 + 9k^2 + 13k + 6)$
 $= \frac{1}{6}(k + 1)(k + 2)(2k + 3) = \frac{1}{6}(k + 1)[(k + 1) + 1][2(k + 1) + 1]$.

15. (I) $n = 1$: $5 - 1 = 4$ and $\frac{1}{2}(1)(9 - 1) = \frac{1}{2} \cdot 8 = 4$

(II) If $4 + 3 + 2 + \dots + (5 - k) = \frac{1}{2}k(9 - k)$, then $4 + 3 + 2 + \dots + (5 - k) + [5 - (k + 1)]$
 $= [4 + 3 + 2 + \dots + (5 - k)] + 4 - k = \frac{1}{2}k(9 - k) + 4 - k = \frac{1}{2}(9k - k^2 + 8 - 2k) = \frac{1}{2}(-k^2 + 7k + 8)$
 $= \frac{1}{2}(k + 1)(8 - k) = \frac{1}{2}(k + 1)[9 - (k + 1)]$.

17. (I) $n = 1: 1 \cdot (1 + 1) = 2$ and $\frac{1}{3} \cdot 1 \cdot 2 \cdot 3 = 2$

(II) If $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + k(k + 1) = \frac{1}{3}k(k + 1)(k + 2)$, then $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + k(k + 1) + (k + 1)[(k + 1) + 1] = [1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + k(k + 1)] + (k + 1)(k + 2)$
 $= \frac{1}{3}k(k + 1)(k + 2) + \frac{1}{3} \cdot 3(k + 1)(k + 2) = \frac{1}{3}(k + 1)(k + 2)(k + 3) = \frac{1}{3}(k + 1)[(k + 1) + 1][(k + 1) + 2].$

19. (I) $n = 1: 1^2 + 1 = 2$, which is divisible by 2.

(II) If $k^2 + k$ is divisible by 2, then $(k + 1)^2 + (k + 1) = k^2 + 2k + 1 + k + 1 = (k^2 + k) + 2k + 2$. Since $k^2 + k$ is divisible by 2 and $2k + 2$ is divisible by 2, $(k + 1)^2 + (k + 1)$ is divisible by 2.

21. (I) $n = 1: 1^2 - 1 + 2 = 2$, which is divisible by 2.

(II) If $k^2 - k + 2$ is divisible by 2, then $(k + 1)^2 - (k + 1) + 2 = k^2 + 2k + 1 - k - 1 + 2 = (k^2 - k + 2) + 2k$. Since $k^2 - k + 2$ is divisible by 2 and $2k$ is divisible by 2, $(k + 1)^2 - (k + 1) + 2$ is divisible by 2.

23. (I) $n = 1$: If $x > 1$, then $x^1 = x > 1$.

(II) Assume, for an arbitrary natural number k , that if $x > 1$ then $x^k > 1$. Multiply both sides of the inequality $x^k > 1$ by x . If $x > 1$, then $x^{k+1} > x > 1$.

25. (I) $n = 1: a - b$ is a factor of $a^1 - b^1 = a - b$.

(II) If $a - b$ is a factor of $a^k - b^k$, then $a^{k+1} - b^{k+1} = a(a^k - b^k) + b^k(a - b)$.

Since $a - b$ is a factor of $a^k - b^k$ and $a - b$ is a factor of $a - b$, then $a - b$ is a factor of $a^{k+1} - b^{k+1}$.

27. (a) $n = 1: (1 + a)^1 = 1 + a \geq 1 + 1 \cdot a$

(b) Assume that there is an integer k for which the inequality holds. So $(1 + a)^k \geq 1 + ka$. We need to show that $(1 + a)^{k+1} \geq 1 + (k + 1)a$.

$(1 + a)^{k+1} = (1 + a)^k(1 + a) \geq (1 + ka)(1 + a) = 1 + ka^2 + a + ka = 1 + (k + 1)a + ka^2 \geq 1 + (k + 1)a.$

29. If $2 + 4 + 6 + \cdots + 2k = k^2 + k + 2$, then $2 + 4 + 6 + \cdots + 2k + 2(k + 1)$

$= (2 + 4 + 6 + \cdots + 2k) + 2k + 2 = k^2 + k + 2 + 2k + 2 = k^2 + 3k + 4 = (k^2 + 2k + 1) + (k + 1) + 2$

$= (k + 1)^2 + (k + 1) + 2.$

But $2 \cdot 1 = 2$ and $1^2 + 1 + 2 = 4$. The fact is that $2 + 4 + 6 + \cdots + 2n = n^2 + n$, not $n^2 + n + 2$ (Problem 1).

31. (I) $n = 1: [a + (1 - 1)d] = a$ and $1 \cdot a + d \frac{1 \cdot (1 - 1)}{2} = a.$

(II) If $a + (a + d) + (a + 2d) + \cdots + [a + (k - 1)d] = ka + d \frac{k(k - 1)}{2}$, then

$a + (a + d) + (a + 2d) + \cdots + [a + (k - 1)d] + [a + ((k + 1) - 1)d] = ka + d \frac{k(k - 1)}{2} + a + kd$

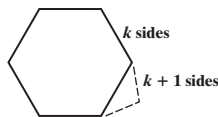
$= (k + 1)a + d \frac{k(k - 1) + 2k}{2} = (k + 1)a + d \frac{(k + 1)(k)}{2} = (k + 1)a + d \frac{(k + 1)[(k + 1) - 1]}{2}.$

33. (I) $n = 3$: The sum of the angles of a triangle is $(3 - 2) \cdot 180^\circ = 180^\circ$.

(II) Assume that for some $k \geq 3$, the sum of the angles of a convex polygon of k sides is $(k - 2) \cdot 180^\circ$.

A convex polygon of $k + 1$ sides consists of a convex polygon of k sides plus a triangle (see the illustration).

The sum of the angles is $(k - 2) \cdot 180^\circ + 180^\circ = (k - 1) \cdot 180^\circ = [(k + 1) - 2] \cdot 180^\circ.$



35. {251} 36. Left: 448.3 kg; right: 366.0 kg 37. $x = \frac{1}{2}, y = -3; (\frac{1}{2}, -3)$ 38. $\begin{bmatrix} 7 & -3 \\ -7 & 8 \end{bmatrix}$

12.5 Assess Your Understanding (page 863)

1. Pascal triangle 2. 1; n 3. F 4. Binomial Theorem 5. 10 7. 21 9. 50 11. 1 13. $\approx 1.8664 \times 10^{15}$ 15. $\approx 1.4834 \times 10^{13}$

17. $x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$ 19. $x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64$ 21. $81x^4 + 108x^3 + 54x^2 + 12x + 1$

23. $x^{10} + 5x^8y^2 + 10x^6y^4 + 10x^4y^6 + 5x^2y^8 + y^{10}$ 25. $x^3 + 6\sqrt{2}x^{5/2} + 30x^2 + 40\sqrt{2}x^{3/2} + 60x + 24\sqrt{2}x^{1/2} + 8$

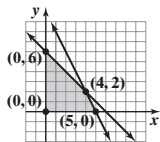
27. $a^5x^5 + 5a^4bx^4y + 10a^3b^2x^3y^2 + 10a^2b^3x^2y^3 + 5ab^4xy^4 + b^5y^5$ 29. 17010 31. -101,376 33. 41,472 35. $2835x^3$ 37. $314,928x^7$ 39. 495

41. 3360 43. 1.00501 45. $\binom{n}{n-1} = \frac{n!}{(n-1)! [n - (n-1)]!} = \frac{n!}{(n-1)! 1!} = \frac{n \cdot (n-1)!}{(n-1)!} = n; \binom{n}{n} = \frac{n!}{n! (n-n)!} = \frac{n!}{n! 0!} = \frac{n!}{n!} = 1$

47. $2^n = (1 + 1)^n = \binom{n}{0}1^n + \binom{n}{1}(1)^{n-1}(1) + \cdots + \binom{n}{n}1^n = \binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n}$ 49. 1

51. $\left\{ \frac{\ln 5}{\ln 6 - \ln 5} \right\} \approx \{8.827\}$ 52. (a) 0 (b) 90° (c) Orthogonal 53. $x = 1, y = 3, z = -2; (1, 3, -2)$

54. Bounded



Review Exercises (page 865)

1. $a_1 = -\frac{4}{3}, a_2 = \frac{5}{4}, a_3 = -\frac{6}{5}, a_4 = \frac{7}{6}, a_5 = -\frac{8}{7}$ 2. $c_1 = 2, c_2 = 1, c_3 = \frac{8}{9}, c_4 = 1, c_5 = \frac{32}{25}$ 3. $a_1 = 3, a_2 = 2, a_3 = \frac{4}{3}, a_4 = \frac{8}{9}, a_5 = \frac{16}{27}$
 4. $a_1 = 2, a_2 = 0, a_3 = 2, a_4 = 0, a_5 = 2$ 5. $6 + 10 + 14 + 18 = 48$ 6. $\sum_{k=1}^{13} (-1)^{k+1} \frac{1}{k}$ 7. Arithmetic; $d = 1; S_n = \frac{n}{2}(n + 1)$ 8. Neither
 9. Geometric; $r = 8; S_n = \frac{8}{7}(8^n - 1)$ 10. Arithmetic; $d = 4; S_n = 2n(n - 1)$ 11. Geometric; $r = \frac{1}{2}; S_n = 6\left[1 - \left(\frac{1}{2}\right)^n\right]$ 12. Neither
 13. 9515 14. -1320 15. $\frac{1093}{2187} \approx 0.49977$ 16. 682 17. 35 18. $\frac{1}{10^{10}}$ 19. $9\sqrt{2}$ 20. $\{a_n\} = \{5n - 4\}$ 21. $\{a_n\} = \{n - 10\}$ 22. Converges; $\frac{9}{2}$
 23. Converges; $\frac{4}{3}$ 24. Diverges 25. Converges; 8
 26. (I) $n = 1: 3 \cdot 1 = 3$ and $\frac{3 \cdot 1}{2}(1 + 1) = 3$

(II) If $3 + 6 + 9 + \dots + 3k = \frac{3k}{2}(k + 1)$, then $3 + 6 + 9 + \dots + 3k + 3(k + 1) = (3 + 6 + 9 + \dots + 3k) + (3k + 3)$

$$= \frac{3k}{2}(k + 1) + (3k + 3) = \frac{3k^2}{2} + \frac{3k}{2} + \frac{6k}{2} + \frac{6}{2} = \frac{3}{2}(k^2 + 3k + 2) = \frac{3}{2}(k + 1)(k + 2) = \frac{3(k + 1)}{2}[(k + 1) + 1].$$

27. (I) $n = 1: 2 \cdot 3^{1-1} = 2$ and $3^1 - 1 = 2$

(II) If $2 + 6 + 18 + \dots + 2 \cdot 3^{k-1} = 3^k - 1$, then $2 + 6 + 18 + \dots + 2 \cdot 3^{k-1} + 2 \cdot 3^{(k+1)-1} = (2 + 6 + 18 + \dots + 2 \cdot 3^{k-1}) + 2 \cdot 3^k = 3^k - 1 + 2 \cdot 3^k = 3 \cdot 3^k - 1 = 3^{k+1} - 1$.

28. (I) $n = 1: (3 \cdot 1 - 2)^2 = 1$ and $\frac{1}{2} \cdot 1 \cdot [6(1)^2 - 3(1) - 1] = 1$

(II) If $1^2 + 4^2 + 7^2 + \dots + (3k - 2)^2 = \frac{1}{2}k(6k^2 - 3k - 1)$, then $1^2 + 4^2 + 7^2 + \dots + (3k - 2)^2 + [3(k + 1) - 2]^2$

$$= [1^2 + 4^2 + 7^2 + \dots + (3k - 2)^2] + (3k + 1)^2 = \frac{1}{2}k(6k^2 - 3k - 1) + (3k + 1)^2 = \frac{1}{2}(6k^3 - 3k^2 - k) + (9k^2 + 6k + 1)$$

$$= \frac{1}{2}(6k^3 + 15k^2 + 11k + 2) = \frac{1}{2}(k + 1)(6k^2 + 9k + 2) = \frac{1}{2}(k + 1)[6(k + 1)^2 - 3(k + 1) - 1].$$

29. 10 30. $x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$ 31. $81x^4 - 432x^3 + 864x^2 - 768x + 256$ 32. 144 33. 84

34. (a) 8 bricks (b) 1100 bricks 35. 360 36. (a) $20\left(\frac{3}{4}\right)^3 = \frac{135}{16}$ ft (b) $20\left(\frac{3}{4}\right)^n$ ft (c) 13 times (d) 140 ft 37. \$151,873.77 38. \$58,492.93

Chapter Test (page 867)

1. $0, \frac{3}{10}, \frac{8}{11}, \frac{5}{4}, \frac{24}{13}$ 2. 4, 14, 44, 134, 404 3. $2 - \frac{3}{4} + \frac{4}{9} = \frac{61}{36}$ 4. $-\frac{1}{3} - \frac{14}{9} - \frac{73}{27} - \frac{308}{81} = -\frac{680}{81}$ 5. $\sum_{k=1}^{10} (-1)^k \left(\frac{k+1}{k+4}\right)$ 6. Neither

7. Geometric; $r = 4; S_n = \frac{2}{3}(1 - 4^n)$ 8. Arithmetic; $d = -8; S_n = n(2 - 4n)$ 9. Arithmetic; $d = -\frac{1}{2}; S_n = \frac{n}{4}(27 - n)$

10. Geometric; $r = \frac{2}{5}; S_n = \frac{125}{3}\left[1 - \left(\frac{2}{5}\right)^n\right]$ 11. Neither 12. Converges; $\frac{1024}{5}$ 13. $243m^5 + 810m^4 + 1080m^3 + 720m^2 + 240m + 32$

14. First we show that the statement holds for $n = 1$. $\left(1 + \frac{1}{1}\right) = 1 + 1 = 2$. The equality is true for $n = 1$, so Condition I holds. Next we assume that

$$\left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right) \cdots \left(1 + \frac{1}{n}\right) = n + 1$$

is true for some k , and we determine whether the formula then holds for $k + 1$. We assume that

$$\left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right) \cdots \left(1 + \frac{1}{k}\right) = k + 1.$$

Now we need to show that $\left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right) \cdots \left(1 + \frac{1}{k}\right)\left(1 + \frac{1}{k+1}\right)$

$= (k + 1) + 1 = k + 2$. We do this as follows:

$$\left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right) \cdots \left(1 + \frac{1}{k}\right)\left(1 + \frac{1}{k+1}\right) = \left[\left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right) \cdots \left(1 + \frac{1}{k}\right)\right]\left(1 + \frac{1}{k+1}\right)$$

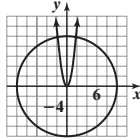
$$= (k + 1)\left(1 + \frac{1}{k+1}\right) \text{ (induction assumption)} = (k + 1) \cdot 1 + (k + 1) \cdot \frac{1}{k+1} = k + 1 + 1 = k + 2$$

Condition II also holds. Thus, the formula holds true for all natural numbers.

15. After 10 years, the Durango will be worth \$6103.11. 16. The weightlifter will have lifted a total of 8000 pounds after 5 sets.

Cumulative Review (page 867)

1. $\{-3, 3, -3i, 3i\}$ 2. (a)



(b) $\left\{ \left(\sqrt{\frac{-1 + \sqrt{3601}}{18}}, \frac{-1 + \sqrt{3601}}{6} \right), \left(-\sqrt{\frac{-1 + \sqrt{3601}}{18}}, \frac{-1 + \sqrt{3601}}{6} \right) \right\}$

(c) The circle and the parabola intersect at

$\left(\sqrt{\frac{-1 + \sqrt{3601}}{18}}, \frac{-1 + \sqrt{3601}}{6} \right), \left(-\sqrt{\frac{-1 + \sqrt{3601}}{18}}, \frac{-1 + \sqrt{3601}}{6} \right).$

3. $\left\{ \ln\left(\frac{5}{2}\right) \right\}$ 4. $y = 5x - 10$ 5. $(x + 1)^2 + (y - 2)^2 = 25$ 6. (a) 5 (b) 13 (c) $\frac{6x + 3}{2x - 1}$ (d) $\left\{ x \mid x \neq \frac{1}{2} \right\}$ (e) $\frac{7x - 2}{x - 2}$ (f) $\{x \mid x \neq 2\}$

(g) $g^{-1}(x) = \frac{1}{2}(x - 1)$; all reals (h) $f^{-1}(x) = \frac{2x}{x - 3}$; $\{x \mid x \neq 3\}$ 7. $\frac{x^2}{7} + \frac{y^2}{16} = 1$ 8. $(x + 1)^2 = 4(y - 2)$

9. $r = 8 \sin \theta$; $x^2 + (y - 4)^2 = 16$ 10. $\left\{ \frac{3\pi}{2} \right\}$ 11. $\frac{2\pi}{3}$ 12. (a) $-\frac{\sqrt{15}}{4}$ (b) $-\frac{\sqrt{15}}{15}$ (c) $-\frac{\sqrt{15}}{8}$ (d) $\frac{7}{8}$ (e) $\sqrt{\frac{1 + \frac{\sqrt{15}}{4}}{2}} = \frac{\sqrt{4 + \sqrt{15}}}{2\sqrt{2}}$

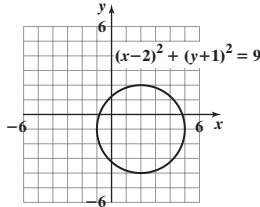
CHAPTER 13 Counting and Probability

13.1 Assess Your Understanding (page 874)

5. subset; \subseteq 6. finite 7. $n(A) + n(B) - n(A \cap B)$ 8. T 9. $\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{a, b, c, d\}$ 11. 25 13. 40 15. 25 17. 37 19. 18 21. 5 23. 15 different arrangements 25. 9000 numbers

27. 175; 125 29. (a) 15 (b) 15 (c) 15 (d) 25 (e) 40 31. (a) 13.8 million (b) 79.5 million 33. 480 portfolios

36. 37. $A \approx 41.4^\circ, B \approx 41.4^\circ, C \approx 97.2^\circ$ 38. 2, 5, -2 39. $\left\{ \frac{1}{18} \right\}$



13.2 Assess Your Understanding (page 881)

3. permutation 4. combination 5. $\frac{n!}{(n-r)!}$ 6. $\frac{n!}{(n-r)!r!}$ 7. 30 9. 24 11. 1 13. 1680 15. 28 17. 35 19. 1 21. 10,400,600

23. $\{abc, abd, abe, acb, acd, ace, adb, adc, ade, aeb, aec, aed, bac, bad, bae, bca, bcd, bce, bda, bdc, bde, bea, bec, bed, cab, cad, cae, cba, cbd, cbe, cda, cdb, cde, cea, ceb, ced, dab, dac, dae, dba, dbc, dbe, dca, dcb, dce, dea, deb, dec, eab, eac, ead, eba, ebc, ebd, eca, ecb, ecd, eda, edb, edc\}$; 60

25. $\{123, 124, 132, 134, 142, 143, 213, 214, 231, 234, 241, 243, 312, 314, 321, 324, 341, 342, 412, 413, 421, 423, 431, 432\}$; 24

27. $\{abc, abd, abe, acd, ace, ade, bcd, bce, bde, cde\}$; 10 29. $\{123, 124, 134, 234\}$; 4 31. 16 33. 8 35. 24 37. 60 39. 18,278 41. 35 43. 1024

45. 120 47. 132,860 49. 336 51. 90,720 53. (a) 63 (b) 35 (c) 1 55. 1.157×10^{76} 57. 362,880 59. 660 61. 15

63. (a) 125,000; 117,600 (b) A better name for a combination lock would be a permutation lock because the order of the numbers matters.

67. 10 sq. ft 68. $(g \circ f)(x) = 4x^2 - 2x - 2$ 69. $\sin 75^\circ = \frac{\sqrt{2} + \sqrt{6}}{4}$; $\cos 15^\circ = \frac{1}{2}\sqrt{2 + \sqrt{3}}$ or $\cos 15^\circ = \frac{\sqrt{2} + \sqrt{6}}{4}$ 70. $a_5 = 80$

Historical Problem (page 891)

1. (a) $\{AAAA, AAAB, AABA, AABB, ABAA, ABAB, ABBA, ABBA, BAAA, BAAB, BABA, BABB, BBAA, BBAB, BBBA, BBBB\}$

(b) $P(A \text{ wins}) = \frac{C(4,2) + C(4,3) + C(4,4)}{2^4} = \frac{6 + 4 + 1}{16} = \frac{11}{16}$; $P(B \text{ wins}) = \frac{C(4,3) + C(4,4)}{2^4} = \frac{4 + 1}{16} = \frac{5}{16}$

13.3 Assess Your Understanding (page 891)

1. equally likely 2. complement 3. F 4. T 5. 0, 0.01, 0.35, 1 7. Probability model 9. Not a probability model

11. $S = \{HH, HT, TH, TT\}$; $P(HH) = \frac{1}{4}$, $P(HT) = \frac{1}{4}$, $P(TH) = \frac{1}{4}$, $P(TT) = \frac{1}{4}$

13. $S = \{HH1, HH2, HH3, HH4, HH5, HH6, HT1, HT2, HT3, HT4, HT5, HT6, TH1, TH2, TH3, TH4, TH5, TH6, TT1, TT2, TT3, TT4, TT5, TT6\}$; each outcome has the probability of $\frac{1}{24}$.

15. $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$; each outcome has the probability of $\frac{1}{8}$.

17. $S = \{1 \text{ Yellow, 1 Red, 1 Green, 2 Yellow, 2 Red, 2 Green, 3 Yellow, 3 Red, 3 Green, 4 Yellow, 4 Red, 4 Green}\}$; each outcome has the probability of $\frac{1}{12}$; thus, $P(2 \text{ Red}) + P(4 \text{ Red}) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$.

19. $S = \{1 \text{ Yellow Forward}, 1 \text{ Yellow Backward}, 1 \text{ Red Forward}, 1 \text{ Red Backward}, 1 \text{ Green Forward}, 1 \text{ Green Backward}, 2 \text{ Yellow Forward}, 2 \text{ Yellow Backward}, 2 \text{ Red Forward}, 2 \text{ Red Backward}, 2 \text{ Green Forward}, 2 \text{ Green Backward}, 3 \text{ Yellow Forward}, 3 \text{ Yellow Backward}, 3 \text{ Red Forward}, 3 \text{ Red Backward}, 3 \text{ Green Forward}, 3 \text{ Green Backward}, 4 \text{ Yellow Forward}, 4 \text{ Yellow Backward}, 4 \text{ Red Forward}, 4 \text{ Red Backward}, 4 \text{ Green Forward}, 4 \text{ Green Backward}\}$; each outcome has the probability of $\frac{1}{24}$; thus, $P(1 \text{ Red Backward}) + P(1 \text{ Green Backward}) = \frac{1}{24} + \frac{1}{24} = \frac{1}{12}$.

21. $S = \{11 \text{ Red}, 11 \text{ Yellow}, 11 \text{ Green}, 12 \text{ Red}, 12 \text{ Yellow}, 12 \text{ Green}, 13 \text{ Red}, 13 \text{ Yellow}, 13 \text{ Green}, 14 \text{ Red}, 14 \text{ Yellow}, 14 \text{ Green}, 21 \text{ Red}, 21 \text{ Yellow}, 21 \text{ Green}, 22 \text{ Red}, 22 \text{ Yellow}, 22 \text{ Green}, 23 \text{ Red}, 23 \text{ Yellow}, 23 \text{ Green}, 24 \text{ Red}, 24 \text{ Yellow}, 24 \text{ Green}, 31 \text{ Red}, 31 \text{ Yellow}, 31 \text{ Green}, 32 \text{ Red}, 32 \text{ Yellow}, 32 \text{ Green}, 33 \text{ Red}, 33 \text{ Yellow}, 33 \text{ Green}, 34 \text{ Red}, 34 \text{ Yellow}, 34 \text{ Green}, 41 \text{ Red}, 41 \text{ Yellow}, 41 \text{ Green}, 42 \text{ Red}, 42 \text{ Yellow}, 42 \text{ Green}, 43 \text{ Red}, 43 \text{ Yellow}, 43 \text{ Green}, 44 \text{ Red}, 44 \text{ Yellow}, 44 \text{ Green}\}$; each outcome has the probability of $\frac{1}{48}$; thus, $E = \{22 \text{ Red}, 22 \text{ Green}, 24 \text{ Red}, 24 \text{ Green}\}$;

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{48} = \frac{1}{12}$$

23. A, B, C, F 25. B 27. $P(H) = \frac{4}{5}$; $P(T) = \frac{1}{5}$ 29. $P(1) = P(3) = P(5) = \frac{2}{9}$; $P(2) = P(4) = P(6) = \frac{1}{9}$ 31. $\frac{3}{10}$ 33. $\frac{1}{2}$ 35. $\frac{1}{6}$ 37. $\frac{1}{8}$ 39. $\frac{1}{4}$

41. $\frac{1}{6}$ 43. $\frac{1}{18}$ 45. 0.55 47. 0.70 49. 0.30 51. 0.858 53. 0.56 55. 0.936 57. $\frac{17}{20}$ 59. $\frac{11}{20}$ 61. $\frac{1}{2}$ 63. $\frac{17}{50}$ 65. $\frac{12}{25}$

67. (a) 0.71 (b) 0.91 (c) 0.94 (d) 0.20 (e) 0.43 (f) 0.09 (g) 0.85 (h) 0.57 69. (a) $\frac{25}{33}$ (b) $\frac{25}{33}$ 71. 0.167

73. $\frac{1}{292,201,338} \approx 0.00000000342$ 74. 2; left; 3; down 75. $(-3, 3\sqrt{3})$ 76. $\{22\}$ 77. $(2, -3, -1)$

Review Exercises (page 895)

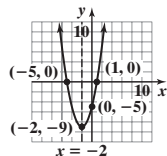
1. $\emptyset, \{\text{Dave}\}, \{\text{Joanne}\}, \{\text{Erica}\}, \{\text{Dave, Joanne}\}, \{\text{Dave, Erica}\}, \{\text{Joanne, Erica}\}, \{\text{Dave, Joanne, Erica}\}$ 2. 17 3. 24 4. 29 5. 34 6. 7 7. 45
 8. 25 9. 7 10. 336 11. 56 12. 60 13. 128 14. 3024 15. 1680 16. 91 17. 1,600,000 18. 216,000
 19. 256 (allowing numbers with initial zeros, such as 011) 20. 12,600 21. (a) 381,024 (b) 1260
 22. (a) $8.634628387 \times 10^{45}$ (b) 0.6531 (c) 0.3469 23. (a) 0.062 (b) 0.938 24. $\frac{4}{9}$ 25. 0.2; 0.26 26. (a) 0.68 (b) 0.58 (c) 0.32

Chapter Test (page 896)

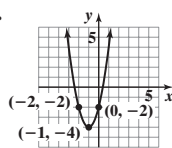
1. 22 2. 3 3. 8 4. 45 5. 5040 6. 151,200 7. 462 8. There are 54,264 ways to choose 6 different colors from the 21 available colors.
 9. There are 840 distinct arrangements of the letters in the word REDEEMED. 10. There are 56 different exacta bets for an 8-horse race.
 11. There are 155,480,000 possible license plates using the new format. 12. (a) 0.95 (b) 0.30 13. (a) 0.25 (b) 0.55 14. 0.19 15. 0.000033069
 16. $P(\text{exactly 2 fours}) = \frac{625}{3888} \approx 0.1608$

Cumulative Review (page 897)

1. $\left\{ \frac{1}{3} - \frac{\sqrt{2}}{3}i, \frac{1}{3} + \frac{\sqrt{2}}{3}i \right\}$ 2.

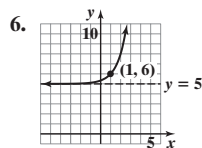


3.



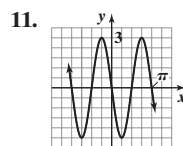
4. $\{x | 3.99 \leq x \leq 4.01\}$ or $[3.99, 4.01]$

5. $\left\{ -\frac{1}{2} + \frac{\sqrt{7}}{2}i, -\frac{1}{2} - \frac{\sqrt{7}}{2}i, -\frac{1}{5}, 3 \right\}$ 6.



Domain: all real numbers
 Range: $\{y | y > 5\}$
 Horizontal asymptote: $y = 5$

7. 2 8. $\left\{ \frac{8}{3} \right\}$ 9. $x = 2, y = -5, z = 3$ 10. 125; 700

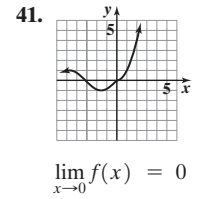
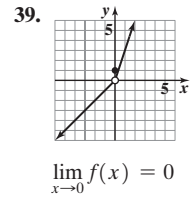
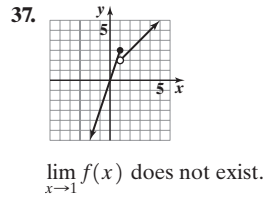
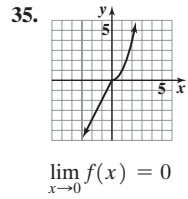
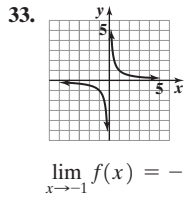
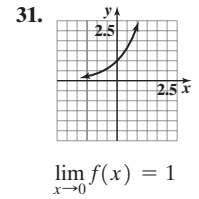
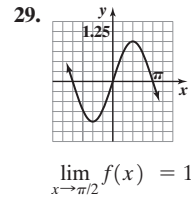
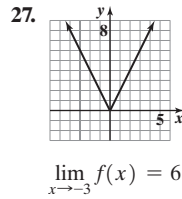
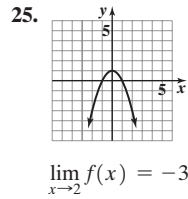
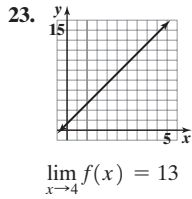


12. $a \approx 6.09, B \approx 31.9^\circ, C \approx 108.1^\circ$;
 area ≈ 14.46 square units

CHAPTER 14 A Preview of Calculus: The Limit, Derivative, and Integral of a Function

14.1 Assess Your Understanding (page 903)

3. a 4. does not exist 5. True 6. False 7. 32 9. 1 11. 4 13. 2 15. 0 17. 3 19. 4 21. Does not exist



43. 0.67 45. 1.6 47. 0 49. $d = 4\sqrt{5}$; $M = (4, -7)$ 50. Center: $(2, -1)$; foci: $(2, -3)$, $(2, 1)$; vertices: $(2, \sqrt{13} - 1)$, $(2, -\sqrt{13} - 1)$

51. \$7288.48 52. $(4, \frac{2\pi}{3})$

14.2 Assess Your Understanding (page 911)

1. product 2. b 3. b 4. True 5. False 6. False 7. 5 9. 4 11. -10 13. 80 15. 8 17. 8 19. -1 21. 8 23. 3 25. -1 27. 32 29. 2 31. $\frac{7}{6}$

33. 3 35. 0 37. $\frac{2}{3}$ 39. $\frac{8}{5}$ 41. 0 43. 5 45. 6 47. 0 49. 0 51. -1 53. 1 55. $\frac{3}{4}$

57. $f(x) = x^3 + x^2 + 1$ 58. $g^{-1}(x) = \frac{3-x}{x-2}$ 59. 60° or $\frac{\pi}{3}$ 60. $x^4 + 8x^3 + 24x^2 + 32x + 16$

14.3 Assess Your Understanding (page 917)

7. one-sided 8. $\lim_{x \rightarrow c} f(x) = R$ 9. continuous; c 10. False 11. d 12. True 13. $\{x | -8 \leq x < -6 \text{ or } -6 < x < 4 \text{ or } 4 < x \leq 6\}$ 15. -8, -5, -3

17. $f(-8) = 0$; $f(-4) = 2$ 19. ∞ 21. 2 23. 1 25. Limit exists; 0 27. No 29. Yes 31. No 33. 5 35. 7 37. 1 39. 4 41. $-\frac{2}{3}$ 43. $\frac{3}{2}$

45. Continuous 47. Continuous 49. Not continuous 51. Not continuous 53. Not continuous 55. Continuous 57. Not continuous

59. Continuous 61. Continuous for all real numbers 63. Continuous for all real numbers 65. Continuous for all real numbers

67. Continuous for all real numbers except $x = \frac{k\pi}{2}$, where k is an odd integer 69. Continuous for all real numbers except $x = -2$ and $x = 2$

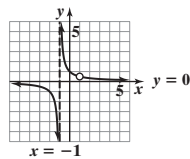
71. Continuous for all positive real numbers except $x = 1$

73. Discontinuous at $x = -1$ and $x = 1$;

$$\lim_{x \rightarrow 1} R(x) = \frac{1}{2}: \text{hole at } \left(1, \frac{1}{2}\right)$$

$$\lim_{x \rightarrow -1^-} R(x) = -\infty; \lim_{x \rightarrow -1^+} R(x) = \infty;$$

vertical asymptote at $x = -1$

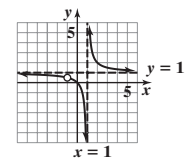


75. Discontinuous at $x = -1$ and $x = 1$;

$$\lim_{x \rightarrow -1} R(x) = \frac{1}{2}: \text{hole at } \left(-1, \frac{1}{2}\right)$$

$$\lim_{x \rightarrow 1^-} R(x) = -\infty; \lim_{x \rightarrow 1^+} R(x) = \infty;$$

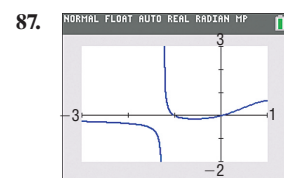
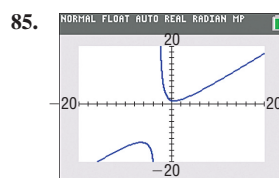
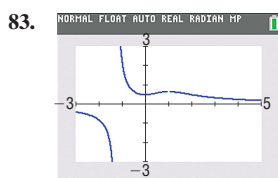
vertical asymptote at $x = 1$



77. $x = -\sqrt[3]{2}$: asymptote; $x = 1$: hole

79. $x = -3$: asymptote; $x = 2$: hole

81. $x = -\sqrt[3]{2}$: asymptote; $x = -1$: hole

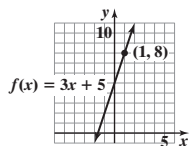


91. Vertical: $x = 4$; horizontal: $y = 3$ 92. 60 93. $\ln\left(\frac{x^5 y^2}{z^4}\right)$ 94. $\begin{bmatrix} 3 & 1 & 2 & 4 \\ 1 & 0 & 2 & 5 \\ 0 & 1 & -3 & -2 \end{bmatrix}$

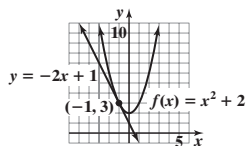
14.4 Assess Your Understanding (page 925)

3. tangent line 4. derivative 5. c 6. True 7. True 8. True

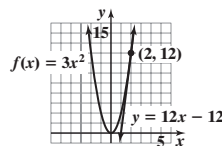
9. $m_{\tan} = 3$



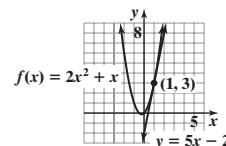
11. $m_{\tan} = -2$



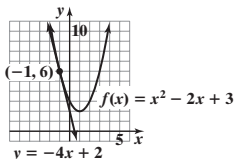
13. $m_{\tan} = 12$



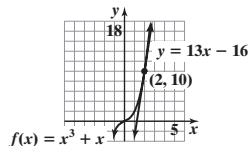
15. $m_{\tan} = 5$



17. $m_{\tan} = -4$



19. $m_{\tan} = 13$

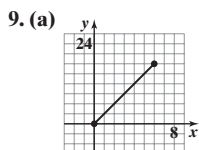


21. -4 23. 0 25. 7 27. 7 29. 3 31. 1 33. 60 35. -0.8587776956
 37. 1.389623659 39. 2.362110222 41. 3.643914112 43. $18\pi \text{ ft}^3/\text{ft}$
 45. $16\pi \text{ ft}^3/\text{ft}$ 47. (a) 6 sec (b) 64 ft/sec (c) $(-32t + 96) \text{ ft/sec}$
 (d) 32 ft/sec (e) 3 sec (f) 144 ft (g) -96 ft/sec
 49. (a) $-23\frac{1}{3} \text{ ft/sec}$ (b) -21 ft/sec (c) -18 ft/sec
 (d) $s(t) = -2.631t^2 - 10.269t + 999.933$
 (e) Approximately -15.531 ft/sec

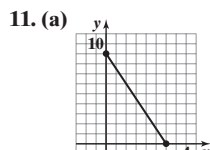
51. Vertex: $(1, \frac{7}{2})$ 52. $(2, 6), (-1, 3)$ 53. 10 54. 23.66 sq. units

14.5 Assess Your Understanding (page 932)

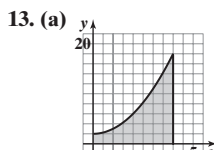
3. $\int_a^b f(x) dx$ 4. b 5. 3 7. 56



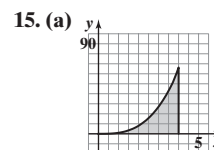
- (b) 36 (c) 72
 (d) 45 (e) 63 (f) 54



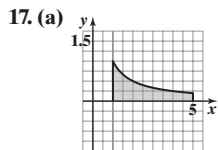
- (b) 18 (c) 9
 (d) $\frac{63}{4}$ (e) $\frac{45}{4}$ (f) $\frac{27}{2}$



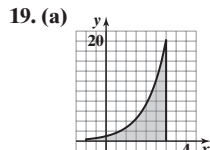
- (b) 22 (c) $\frac{51}{2}$
 (d) $\int_0^4 (x^2 + 2) dx$ (e) $\frac{88}{3}$



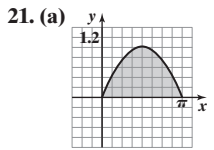
- (b) 36 (c) 49
 (d) $\int_0^4 x^3 dx$ (e) 64



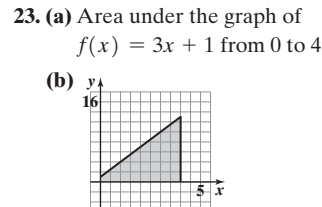
- (b) $\frac{25}{12}$ (c) $\frac{4609}{2520}$
 (d) $\int_1^5 \frac{1}{x} dx$ (e) 1.609



- (b) 11.475 (c) 15.197
 (d) $\int_{-1}^3 e^x dx$ (e) 19.718

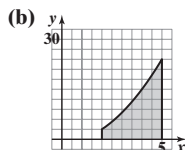


- (b) 1.896 (c) 1.974
 (d) $\int_0^\pi \sin x dx$ (e) 2



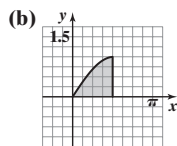
(c) 28

25. (a) Area under the graph of $f(x) = x^2 - 1$ from 2 to 5



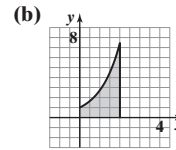
(c) 36

27. (a) Area under the graph of $f(x) = \sin x$ from 0 to $\frac{\pi}{2}$



(c) 1

29. (a) Area under the graph of $f(x) = e^x$ from 0 to 2



(c) 6.389

31. Using left endpoints: $n = 2: 0 + 0.5 = 0.5$;

$n = 4: 0 + 0.125 + 0.25 + 0.375 = 0.75$;

$n = 10: 0 + 0.02 + 0.04 + 0.06 + \dots + 0.18 = \frac{10}{2}(0 + 0.18) = 0.9$;

$n = 100: 0 + 0.0002 + 0.0004 + 0.0006 + \dots + 0.0198$

$= \frac{100}{2}(0 + 0.0198) = 0.99$;

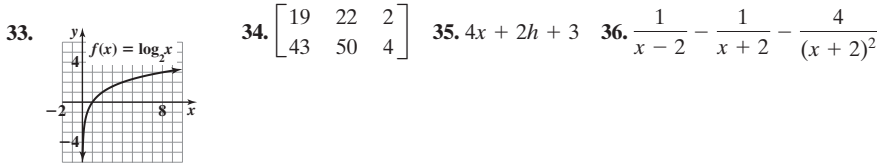
Using right endpoints:

$n = 2: 0.5 + 1 = 1.5$; $n = 4: 0.125 + 0.25 + 0.375 + 0.5 = 1.25$;

$n = 10: 0.02 + 0.04 + 0.06 + \dots + 0.20 = \frac{10}{2}(0.02 + 0.20) = 1.1$;

$n = 100: 0.0002 + 0.0004 + 0.0006 + \dots + 0.02$

$= \frac{100}{2}(0.0002 + 0.02) = 1.01$



Review Exercises (page 935)

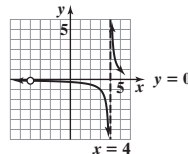
1. 9 2. 25 3. 4 4. 0 5. 64 6. $-\frac{1}{4}$ 7. $\frac{1}{3}$ 8. $\frac{6}{7}$ 9. 0 10. $\frac{3}{2}$ 11. $\frac{28}{11}$ 12. Continuous 13. Not continuous 14. Not continuous 15. Continuous
 16. $\{x | -6 \leq x < 2 \text{ or } 2 < x < 5 \text{ or } 5 < x \leq 6\}$ 17. All real numbers 18. 1, 6 19. 4 20. $f(-6) = 2; f(-4) = 1$ 21. 4 22. -2 23. $-\infty$ 24. ∞
 25. Does not exist 26. No 27. Yes

28. R is discontinuous at $x = -4$ and $x = 4$.

$\lim_{x \rightarrow -4} R(x) = -\frac{1}{8}$: hole at $(-4, -\frac{1}{8})$

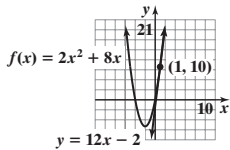
$\lim_{x \rightarrow 4^-} R(x) = -\infty$; $\lim_{x \rightarrow 4^+} R(x) = \infty$:

The graph of R has a vertical asymptote at $x = 4$.

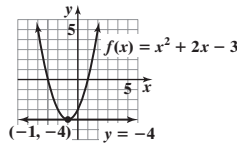


29. Undefined at $x = 2$ and $x = 9$; R has a hole at $x = 2$ and a vertical asymptote at $x = 9$.

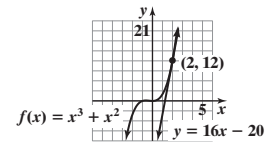
30. $m_{\tan} = 12$



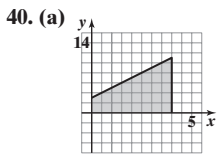
31. $m_{\tan} = 0$



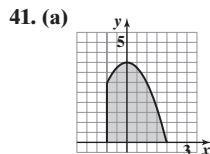
32. $m_{\tan} = 16$



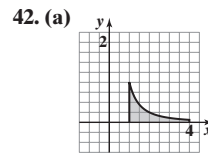
33. -24 34. -3 35. 7 36. -158 37. 0.6662517653 38. (a) 7 sec (b) 6 sec (c) 64 ft/sec (d) $(-32t + 96)$ ft/sec (e) 32 ft/sec (f) At $t = 3$ sec
 (g) -96 ft/sec (h) -128 ft/sec 39. (a) \$61.29/watch (b) \$71.31/watch (c) \$81.40/watch (d) $R(x) = -0.25x^2 + 100.01x - 1.24$
 (e) Approximately \$87.51/watch



- (b) 24 (c) 32
 (d) 26 (e) 30 (f) 28

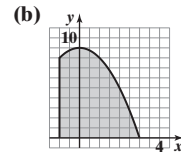


- (b) 10 (c) $\frac{77}{8}$
 (d) $\int_{-1}^2 (4 - x^2) dx$ (e) 9



- (b) $\frac{49}{36} \approx 1.36$ (c) 1.02
 (d) $\int_1^4 \frac{1}{x^2} dx$ (e) 0.75

43. (a) Area under the graph of $f(x) = 9 - x^2$ from -1 to 3



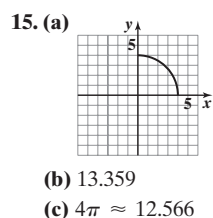
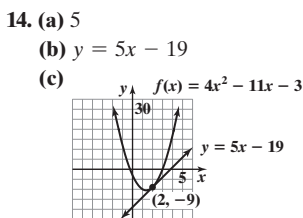
(c) $\frac{80}{3}$

44. (a) Area under the graph of $f(x) = e^x$ from -1 to 1

(b)  (c) 2.35

Chapter Test (page 936)

1. -5 2. $\frac{1}{3}$ 3. 5 4. -2 5. 135 6. $\frac{2}{3}$ 7. -1 8. -3 9. 5 10. 2 11. Limit exists; 2
 12. (a) Yes (b) No; $\lim_{x \rightarrow 1} f(x) \neq f(1)$ (c) No; $\lim_{x \rightarrow 3} f(x) \neq f(3)$ (d) Yes 13. $x = -7$: asymptote; $x = 2$: hole

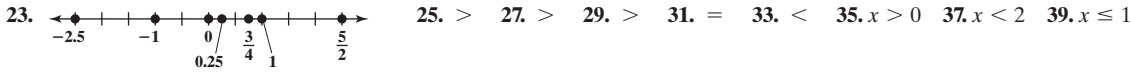


16. $\int_1^4 (-x^2 + 5x + 3) dx$ 17. $35\frac{1}{3}$ ft/sec

APPENDIX A Review

A.1 Assess Your Understanding (page A10)

1. variable 2. origin 3. strict 4. base; exponent or power 5. d 6. b 7. T 8. T 9. F 10. F 11. {1, 2, 3, 4, 5, 6, 7, 8, 9}
 13. {4} 15. {1, 3, 4, 6} 17. {0, 2, 6, 7, 8} 19. {0, 1, 2, 3, 5, 6, 7, 8, 9} 21. {0, 1, 2, 3, 5, 6, 7, 8, 9}



65. 22 67. 2 69. $x = 0$ 71. $x = 3$ 73. None 75. $x = 0, x = 1, x = -1$ 77. $\{x|x \neq 5\}$ 79. $\{x|x \neq -4\}$ 81. 0°C 83. 25°C 85. 16
 87. $\frac{1}{16}$ 89. $\frac{1}{9}$ 91. 9 93. 5 95. 4 97. $64x^6$ 99. $\frac{x^4}{y^2}$ 101. $\frac{x}{y}$ 103. $-\frac{8x^2z}{9y}$ 105. $\frac{16x^2}{9y^2}$ 107. -4 109. 5 111. 4 113. 2 115. $\sqrt{5}$ 117. $\frac{1}{2}$ 119. 10; 0
 121. 81 123. 304,006.671 125. 0.004 127. 481.890 129. 0.000 131. $A = lw$ 133. $C = \pi d$ 135. $A = \frac{\sqrt{3}}{4}x^2$ 137. $V = \frac{4}{3}\pi r^3$ 139. $V = x^3$
 141. (a) \$6000 (b) \$8000 143. $|x - 4| \geq 6$ 145. (a) $2 \leq 5$ (b) $6 > 5$ 147. (a) Yes (b) No 149. No; $\frac{1}{3}$ is larger; 0.000333... 151. No

A.2 Assess Your Understanding (page A19)

1. right; hypotenuse 2. $A = \frac{1}{2}bh$ 3. $C = 2\pi r$ 4. similar 5. c 6. b 7. T 8. T 9. F 10. T 11. T 12. F 13. 13 15. 26 17. 25
 19. Right triangle; 5 21. Not a right triangle 23. Right triangle; 25 25. Not a right triangle 27. 8 in.² 29. 4 in.² 31. $A = 25\pi \text{ m}^2$; $C = 10\pi \text{ m}$
 33. $V = 224 \text{ ft}^3$; $S = 232 \text{ ft}^2$ 35. $V = \frac{256}{3}\pi \text{ cm}^3$; $S = 64\pi \text{ cm}^2$ 37. $V = 648\pi \text{ in.}^3$; $S = 306\pi \text{ in.}^2$ 39. π square units 41. 2π square units
 43. $x = 4$ units; $A = 90^\circ$; $B = 60^\circ$; $C = 30^\circ$ 45. $x = 67.5$ units; $A = 60^\circ$; $B = 95^\circ$; $C = 25^\circ$ 47. About 16.8 ft 49. 64 ft²
 51. $24 + 2\pi \approx 30.28 \text{ ft}^2$; $16 + 2\pi \approx 22.28 \text{ ft}$ 53. 160 paces 55. About 5.477 mi 57. From 100 ft: 12.2 mi; From 150 ft: 15.0 mi

A.3 Assess Your Understanding (page A29)

1. 4; 3 2. $x^4 - 16$ 3. $x^3 - 8$ 4. F 5. F 6. T 7. F 8. add; $\frac{25}{4}$ 9. quotient; divisor; remainder 10. a 11. c 12. b 13. d 14. c
 15. Monomial; variable: x ; coefficient: 2; degree: 3 17. Not a monomial; the exponent of the variable is not a nonnegative integer
 19. Not a monomial; it has more than one term 21. Not a monomial; the exponent of one of the variables is not a nonnegative integer
 23. Not a monomial; it has more than one term 25. Yes; 2 27. Yes; 0 29. No; the variable of one of the terms is not a nonnegative integer
 31. Yes; 3 33. No; the polynomial of the denominator has a degree greater than 0 35. $x^2 + 7x + 2$ 37. $x^3 - 4x^2 + 9x + 7$ 39. $-2x^3 + 18x^2 - 18$
 41. $15y^2 - 27y + 30$ 43. $x^3 + x^2 - 4x$ 45. $x^2 + 6x + 8$ 47. $2x^2 + 9x + 10$ 49. $x^2 - 49$ 51. $4x^2 - 9$ 53. $x^2 + 8x + 16$ 55. $4x^2 - 12x + 9$
 57. $x^3 - 6x^2 + 12x - 8$ 59. $8x^3 + 12x^2 + 6x + 1$ 61. $4x^2 - 11x + 23$; remainder -45 63. $4x - 3$; remainder $x + 1$ 65. $5x^2 - 13$; remainder $x + 27$
 67. $2x^2$; remainder $-x^2 + x + 1$ 69. $x^2 - 2x + \frac{1}{2}$; remainder $\frac{5}{2}x + \frac{1}{2}$ 71. $-4x^2 - 3x - 3$; remainder -7 73. $x^2 - x - 1$; remainder $2x + 2$
 75. $x^2 + ax + a^2$; remainder 0 77. $(x + 6)(x - 6)$ 79. $2(1 + 2x)(1 - 2x)$ 81. $(x + 1)(x + 10)$ 83. $(x - 7)(x - 3)$
 85. $4(x^2 - 2x + 8)$ 87. Prime 89. $-(x - 5)(x + 3)$ 91. $3(x + 2)(x - 6)$ 93. $y^2(y + 5)(y + 6)$ 95. $(2x + 3)^2$ 97. $2(3x + 1)(x + 1)$
 99. $(x - 3)(x + 3)(x^2 + 9)$ 101. $(x - 1)^2(x^2 + x + 1)^2$ 103. $x^5(x - 1)(x + 1)$ 105. $(4x + 3)^2$ 107. $-(4x - 5)(4x + 1)$
 109. $(2y - 5)(2y - 3)$ 111. $-(3x - 1)(3x + 1)(x^2 + 1)$ 113. $(x + 3)(x - 6)$ 115. $(x + 2)(x - 3)$ 117. $(3x - 5)(9x^2 - 3x + 7)$
 119. $(x + 5)(3x + 11)$ 121. $(x - 1)(x + 1)(x + 2)$ 123. $(x - 1)(x + 1)(x^2 - x + 1)$ 125. 25; $(x + 5)^2$ 127. 9; $(y - 3)^2$
 129. $\frac{1}{16}; \left(x - \frac{1}{4}\right)^2$ 131. $2(3x + 4)(9x + 13)$ 133. $2x(3x + 5)$ 135. $5(x + 3)(x - 2)^2(x + 1)$ 137. $3(4x - 3)(4x - 1)$
 139. $6(3x - 5)(2x + 1)^2(5x - 4)$ 141. The possibilities are $(x \pm 1)(x \pm 4) = x^2 \pm 5x + 4$ or $(x \pm 2)(x \pm 2) = x^2 \pm 4x + 4$, none of which equals $x^2 + 4$.

A.4 Assess Your Understanding (page A34)

1. quotient; divisor; remainder 2. $-3 \overline{)20-51}$ 3. d 4. a 5. T 6. T 7. $x^2 + x + 4$; remainder 12 9. $3x^2 + 11x + 32$; remainder 99
 11. $x^4 - 3x^3 + 5x^2 - 15x + 46$; remainder -138 13. $4x^5 + 4x^4 + x^3 + x^2 + 2x + 2$; remainder 7 15. $0.1x^2 - 0.11x + 0.321$; remainder -0.3531
 17. $x^4 + x^3 + x^2 + x + 1$; remainder 0 19. No 21. Yes 23. Yes 25. No 27. Yes 29. -9

A.5 Assess Your Understanding (page A41)

1. lowest terms 2. least common multiple 3. T 4. F 5. d 6. a 7. $\frac{3}{x-3}$ 9. $\frac{x}{3}$ 11. $\frac{4x}{2x-1}$ 13. $\frac{y+5}{2(y+1)}$ 15. $\frac{3}{5x(x-2)}$
 17. $\frac{2x(x^2 + 4x + 16)}{x + 4}$ 19. $\frac{4}{5(x-1)}$ 21. $-\frac{(x-4)^2}{4x}$ 23. $\frac{(x-2)(x+2)}{2x-3}$ 25. $\frac{2(x^2-2)}{x(x-2)(x+2)}$ 27. $\frac{5x}{(x-6)(x-1)(x+4)}$

29. $\frac{2(2x^2 + 5x - 2)}{(x - 2)(x + 2)(x + 3)}$ 31. $\frac{5x + 1}{(x - 1)^2(x + 1)^2}$ 33. $\frac{x + 1}{x - 1}$ 35. $\frac{-2x(x^2 - 2)}{(x + 2)(x^2 - x - 3)}$ 37. $\frac{19}{(3x - 5)^2}$ 39. $\frac{(x + 1)(x - 1)}{(x^2 + 1)^2}$
 41. $\frac{x(3x + 2)}{(3x + 1)^2}$ 43. $-\frac{(x + 3)(3x - 1)}{(x^2 + 1)^2}$ 45. $f = \frac{R_1 \cdot R_2}{(n - 1)(R_1 + R_2)}; \frac{2}{15} \text{ m}$

A.6 Assess Your Understanding (page A54)

5. equivalent equations 6. identity 7. F 8. T 9. add; $\frac{81}{4}$ 10. discriminant; negative 11. F 12. F 13. b 14. d 15. {7} 17. {-3} 19. {4}
 21. $\left\{\frac{5}{4}\right\}$ 23. {-1} 25. {-18} 27. {-3} 29. {-16} 31. {0.5} 33. {2} 35. {2} 37. {3} 39. {0, 9} 41. {0, 9} 43. {21}
 45. {-2, 2} 47. {6} 49. {-3, 3} 51. {-4, 1} 53. $\left\{-1, \frac{3}{2}\right\}$ 55. {-4, 4} 57. {2} 59. No real solution 61. {-2, 2}
 63. {-1, 3} 65. {-2, -1, 0, 1} 67. {0, 4} 69. {-6, 2} 71. $\left\{-\frac{1}{2}, 3\right\}$ 73. {3, 4} 75. $\left\{\frac{3}{2}\right\}$ 77. $\left\{-\frac{2}{3}, \frac{3}{2}\right\}$ 79. $\left\{-\frac{3}{4}, 2\right\}$ 81. {-6}
 83. {-2, -1, 1, 2} 85. {-6, -5} 87. $\left\{-\frac{3}{2}, 2\right\}$ 89. {-5, 0, 4} 91. {-1, 1} 93. $\left\{-2, \frac{1}{2}, 2\right\}$ 95. {-5, 5} 97. {-1, 3} 99. {-3, 0}
 101. 16 103. $\frac{1}{16}$ 105. $\frac{1}{9}$ 107. {-7, 3} 109. $\left\{-\frac{1}{4}, \frac{3}{4}\right\}$ 111. $\left\{\frac{-1 - \sqrt{7}}{6}, \frac{-1 + \sqrt{7}}{6}\right\}$ 113. $[2 - \sqrt{2}, 2 + \sqrt{2}]$
 115. $\left\{\frac{5 - \sqrt{29}}{2}, \frac{5 + \sqrt{29}}{2}\right\}$ 117. $\left\{1, \frac{3}{2}\right\}$ 119. No real solution 121. $\left\{\frac{-1 - \sqrt{5}}{4}, \frac{-1 + \sqrt{5}}{4}\right\}$ 123. $\left\{\frac{-\sqrt{3} - \sqrt{15}}{2}, \frac{-\sqrt{3} + \sqrt{15}}{2}\right\}$
 125. No real solution 127. Repeated real solution 129. Two unequal real solutions 131. $R = \frac{R_1 R_2}{R_1 + R_2}$ 133. $R = \frac{mv^2}{F}$ 135. $r = \frac{S - a}{S}$
 137. $\frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-2b}{2a} = \frac{-b}{a}$ 139. $k = -\frac{1}{2}$ or $\frac{1}{2}$
 141. The solutions of $ax^2 - bx + c = 0$ are $\frac{b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{b - \sqrt{b^2 - 4ac}}{2a}$. 143. (b)

A.7 Assess Your Understanding (page A64)

1. T 2. 5 3. F 4. real; imaginary; imaginary unit 5. F 6. T 7. F 8. b 9. a 10. c 11. $8 + 5i$ 13. $-7 + 6i$ 15. $-6 - 11i$ 17. $6 - 18i$
 19. $6 + 4i$ 21. $10 - 5i$ 23. 37 25. $\frac{6}{5} + \frac{8}{5}i$ 27. $1 - 2i$ 29. $\frac{5}{2} - \frac{7}{2}i$ 31. $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$ 33. $2i$ 35. $-i$ 37. i 39. -6 41. $-10i$ 43. $-2 + 2i$
 45. 0 47. 0 49. $2i$ 51. $5i$ 53. $2\sqrt{3}i$ 55. $10\sqrt{2}i$ 57. $5i$ 59. $\{-2i, 2i\}$ 61. $\{-4, 4\}$ 63. $\{3 - 2i, 3 + 2i\}$ 65. $\{3 - i, 3 + i\}$
 67. $\left\{\frac{1}{4} - \frac{1}{4}i, \frac{1}{4} + \frac{1}{4}i\right\}$ 69. $\left\{\frac{1}{5} - \frac{2}{5}i, \frac{1}{5} + \frac{2}{5}i\right\}$ 71. $\left\{-\frac{1}{2} - \frac{\sqrt{3}}{2}i, -\frac{1}{2} + \frac{\sqrt{3}}{2}i\right\}$ 73. $\{2, -1 - \sqrt{3}i, -1 + \sqrt{3}i\}$ 75. $\{-2, 2, -2i, 2i\}$
 77. $\{-3i, -2i, 2i, 3i\}$ 79. Two complex solutions that are conjugates of each other 81. Two unequal real solutions 83. A repeated real solution
 85. $2 - 3i$ 87. 6 89. 25 91. $2 + 3i$ ohms 93. $z + \bar{z} = (a + bi) + (a - bi) = 2a; z - \bar{z} = (a + bi) - (a - bi) = 2bi$
 95. $\bar{z} + \bar{w} = \overline{(a + bi) + (c + di)} = \overline{(a + c) + (b + d)i} = (a + c) - (b + d)i = (a - bi) + (c - di) = \bar{z} + \bar{w}$

A.8 Assess Your Understanding (page A72)

1. mathematical modeling 2. interest 3. uniform motion 4. F 5. T 6. a 7. b 8. c 9. $A = \pi r^2; r = \text{radius}, A = \text{area}$
 11. $A = s^2; A = \text{area}, s = \text{length of a side}$ 13. $F = ma; F = \text{force}, m = \text{mass}, a = \text{acceleration}$ 15. $W = Fd; W = \text{work}, F = \text{force}, d = \text{distance}$
 17. $C = 150x; C = \text{total variable cost}, x = \text{number of dishwashers}$ 19. Invest \$31,250 in bonds and \$18,750 in CDs.
 21. \$11,600 was loaned out at 8%. 23. Mix 75 lb of Earl Grey tea with 25 lb of Orange Pekoe tea. 25. Mix 160 lb of cashews with the almonds.
 27. The speed of the current is 2.286 mi/h. 29. The speed of the current is 5 mi/h. 31. Karen walked at 4.05 ft/sec. 33. A doubles tennis court is 78 feet long and 36 feet wide. 35. Working together, it takes 12 min. 37. (a) The dimensions are 10 ft by 5 ft. (b) The area is 50 sq ft. (c) The dimensions would be 75 ft by 75 ft. (d) The area would be 56.25 sq ft. 39. The defensive back catches up to the tight end at the tight end's 45-yd line.
 41. Add $\frac{2}{3}$ gal of water. 43. Evaporate 10.67 oz of water. 45. 40 g of 12-karat gold should be mixed with 20 g of pure gold.
 47. Mike passes Dan $\frac{1}{3}$ mile from the start, 2 min from the time Mike started to run. 49. Start the auxiliary pump at 9:45 AM. 51. The tub will fill in 1 hr.
 53. Run: 12 miles; bicycle: 75 miles 55. Bolt would beat Burke by 19.25 m. 57. The dimensions should be 4ft by 4ft.
 59. Set the original price at \$40. At 50% off, there will be no profit. 63. The tail wind was 91.47 knots.

A.9 Assess Your Understanding (page A82)

5. negative 6. closed interval 7. -5, 5 8. $-5 < x < 5$ 9. T 10. T 11. b 12. c 13. d 14. a 15. $[0, 2]; 0 \leq x \leq 2$ 17. $[2, \infty); x \geq 2$
 19. $[0, 3); 0 \leq x < 3$ 21. (a) $6 < 8$ (b) $-2 < 0$ (c) $9 < 15$ (d) $-6 > -10$ 23. (a) $7 > 0$ (b) $-1 > -8$ (c) $12 > -9$ (d) $-8 < 6$
 25. (a) $2x + 4 < 5$ (b) $2x - 4 < -3$ (c) $6x + 3 < 6$ (d) $-4x - 2 > -4$
 27. $[0, 4]$ 29. $[4, 6)$ 31. $[4, \infty)$ 33. $(-\infty, -4)$



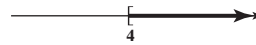
35. $2 \leq x \leq 5$



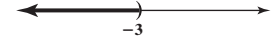
37. $-3 < x < -2$



39. $x \geq 4$

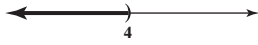


41. $x < -3$

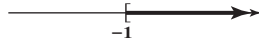


43. $<$ 45. $>$ 47. \geq 49. $<$ 51. \leq 53. $>$ 55. \geq

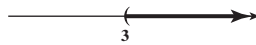
57. $\{x|x < 4\}$ or $(-\infty, 4)$



59. $\{x|x \geq -1\}$ or $[-1, \infty)$



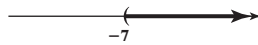
61. $\{x|x > 3\}$ or $(3, \infty)$



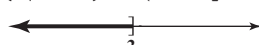
63. $\{x|x \geq 2\}$ or $[2, \infty)$



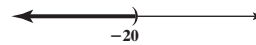
65. $\{x|x > -7\}$ or $(-7, \infty)$



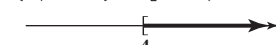
67. $\{x|x \leq \frac{2}{3}\}$ or $(-\infty, \frac{2}{3}]$



69. $\{x|x < -20\}$ or $(-\infty, -20)$



71. $\{x|x \geq \frac{4}{3}\}$ or $[\frac{4}{3}, \infty)$



73. $\{x|3 \leq x \leq 5\}$ or $[3, 5]$



75. $\{x|\frac{2}{3} \leq x \leq 3\}$ or $[\frac{2}{3}, 3]$



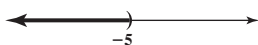
77. $\{x|-\frac{11}{2} < x < \frac{1}{2}\}$ or $(-\frac{11}{2}, \frac{1}{2})$



79. $\{x|-6 < x < 0\}$ or $(-6, 0)$



81. $\{x|x < -5\}$ or $(-\infty, -5)$



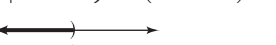
83. $\{x|x \geq -1\}$ or $[-1, \infty)$



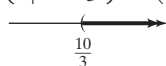
85. $\{x|\frac{1}{2} \leq x < \frac{5}{4}\}$ or $[\frac{1}{2}, \frac{5}{4})$



87. $\{x|x < -\frac{1}{2}\}$ or $(-\infty, -\frac{1}{2})$



89. $\{x|x > \frac{10}{3}\}$ or $(\frac{10}{3}, \infty)$



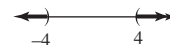
91. $\{x|x > 3\}$ or $(3, \infty)$



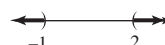
93. $\{x|-4 < x < 4\}$; $(-4, 4)$



95. $\{x|x < -4 \text{ or } x > 4\}$; $(-\infty, -4) \cup (4, \infty)$



97. $\{x|0 \leq x \leq 1\}$; $[0, 1]$ 99. $\{x|x < -1 \text{ or } x > 2\}$; $(-\infty, -1) \cup (2, \infty)$ 101. $\{x|-1 \leq x \leq 1\}$; $[-1, 1]$ 103. $\{x|x \leq -2 \text{ or } x \geq 2\}$; $(-\infty, -2] \cup [2, \infty)$



105. $|x - 2| < \frac{1}{2}$; $\{x|\frac{3}{2} < x < \frac{5}{2}\}$ 107. $|x + 3| > 2$; $\{x|x < -5 \text{ or } x > -1\}$ 109. $\{x|x \geq -2\}$ 111. $21 < \text{Age} < 30$ 113. (a) Male ≥ 82.3 years

(b) Female ≥ 85.8 years (c) A female can expect to live 3.5 years longer. 115. The agent's commission ranges from \$45,000 to \$95,000, inclusive. As a percent of selling price, the commission ranges from 5% to 8.6%, inclusive. 117. The amount withheld varies from \$133.10 to \$183.10, inclusive. 119. The usage varies from 700 kW-h to 2700 kW-h, inclusive. 121. The dealer's cost varies from \$15,254.24 to \$16,071.43, inclusive.

123. (a) You need at least a 74 on the fifth test. (b) You need at least a 77 on the fifth test. 125. $\frac{a+b}{2} - a = \frac{a+b-2a}{2} = \frac{b-a}{2} > 0$;

therefore, $a < \frac{a+b}{2}$. $b - \frac{a+b}{2} = \frac{2b-a-b}{2} = \frac{b-a}{2} > 0$; therefore, $b > \frac{a+b}{2}$. 127. $(\sqrt{ab})^2 - a^2 = ab - a^2 = a(b-a) > 0$;

thus $(\sqrt{ab})^2 > a^2$ and $\sqrt{ab} > a$. $b^2 - (\sqrt{ab})^2 = b^2 - ab = b(b-a) > 0$; thus $b^2 > (\sqrt{ab})^2$ and $b > \sqrt{ab}$.

129. $h - a = \frac{2ab}{a+b} - a = \frac{ab - a^2}{a+b} = \frac{a(b-a)}{a+b} > 0$; thus $h > a$. $b - h = b - \frac{2ab}{a+b} = \frac{b^2 - ab}{a+b} = \frac{b(b-a)}{a+b} > 0$; thus $h < b$.

131. Since $0 < a < b$, then $a - b < 0$ and $\frac{a-b}{ab} < 0$. So $\frac{a}{ab} - \frac{b}{ab} < 0$, or $\frac{1}{b} - \frac{1}{a} < 0$. Therefore, $\frac{1}{b} < \frac{1}{a}$. And $0 < \frac{1}{b}$ because $b > 0$.

A.10 Assess Your Understanding (page A91)

3. index 4. cube root 5. b 6. d 7. c 8. c 9. T 10. F 11. 3 13. -2 15. $2\sqrt{2}$ 17. $-2x\sqrt[3]{x}$ 19. x^3y^2 21. x^2y 23. $6\sqrt{x}$ 25. $3x^2y^3\sqrt[3]{2x}$

27. $6x\sqrt{x}$ 29. $15\sqrt[3]{3}$ 31. $12\sqrt{3}$ 33. $7\sqrt{2}$ 35. $\sqrt{2}$ 37. $2\sqrt{3}$ 39. $-\sqrt[3]{2}$ 41. $x - 2\sqrt{x} + 1$ 43. $(2x - 1)\sqrt[3]{2x}$ 45. $(2x - 15)\sqrt{2x}$

47. $-(x + 5y)\sqrt[3]{2xy}$ 49. $\frac{\sqrt{2}}{2}$ 51. $-\frac{\sqrt{15}}{5}$ 53. $\frac{(5 + \sqrt{2})\sqrt{3}}{23}$ 55. $\frac{8\sqrt{5} - 19}{41}$ 57. $5\sqrt{2} + 5$ 59. $\frac{5\sqrt[3]{4}}{2}$ 61. $\frac{2x + h - 2\sqrt{x^2 + xh}}{h}$ 63. $\left\{\frac{9}{2}\right\}$

65. $\{3\}$ 67. 4 69. -3 71. 64 73. $\frac{1}{27}$ 75. $\frac{27\sqrt{2}}{32}$ 77. $\frac{27\sqrt{2}}{32}$ 79. $-\frac{1}{10}$ 81. $\frac{25}{16}$ 83. $x^{7/12}$ 85. xy^2 87. $x^{2/3}y$ 89. $\frac{8x^{5/4}}{y^{3/4}}$ 91. 1.41 93. 1.59

95. 4.89 97. 2.15 99. $\frac{3x+2}{(1+x)^{1/2}}$ 101. $\frac{x(3x^2+2)}{(x^2+1)^{1/2}}$ 103. $\frac{22x+5}{10\sqrt{x-5}\sqrt{4x+3}}$ 105. $\frac{2+x}{2(1+x)^{3/2}}$ 107. $\frac{4-x}{(x+4)^{3/2}}$ 109. $\frac{1}{x^2(x^2-1)^{1/2}}$

111. $\frac{1-3x^2}{2\sqrt{x}(1+x^2)^2}$ 113. $\frac{1}{2}(5x+2)(x+1)^{1/2}$ 115. $2x^{1/2}(3x-4)(x+1)$ 117. $(x^2+4)^{1/3}(11x^2+12)$ 119. $(3x+5)^{1/3}(2x+3)^{1/2}(17x+27)$

121. $\frac{3(x+2)}{2x^{1/2}}$ 123. (a) 15,660.4 gal (b) 390.7 gal 125. $2\sqrt{2}\pi \approx 8.89$ sec

APPENDIX B The Limit of a Sequence; Infinite Series**Assess Your Understanding** (page B7)

1. a 2. infinite series 3. converges 4. True 5. converges; 2 7. diverges 9. converges; $\frac{1}{2}$ 11. converges; 0 13. diverges
15. converges; 0 17. converges; 3 19. converges; 4 21. converges; 1 23. diverges 25. diverges 27. diverges 29. $1, \frac{4}{3}, \frac{13}{9}, \frac{40}{27}, \frac{121}{81}$
31. 1, 3, 6, 10, 15