

Section 3.1

Linear Functions and Their Properties

1 Graph Linear Functions

DEFINITION

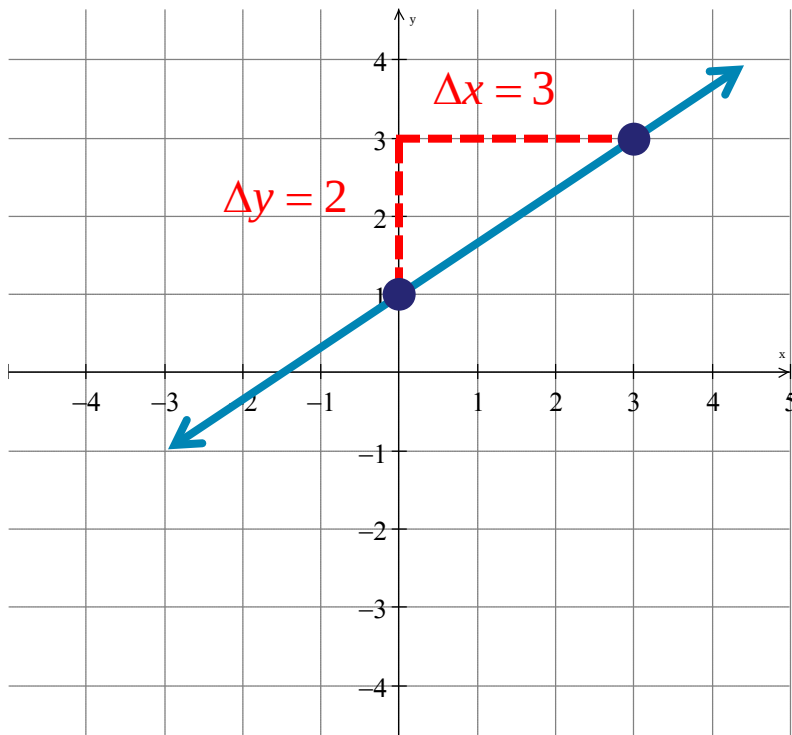
A **linear function** is a function of the form

$$f(x) = mx + b$$

The graph of a linear function is a line with slope m and y -intercept b .

EXAMPLE**Graphing a Linear Function**

Graph the linear function: $f(x) = \frac{2}{3}x + 1$



This linear function has a slope of $\frac{2}{3}$ and a y-intercept of 1 so first plot the point (0,1), the y-intercept.

Next, the slope $\frac{\Delta y}{\Delta x} = \frac{2}{3}$ so change the y value of the point on the graph by 2 and the x value by 3.

2 Use Average Rate of Change to Identify Linear Functions

x	$y = f(x) = -3x + 7$	Average Rate of Change = $\frac{\Delta y}{\Delta x}$
-2	13	$\frac{10 - 13}{-1 - (-2)} = \frac{-3}{1} = -3$
-1	10	
0	7	$\frac{7 - 10}{0 - (-1)} = \frac{-3}{1} = -3$
1	4	-3
2	1	-3
3	-2	-3

Theorem

Average Rate of Change of a Linear Function

Linear functions have a constant average rate of change. That is, the average rate of change of a linear function $f(x) = mx + b$ is

$$\frac{\Delta y}{\Delta x} = m$$

EXAMPLE

Using the Average Rate of Change to Identify Linear Functions

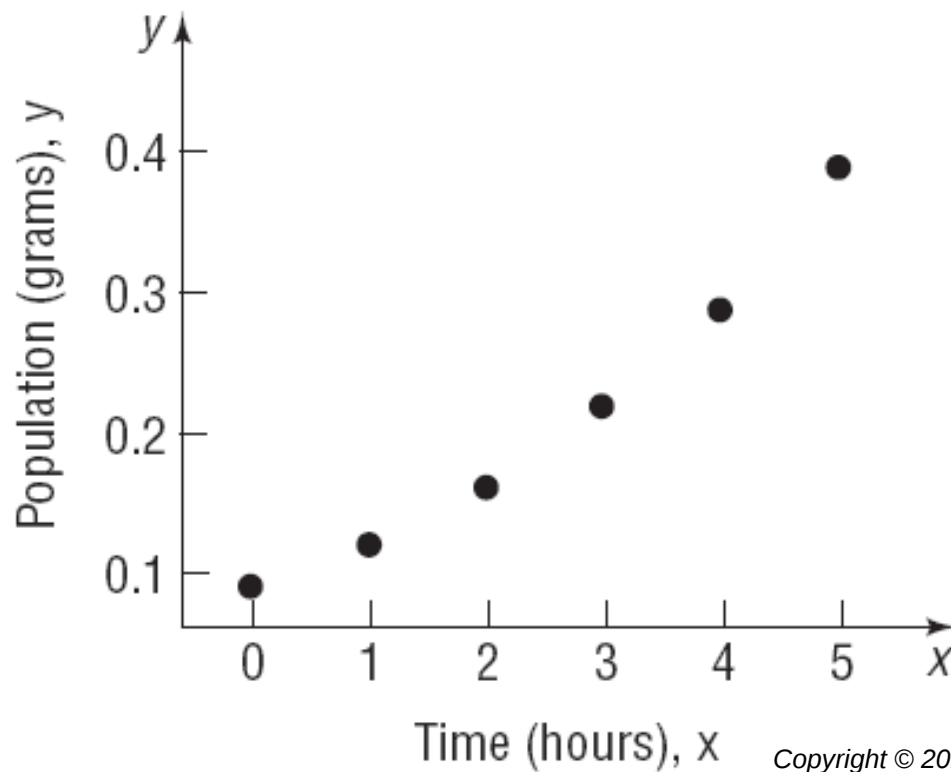
A strain of E-coli Beu 397-recA441 is placed into a Petri dish at 30° Celsius and allowed to grow. The data shown in Table 2 are collected. The population is measured in grams and the time in hours. Plot the ordered pairs (x, y) in the Cartesian plane and use the average rate of change to determine whether the function is linear.



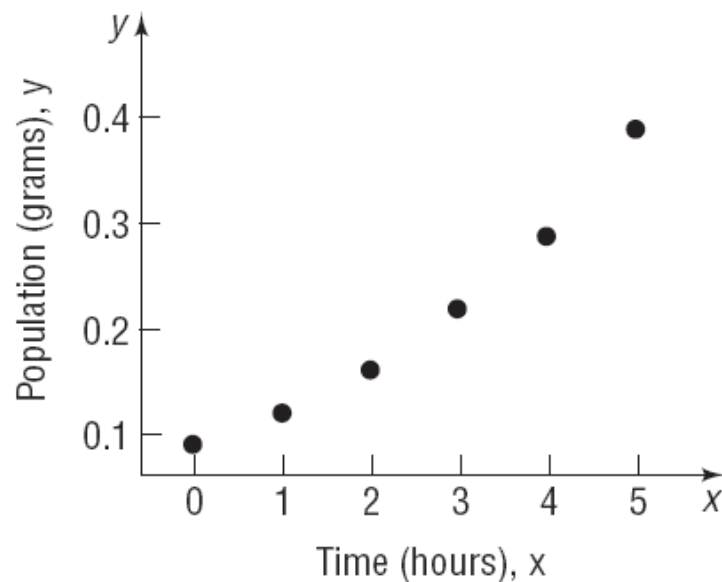
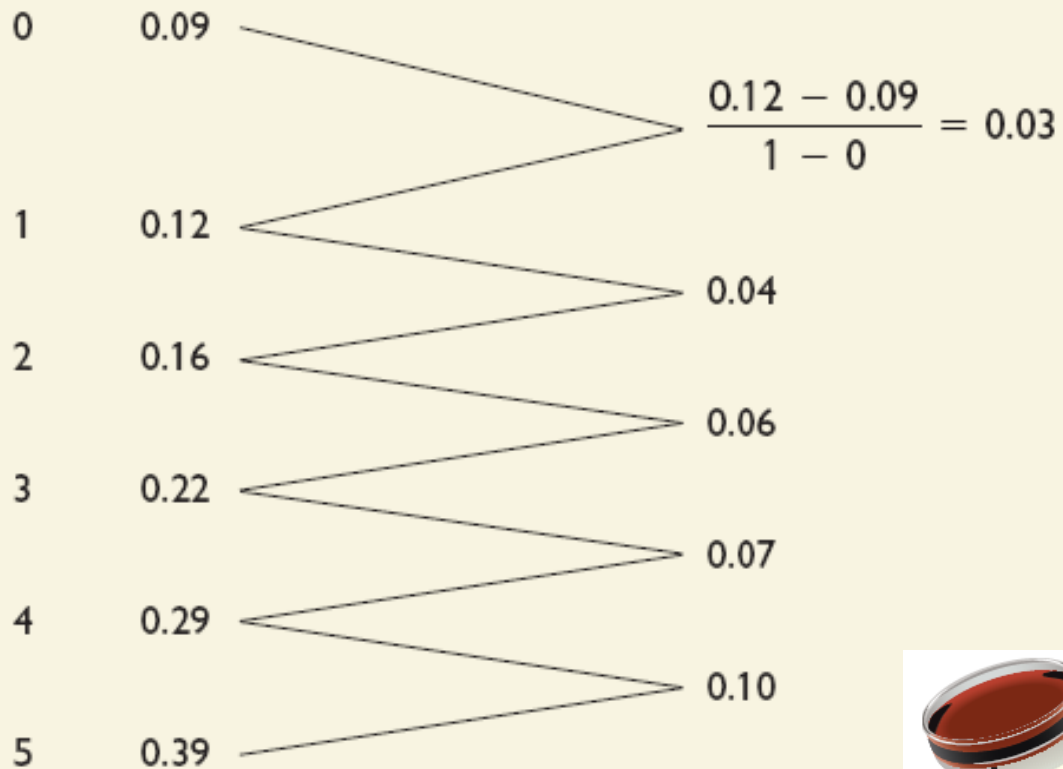
Time (hours), x	Population (grams), y	(x, y)
0	0.09	$(0, 0.09)$
1	0.12	$(1, 0.12)$
2	0.16	$(2, 0.16)$
3	0.22	$(3, 0.22)$
4	0.29	$(4, 0.29)$
5	0.39	$(5, 0.39)$



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Time	Population (grams), y	Average Rate of Change
0	0.09	$\frac{0.12 - 0.09}{1 - 0} = 0.03$
1	0.12	
2	0.16	0.04
3	0.22	0.06
4	0.29	0.07
5	0.39	0.10



Since the average rate of change is not constant, the function is not linear.



Time (hours), x	Population (grams), y	(x, y)
0	0.09	(0, 0.09)
1	0.12	(1, 0.12)
2	0.16	(2, 0.16)
3	0.22	(3, 0.22)
4	0.29	(4, 0.29)
5	0.39	(5, 0.39)

EXAMPLE

Using the Average Rate of Change to Identify Linear Functions

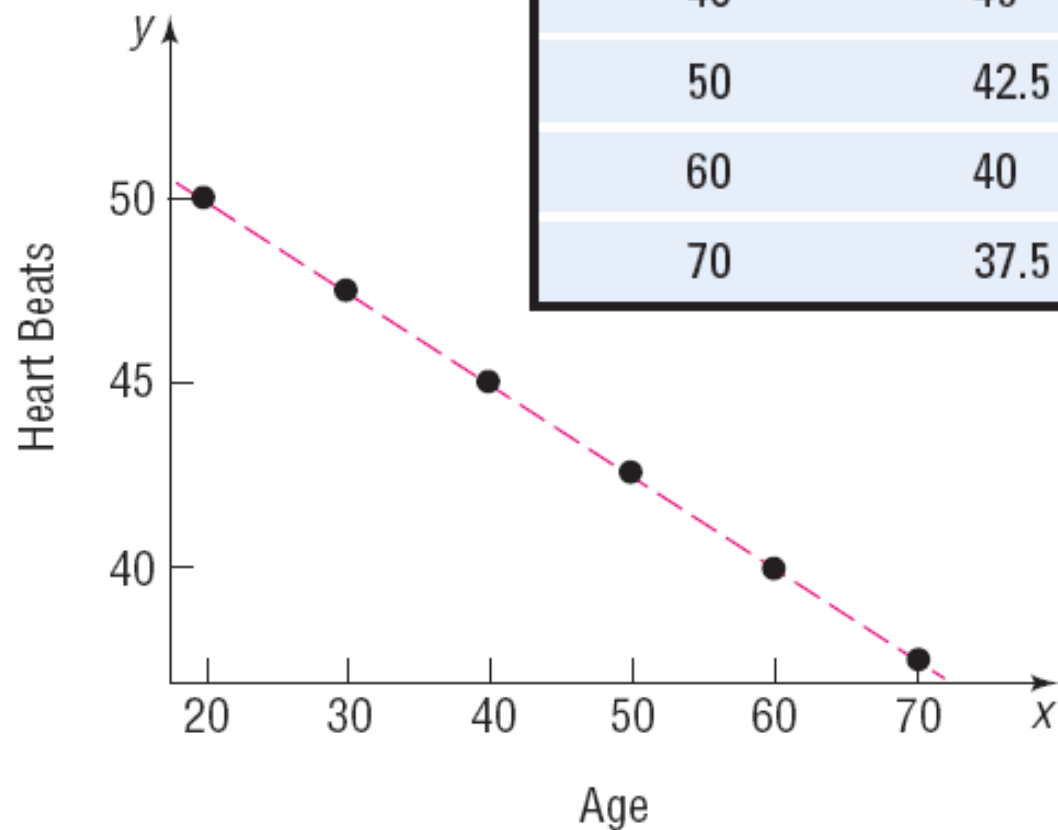
The data in Table 3 represent the maximum number of heartbeats that a healthy individual should have during a 15-second interval of time while exercising for different ages. Plot the ordered pairs (x, y) in the Cartesian plane and use the average rate of change to determine whether the function is linear.

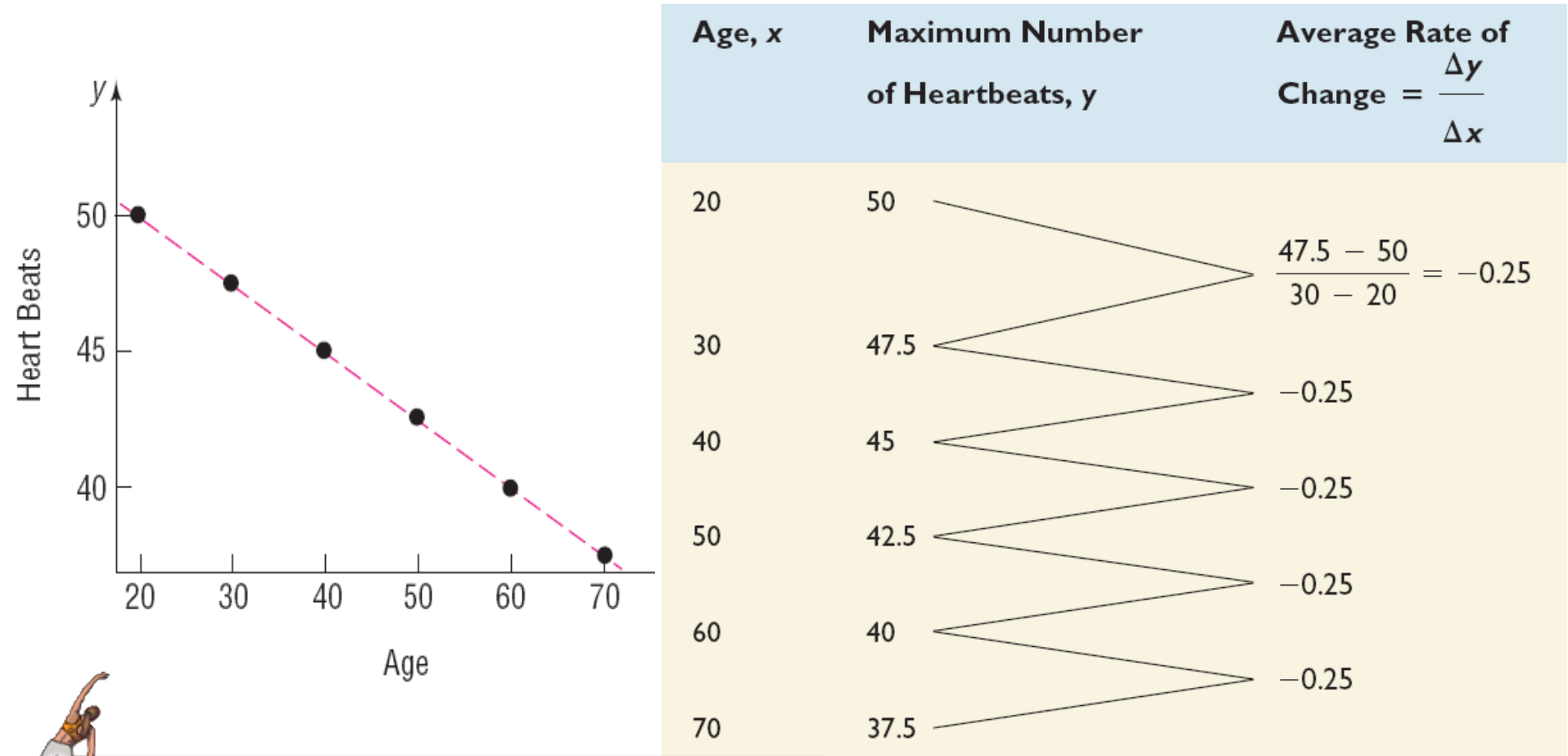


Age, x	Maximum Number of Heart Beats, y	(x, y)
20	50	$(20, 50)$
30	47.5	$(30, 47.5)$
40	45	$(40, 45)$
50	42.5	$(50, 42.5)$
60	40	$(60, 40)$
70	37.5	$(70, 37.5)$



Age, x	Maximum Number of Heart Beats, y	(x, y)
20	50	$(20, 50)$
30	47.5	$(30, 47.5)$
40	45	$(40, 45)$
50	42.5	$(50, 42.5)$
60	40	$(60, 40)$
70	37.5	$(70, 37.5)$





Age, x	Maximum Number of Heart Beats, y	(x, y)
20	50	$(20, 50)$
30	47.5	$(30, 47.5)$
40	45	$(40, 45)$
50	42.5	$(50, 42.5)$
60	40	$(60, 40)$
70	37.5	$(70, 37.5)$

Since the average rate of change is a constant -0.25 , the function is linear.

3 Determine Whether a Linear Function Is Increasing, Decreasing, or Constant

INCREASING

DECREASING

CONSTANT

Theorem

Increasing, Decreasing, and Constant Linear Functions

A linear function $f(x) = mx + b$ is increasing over its domain if its slope, m , is positive. It is decreasing over its domain if its slope, m , is negative. It is constant over its domain if its slope, m , is zero.

EXAMPLE

Determining Whether a Linear Function Is Increasing, Decreasing, or Constant

Determine whether the following linear functions are increasing, decreasing, or constant.

(a) $f(x) = -2x + 4$

(b) $g(x) = 5$

(c) $s(t) = \frac{3}{4}t$

(d) $m(z) = z - 3$

(a) The linear function has a slope of -2 so the function f is decreasing.

(b) This function could be written $g(x) = 0x + 5$ so the function has a slope of 0 and is function g is constant.

(c) The linear function has a slope of $\frac{3}{4}$ so the function s is increasing.

(d) The linear function has a slope of 1 so the function m is increasing.

4 Build Linear Models from Verbal Descriptions

Modeling with a Linear Function

If the average rate of change of a function is a constant m , a linear function f can be used to model the relation between the two variables as follows:

$$f(x) = mx + b$$

where b is the value of f at 0, that is, $b = f(0)$.

EXAMPLE

Straight-line Depreciation

Book value is the value of an asset that a company uses to create its balance sheet. Some companies depreciate their assets using straight-line depreciation so that the value of the asset declines by a fixed amount each year. The amount of the decline depends on the useful life that the company places on the asset. Suppose that a company just purchased a fleet of new cars for its sales force at a cost of \$28,000 per car. The company chooses to depreciate each vehicle using the straight-line method over 7 years. This means that each car will depreciate by $\frac{\$28,000}{7} = \4000 per year.

- (a) Write a linear function that expresses the book value V of each car as a function of its age, x .

Let $V(x)$ represent the value of each car after x years.

$V(0) = \$28,000$ and the slope is -4000 since the car depreciates by that amount per year.

$$V(x) = -4000x + 28000$$

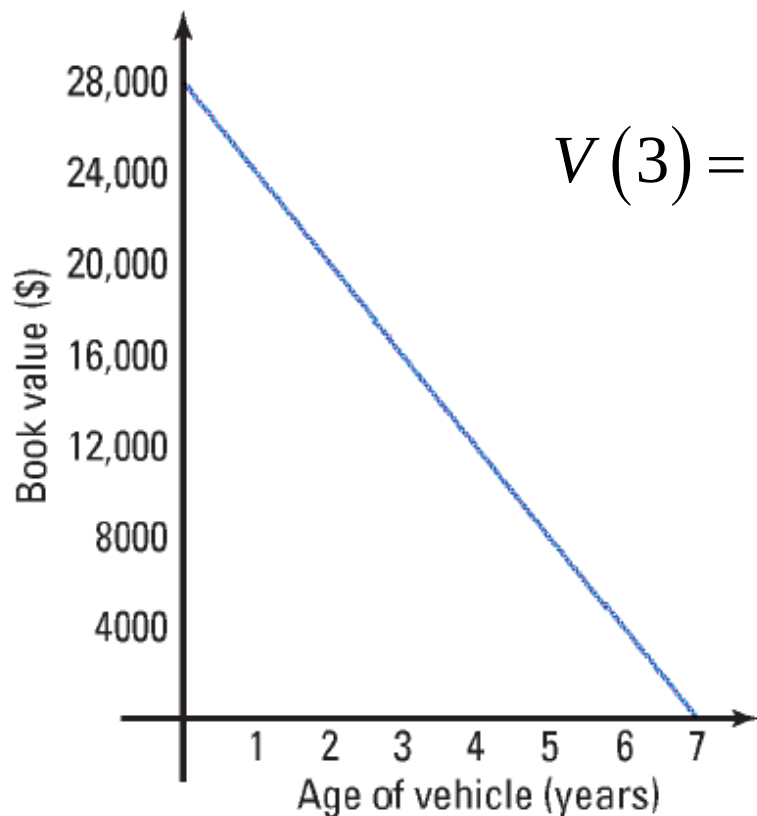
EXAMPLE

Straight-line Depreciation

$$V(x) = -4000x + 28,000$$

(b) Graph the linear function.

(c) What is the book value of each car after 3 years?



$$V(3) = -4000(3) + 2800 = 16000$$

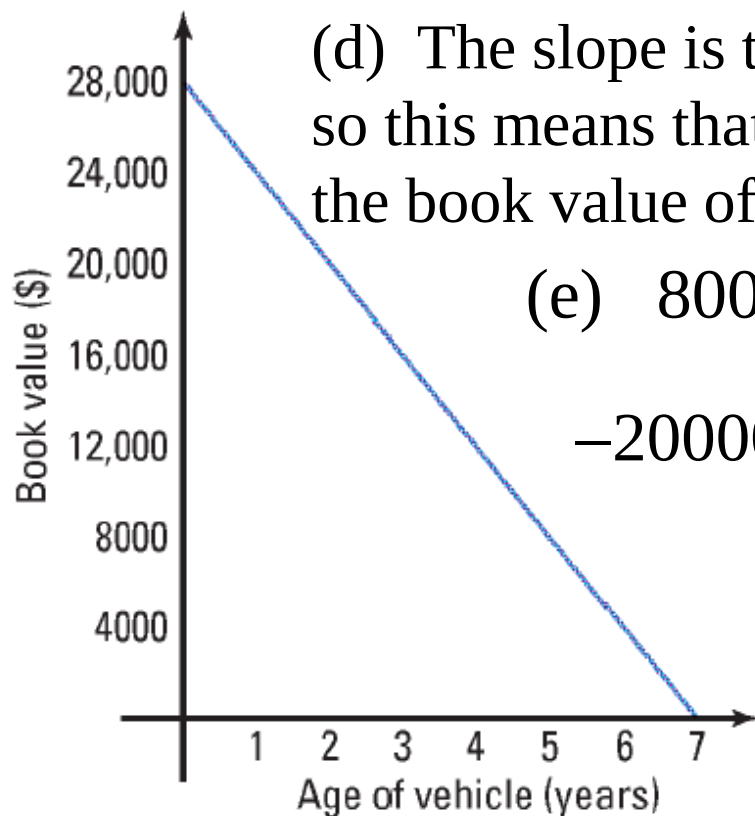
EXAMPLE

Straight-line Depreciation

$$V(x) = -4000x + 28,000$$

(d) Interpret the slope.

(e) When will the book value of each car be \$8000?



(d) The slope is the average rate of change and is -4000 so this means that for each additional year that passes, the book value of the car decreases by \$4000.

$$(e) \quad 8000 = -4000x + 28000$$

$$-20000 = -4000x \quad x = \frac{-20000}{-4000} = 5$$

The car will have a book value of \$8000 when it is 5 years old.

EXAMPLE**Supply and Demand**

The quantity supplied of a good is the amount of a product that a company is willing to make available for sale at a given price. The quantity demanded of a good is the amount of a product that consumers are willing to purchase at a given price. Suppose that the quantity supplied, S , and quantity demanded, D , of cellular telephones each month are given by the following functions:

$$S(p) = 60p - 900$$

$$D(p) = -15p + 2850$$

where p is the price (in dollars) of the telephone.

- (a) The **equilibrium price** of a product is defined as the price at which quantity supplied equals quantity demanded. That is, the equilibrium price is the price at which $S(p) = D(p)$. Find the equilibrium price of cellular telephones. What is the equilibrium quantity, the amount demanded (or supplied), at the equilibrium price?

$$60p - 900 = -15p + 2850$$

$$75p = 3750 \quad p = 50$$

$$s(50) = 60(50) - 900 = 2100$$

So the equilibrium price is \$50 and the equilibrium quantity is 2100 phones.

EXAMPLE**Supply and Demand**

$$S(p) = 60p - 900$$

$$D(p) = -15p + 2850$$

- (b) Determine the prices for which quantity supplied is greater than quantity demanded. That is, solve the inequality $S(p) > D(p)$.

$$60p - 900 > -15p + 2850$$

$$75p > 3750$$

$$p > 50$$

If the company charges more than \$50 per phone, then quantity supplied will exceed quantity demanded. In this case the company will have excess phones in inventory.

EXAMPLE**Supply and Demand**

$$S(p) = 60p - 900$$

$$D(p) = -15p + 2850$$

(c) Graph $S = S(p)$, $D = D(p)$ and label the equilibrium price.

