Section 4.1

Polynomial Functions and Models

1 Identify Polynomial Functions and Their Degree

A **polynomial function** is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \qquad \textbf{(1)}$$

where $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers and n is a nonnegative integer. The domain is the set of all real numbers.

EXAMPLE Identifying Polynomial Functions

Determine which of the following are polynomial functions. For those that are, state the degree; for those that are not, tell why not.

(a) $f(x) = 3x - 4x^3 + x^8$

(a) f is a polynomial of degree 8.

(b) $g(x) = \dfrac{x^2 + 3}{x - 1}$

(b) g is not a polynomial function.

It is the ratio of two distinct polynomials.

(c) $h(x) = 5$

(c) h is a polynomial function of degree 0.

It can be written $h(x) = 5x^0 = 5$.

(d) $F(x) = (x - 3)(x + 2)$

(d) F is a polynomial function of degree 2.

It can be written $F(x) = x^2 - x - 6$.

(e) $G(x) = 3x - 4x^{-1}$

(e) G is not a polynomial function.

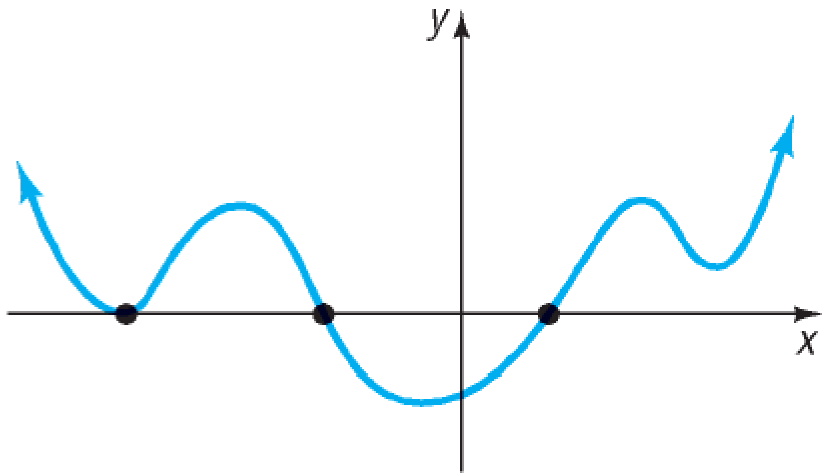
The second term does not have a nonnegative integer exponent.

(f) $H(x) = \dfrac{1}{2}x^3 - \dfrac{2}{3}x^2 + \dfrac{1}{4}x$

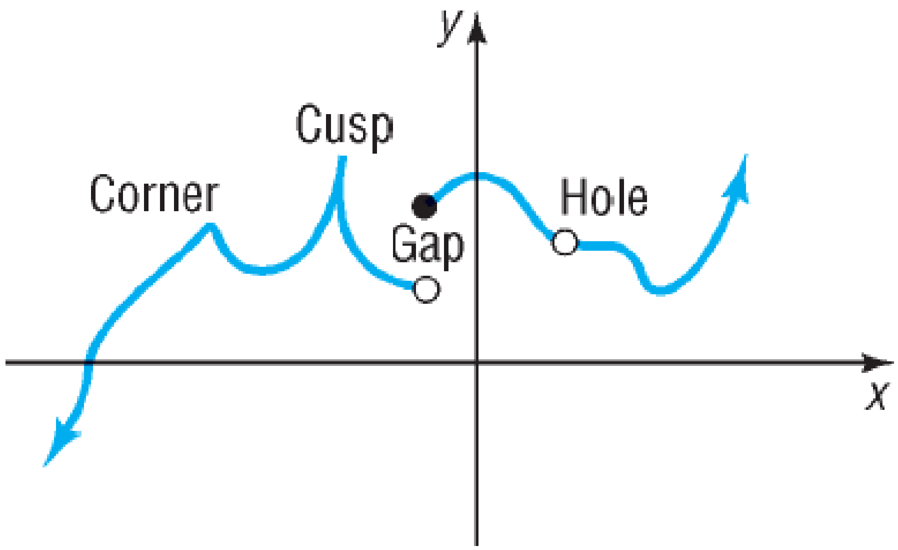
(f) H is a polynomial of degree 3.

Summary of the Properties of the Graphs of Polynomial Functions

Degree	Form	Name	Graph
No degree	$f(x) = 0$	Zero function	The x-axis
0	$f(x) = a_0, \quad a_0 \neq 0$	Constant function	Horizontal line with y-intercept a_0
1	$f(x) = a_1x + a_0, \quad a_1 \neq 0$	Linear function	Nonvertical, nonhorizontal line with slope a_1 and y-intercept a_0
2	$f(x) = a_2x^2 + a_1x + a_0, \quad a_2 \neq 0$	Quadratic function	Parabola: Graph opens up if $a_2 > 0$; graph opens down if $a_2 < 0$



(a) Graph of a polynomial function: smooth, continuous



(b) Cannot be the graph of a polynomial function

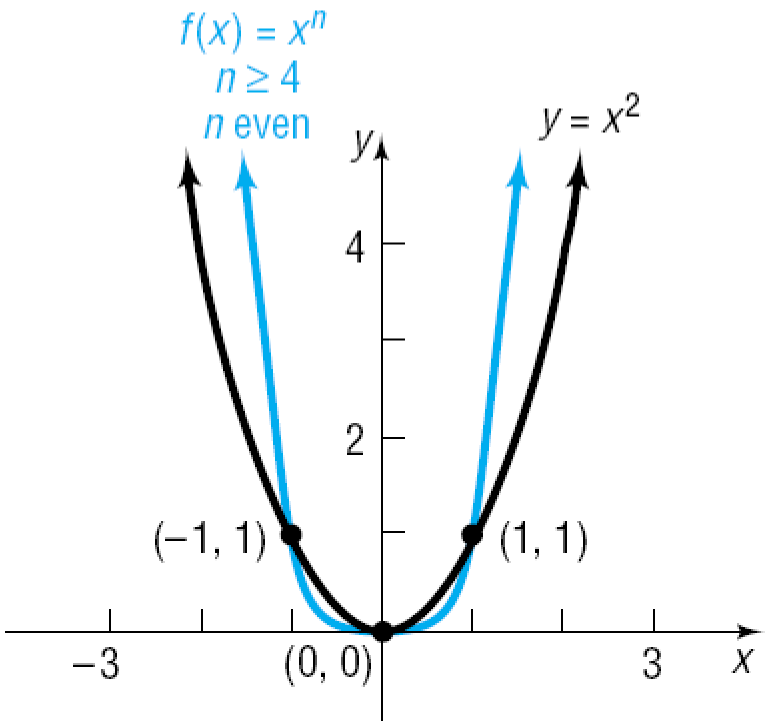
Power Functions

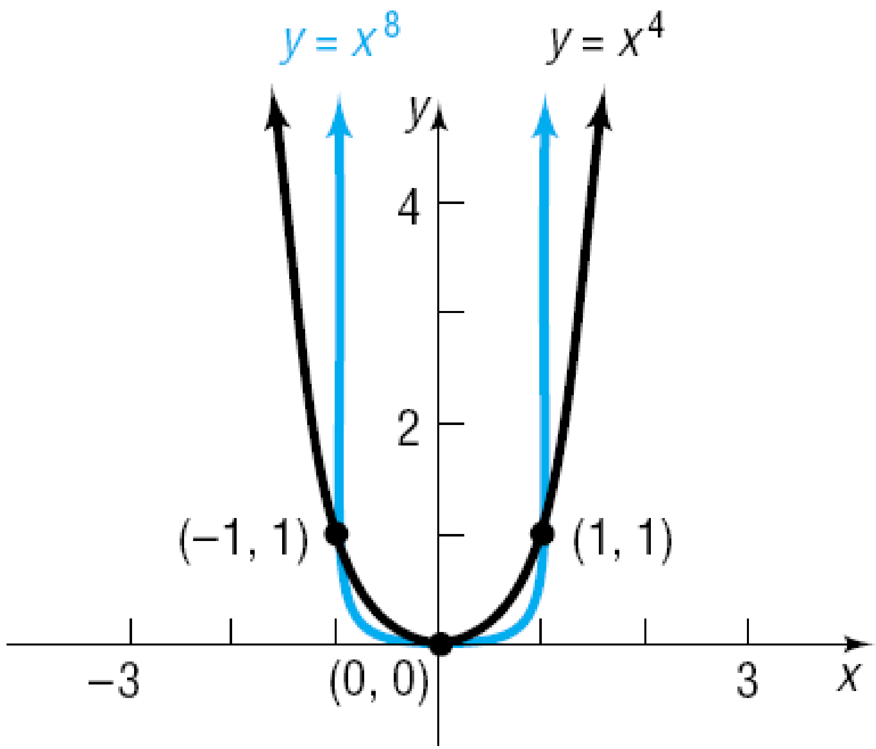
A **power function of degree** n is a function of the form

$$f(x) = ax^n$$

where a is a real number, $a \neq 0$, and $n > 0$ is an integer.

$f(x) = 3x$	$f(x) = -5x^2$	$f(x) = 8x^3$	$f(x) = -5x^4$
degree 1	degree 2	degree 3	degree 4

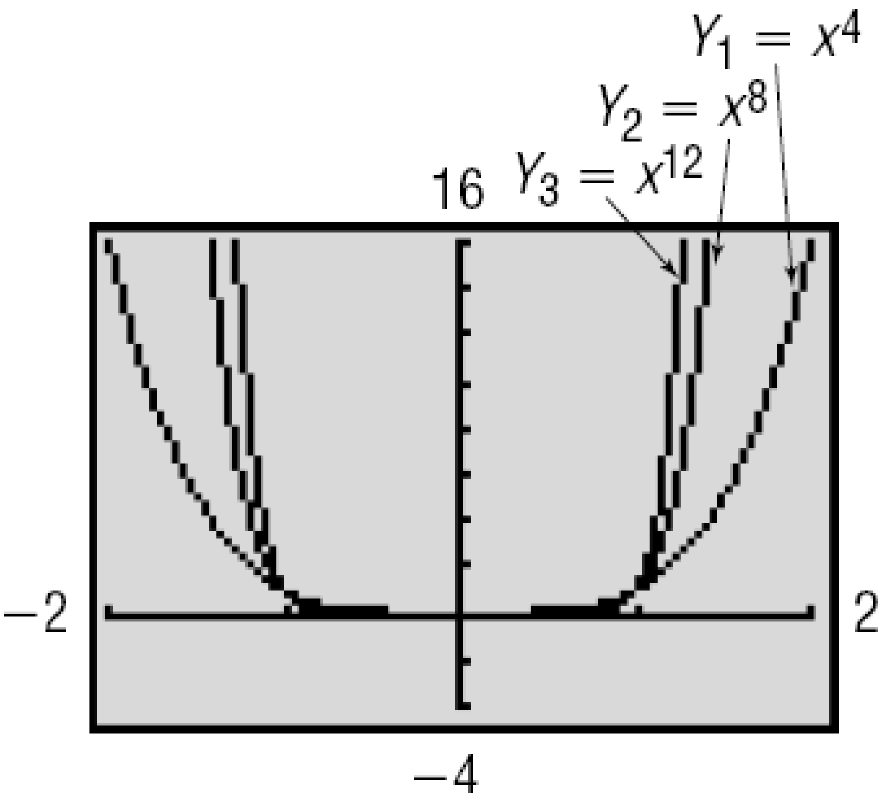


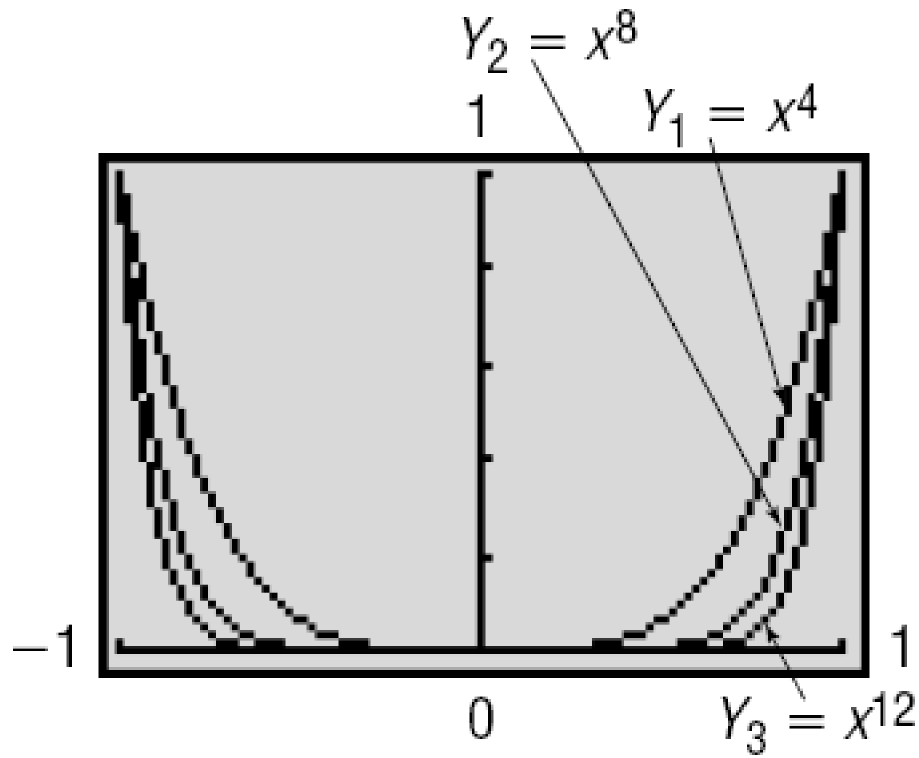


	$x = 0.1$	$x = 0.3$	$x = 0.5$
$f(x) = x^8$	10^{-8}	0.0000656	0.0039063
$f(x) = x^{20}$	10^{-20}	$3.487 \cdot 10^{-11}$	0.000001
$f(x) = x^{40}$	10^{-40}	$1.216 \cdot 10^{-21}$	$9.095 \cdot 10^{-13}$

Seeing the Concept

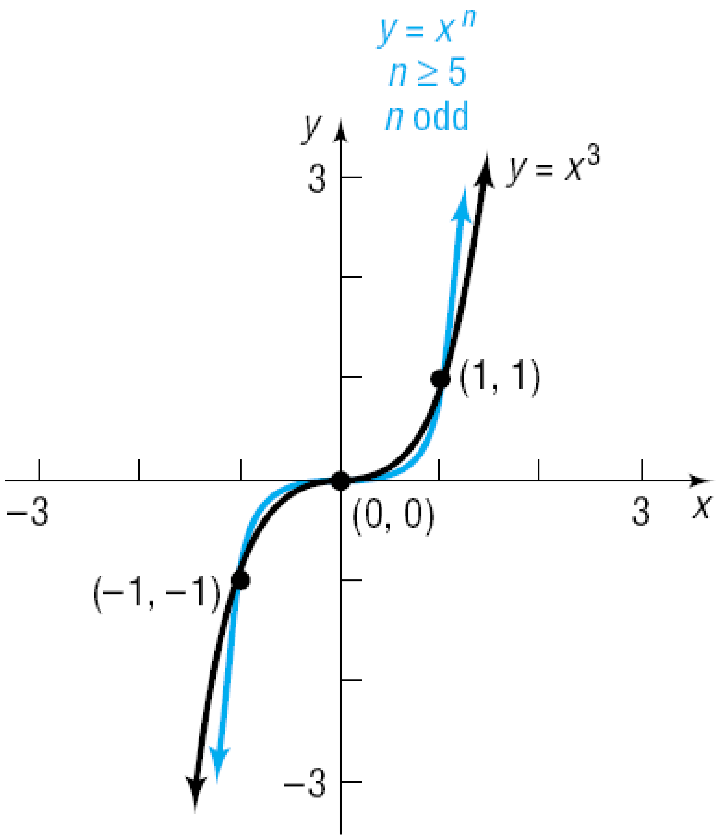
Graph $Y_1 = x^4$, $Y_2 = x^8$, and $Y_3 = x^{12}$ using the viewing rectangle $-2 \le x \le 2$, $-4 \le y \le 16$. Then graph each again using the viewing rectangle $-1 \le x \le 1$, $0 \le y \le 1$. See Figure 3. TRACE along one of the graphs to confirm that for x close to 0 the graph is above the x-axis and that for $x > 0$ the graph is increasing.

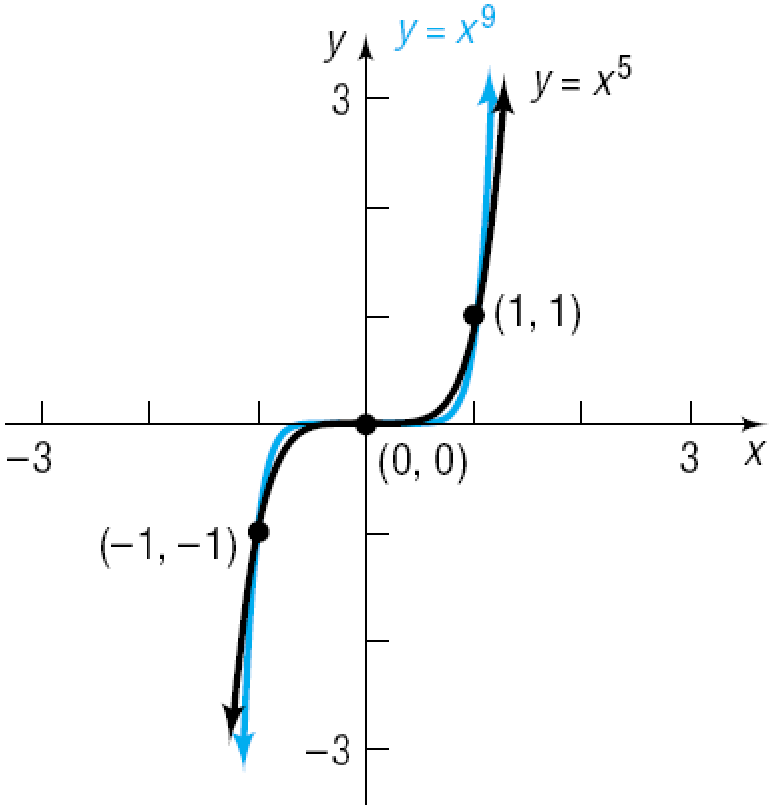




Properties of Power Functions, $f(x) = x^n$, n Is a Positive Even Integer

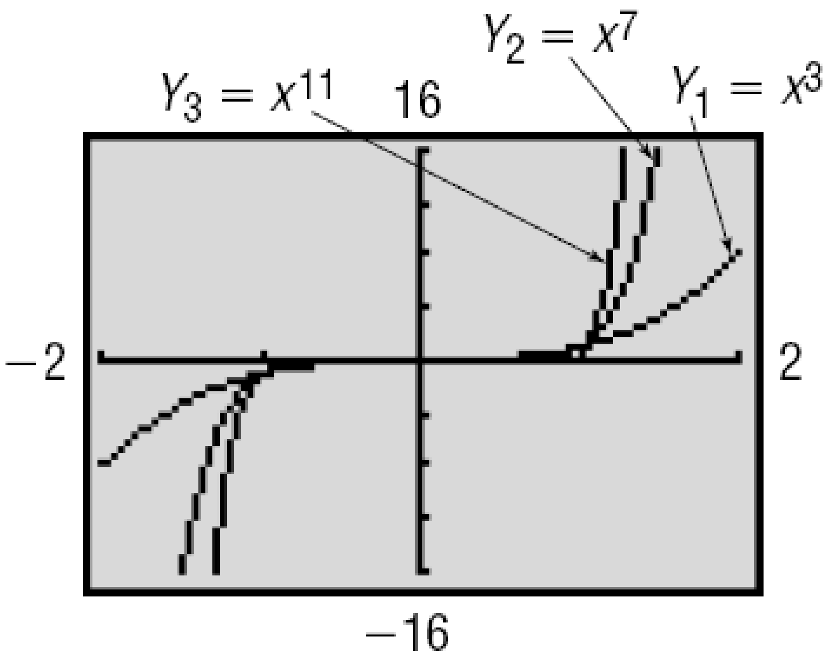
1. f is an even function, so its graph is symmetric with respect to the y-axis.
2. The domain is the set of all real numbers. The range is the set of nonnegative real numbers.
3. The graph always contains the points $(-1, 1)$, $(0, 0)$, and $(1, 1)$.
4. As the exponent n increases in magnitude, the function increases more rapidly when $x < -1$ or $x > 1$; but for x near the origin, the graph tends to flatten out and lie closer to the x-axis.

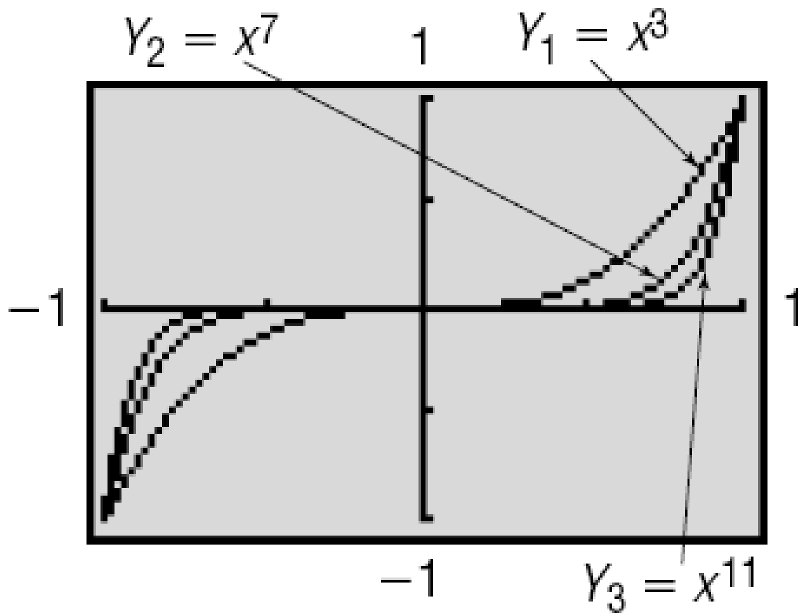

y = x^n
n ≥ 5
n odd
y
3
y = x^3
(1, 1)
−3
(0, 0)
3
x
(−1, −1)
−3


y
y = x^9
3
y = x^5
(1, 1)
−3
(0, 0)
3
x
(−1, −1)
−3

Seeing the Concept

Graph $Y_1 = x^3$, $Y_2 = x^7$, and $Y_3 = x^{11}$ using the viewing rectangle $-2 \le x \le 2$, $-16 \le y \le 16$. Then graph each again using the viewing rectangle $-1 \le x \le 1$, $-1 \le y \le 1$. See Figure 6. TRACE along one of the graphs to confirm that the graph is increasing and crosses the x-axis at the origin.





Properties of Power Functions, $f(x) = x^n$, n Is a Positive Odd Integer

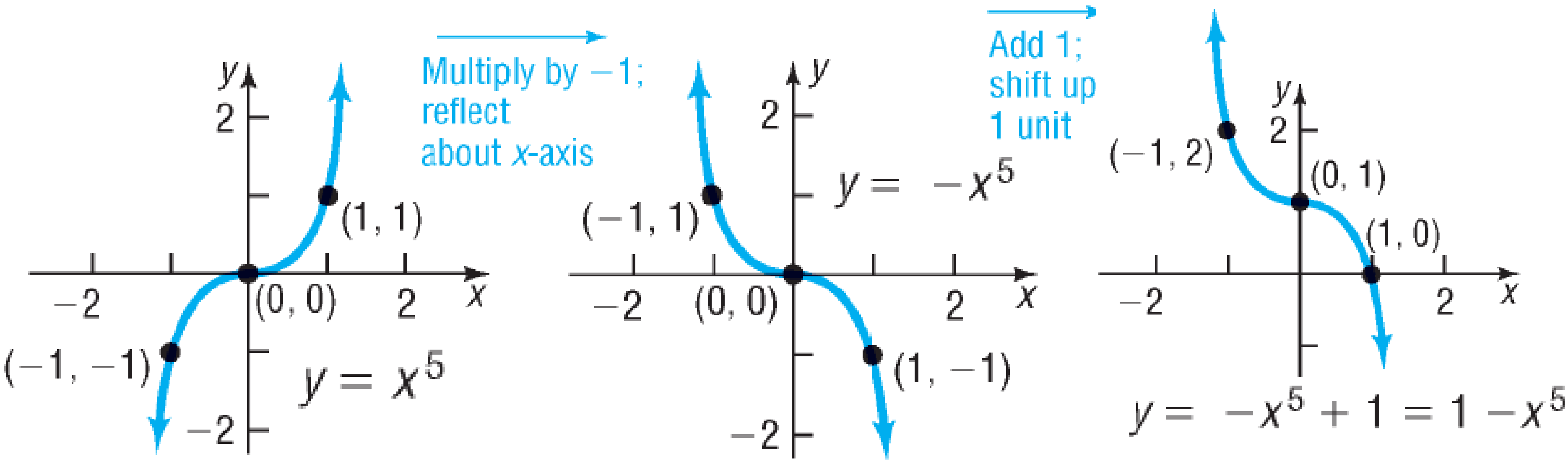
1. f is an odd function, so its graph is symmetric with respect to the origin.
2. The domain and the range are the set of all real numbers.
3. The graph always contains the points $(-1, -1)$, $(0, 0)$, and $(1, 1)$.
4. As the exponent n increases in magnitude, the function increases more rapidly when $x < -1$ or $x > 1$; but for x near the origin, the graph tends to flatten out and lie closer to the x-axis.

2 Graph Polynomial Functions Using Transformations

EXAMPLE

Graphing a Polynomial Function Using Transformations

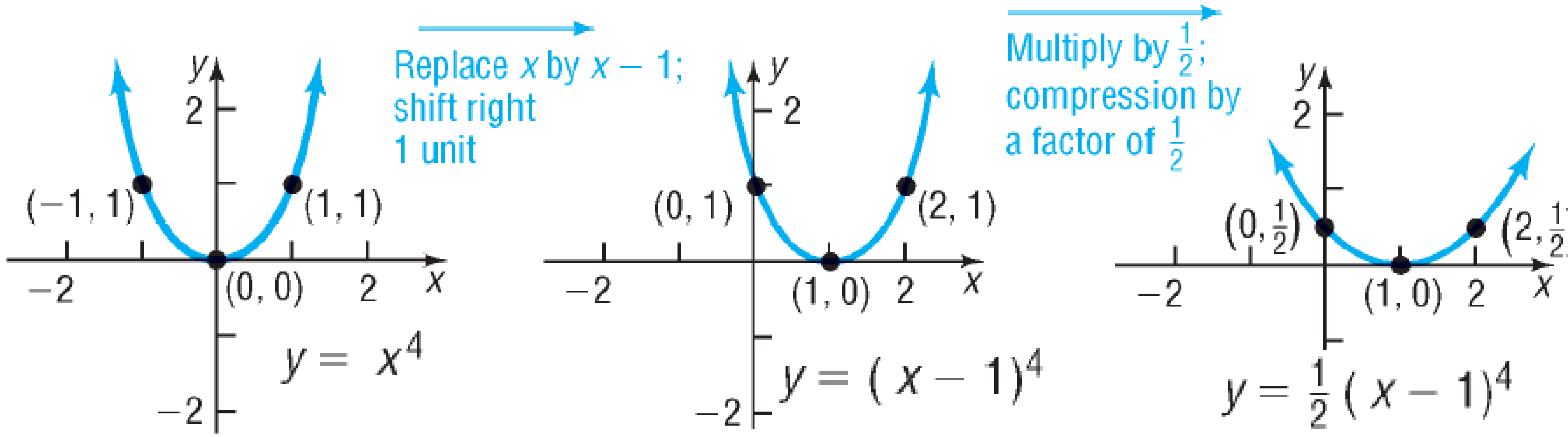
Graph: $f(x) = 1 - x^5$



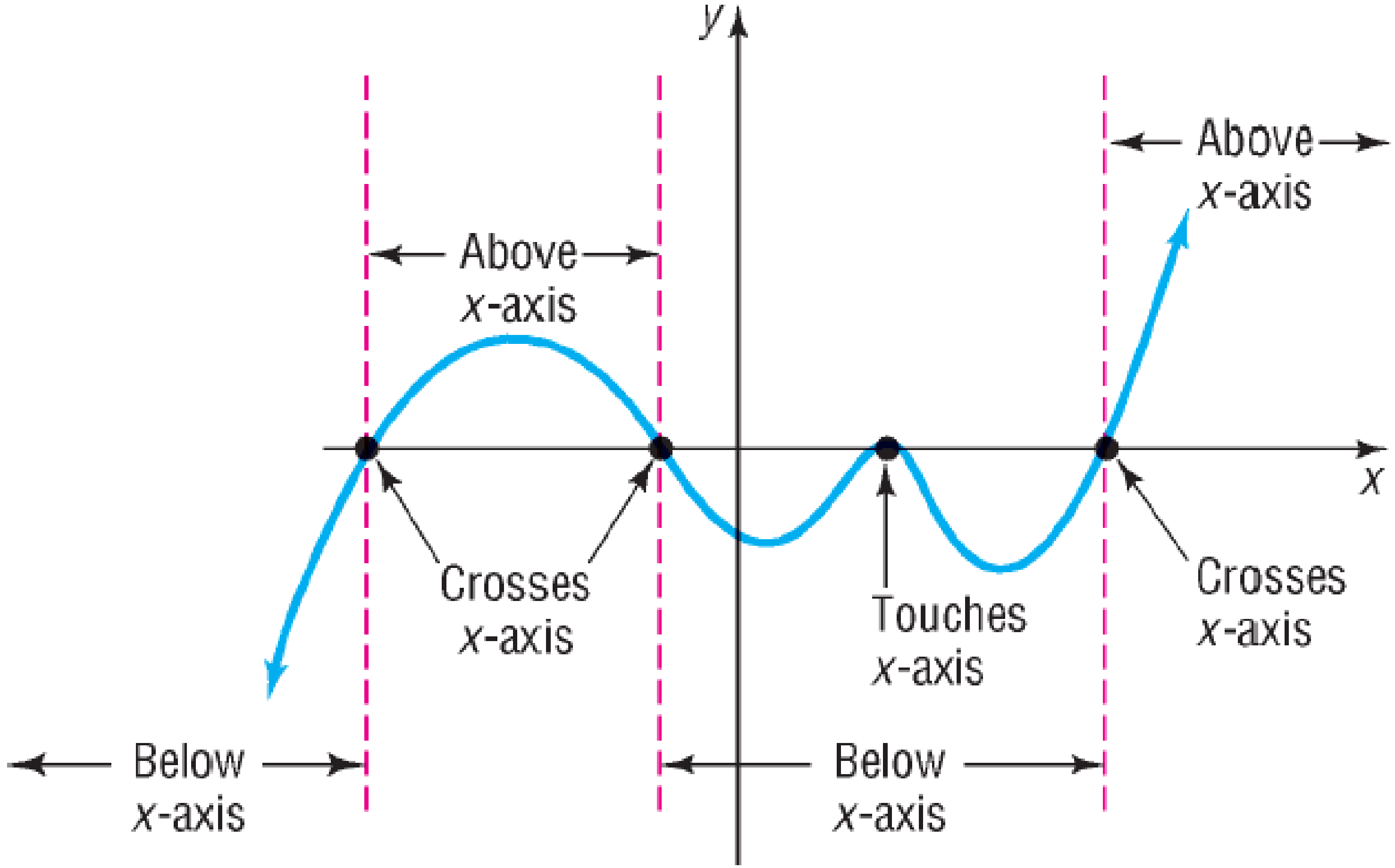
EXAMPLE

Graphing a Polynomial Function Using Transformations

$$\text{Graph: } f(x) = \frac{1}{2}(x-1)^4$$



3 Identify the Real Zeros of a Polynomial Function and Their Multiplicity


y
Above
x-axis
Above
x-axis
x
Crosses
x-axis
Touches
x-axis
Crosses
x-axis
Below
x-axis
Below
x-axis

DEFINITION

If f is a function and r is a real number for which $f(r) = 0$, then r is called a **real zero** of f.

As a consequence of this definition, the following statements are equivalent.

1. r is a real zero of a polynomial function f.
2. r is an x-intercept of the graph of f.
3. $x - r$ is a factor of f.
4. r is a solution to the equation $f(x) = 0$.

EXAMPLE

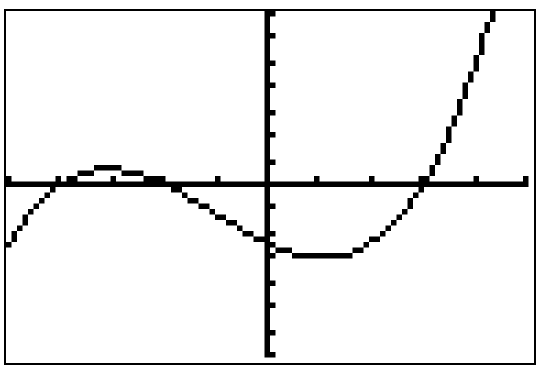
Finding a Polynomial Function from Its Zeros

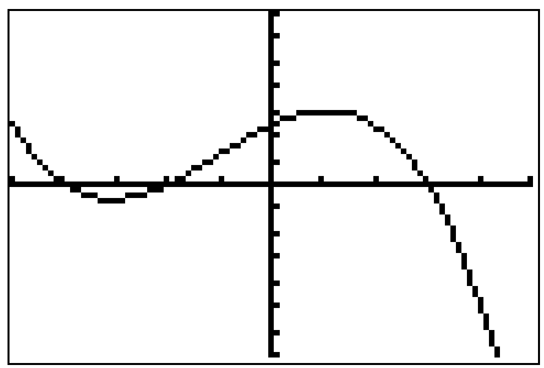
Find a polynomial of degree 3 whose zeros are -4, -2, and 3.

Use a graphing utility to verify your result.

$$f(x) = a(x+4)(x+2)(x-3) = a\left(x^3 + 3x^2 - 10x - 24\right)$$

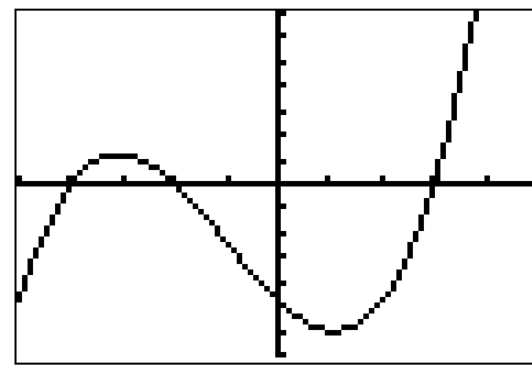
$f(x) = (x+4)(x+2)(x-3)$





$f(x) = -(x+4)(x+2)(x-3)$

$f(x) = 2(x+4)(x+2)(x-3)$



If $(x-r)^m$ is a factor of a polynomial f and $(x-r)^{m+1}$ is not a factor of f, then r is called a **zero of multiplicity m of f**.

EXAMPLE Identifying Zeros and Their Multiplicities

For the polynomial, list all zeros and their multiplicities.

$$f(x) = -2(x-2)(x+1)^3(x-3)^4$$

2 is a zero of multiplicity 2 because the exponent on the factor $x - 2$ is 1.

–1 is a zero of multiplicity 3 because the exponent on the factor $x + 1$ is 3.

3 is a zero of multiplicity 4 because the exponent on the factor $x - 3$ is 4.

EXAMPLE Graphing a Polynomial Using Its *x*-Intercepts

For the polynomial: $f(x) = x(x-3)^2$

(a) Find the *x*- and *y*-intercepts of the graph of *f*.

(b) Use the *x*-intercepts to find the intervals on which the graph of *f* is above the *x*-axis and the intervals on which the graph of *f* is below the *x*-axis.

(c) Locate other points on the graph and connect all the points plotted with a smooth, continuous curve.

(a) *x*-intercepts: $0 = x(x-3)^2$ $\qquad x = 0 \text{ or } (x-3)^2 = 0$

$$x = 0 \text{ or } x = 3$$

y-intercept: $f(0) = 0(0-3)^2 = 0$ $\qquad y = 0$

For the polynomial: $f(x) = x(x-3)^2$

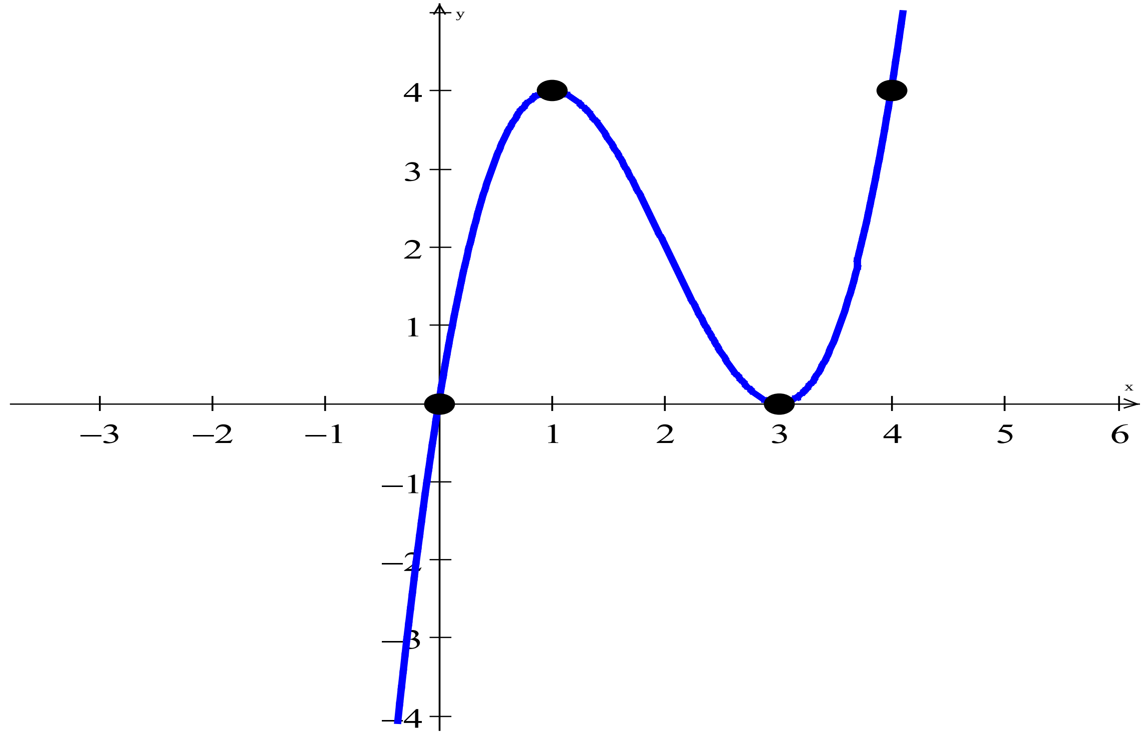
(a) Find the *x*- and *y*-intercepts of the graph of *f*. $(0,0),(3,0)$

(b) Use the *x*-intercepts to find the intervals on which the graph of *f* is above the *x*-axis and the intervals on which the graph of *f* is below the *x*-axis.

(c) Locate other points on the graph and connect all the points plotted with a smooth, continuous curve.

			
Interval	$(-\infty,0)$	$(0,3)$	$(3,\infty)$
Number Chosen	-1	1	4
Value of *f*	$f(-1)=-16$	$f(1)=4$	$f(4)=4$
Location of Graph	Below *x*-axis	Above *x*-axis	Above *x*-axis
Point on Graph	$(-1,-16)$	$(1,4)$	$(4,4)$

$$f(x) = x(x-3)^2$$



	0 3 x		
Interval	$(-\infty, 0)$	$(0, 3)$	$(3, \infty)$
Number Chosen	-1	1	4
Value of *f*	$f(-1) = -16$	$f(1) = 4$	$f(4) = 4$
Location of Graph	Below x-axis	Above x-axis	Above x-axis
Point on Graph	$(-1, -16)$	$(1, 4)$	$(4, 4)$

If r Is a Zero of Even Multiplicity

Sign of $f(x)$ does not change from one side of r to the other side of r.

Graph **touches** x-axis at r.

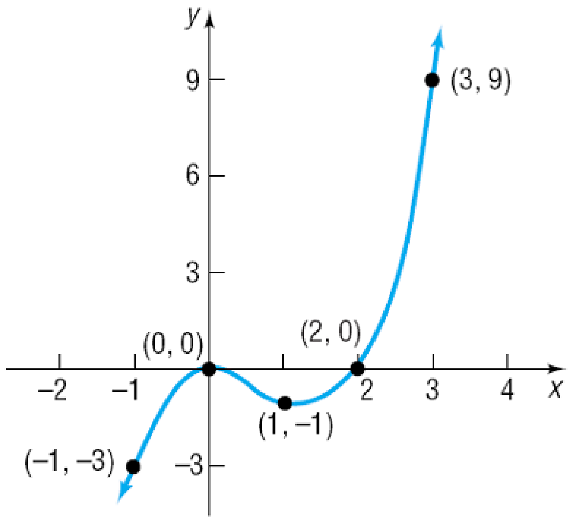
If r Is a Zero of Odd Multiplicity

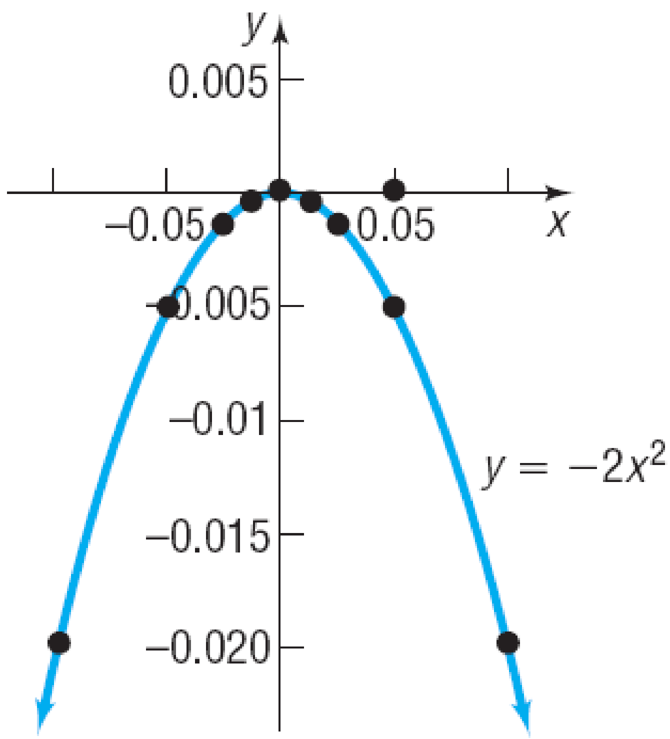
Sign of $f(x)$ changes from one side of r to the other side of r.

Graph **crosses** x-axis at r.

Behavior Near a Zero

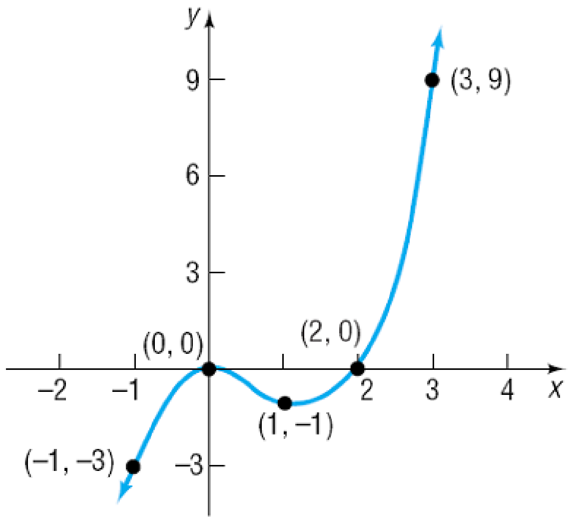
x	$f(x) = x^2(x-2)$	$y = -2x^2$
−0.1	−0.021	−0.02
−0.05	−0.0051	−0.005
−0.03	−0.001827	−0.0018
−0.01	−0.0000201	−0.0002
0	0	0
0.01	−0.000199	−0.0002
0.03	−0.001773	−0.0018
0.05	−0.004875	−0.005
0.1	−0.019	−0.02

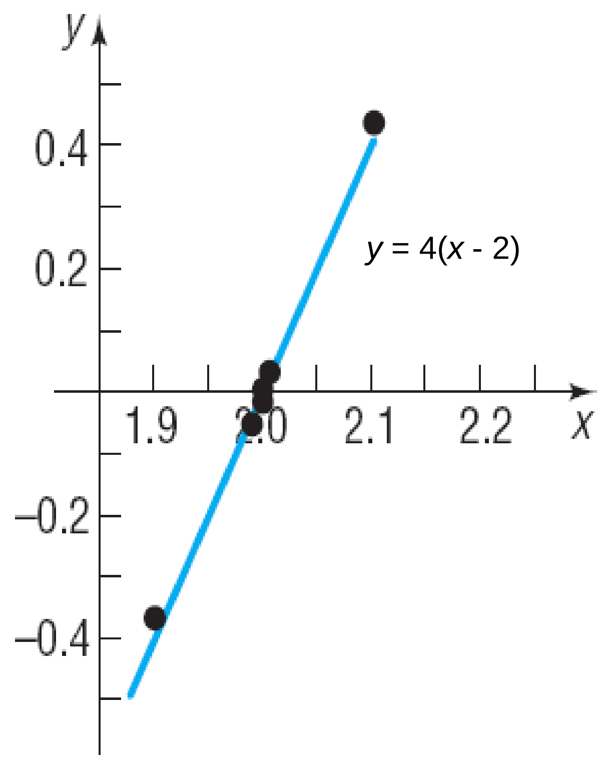




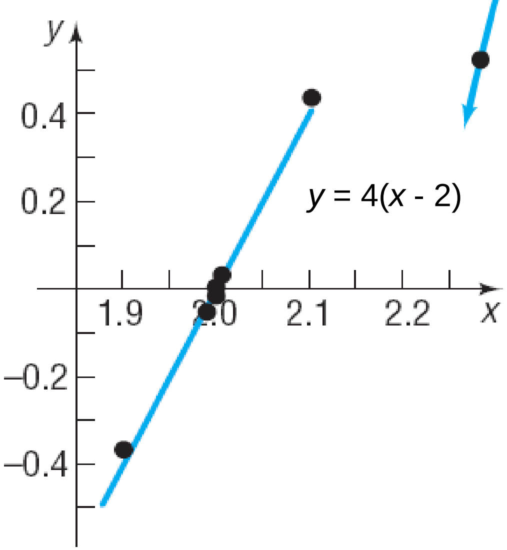
Behavior Near a Zero

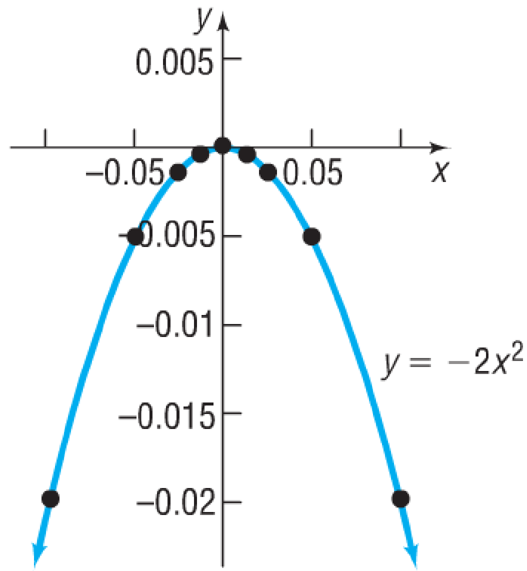
x	$f(x) = x^2(x - 2)$	$y = 4(x - 2)$
1.9	−0.361	−0.4
1.99	−0.0396	−0.04
1.999	−0.003996	−0.004
2	0	0
2.001	0.004004	0.004
2.01	0.0404	0.04
2.1	0.441	0.4

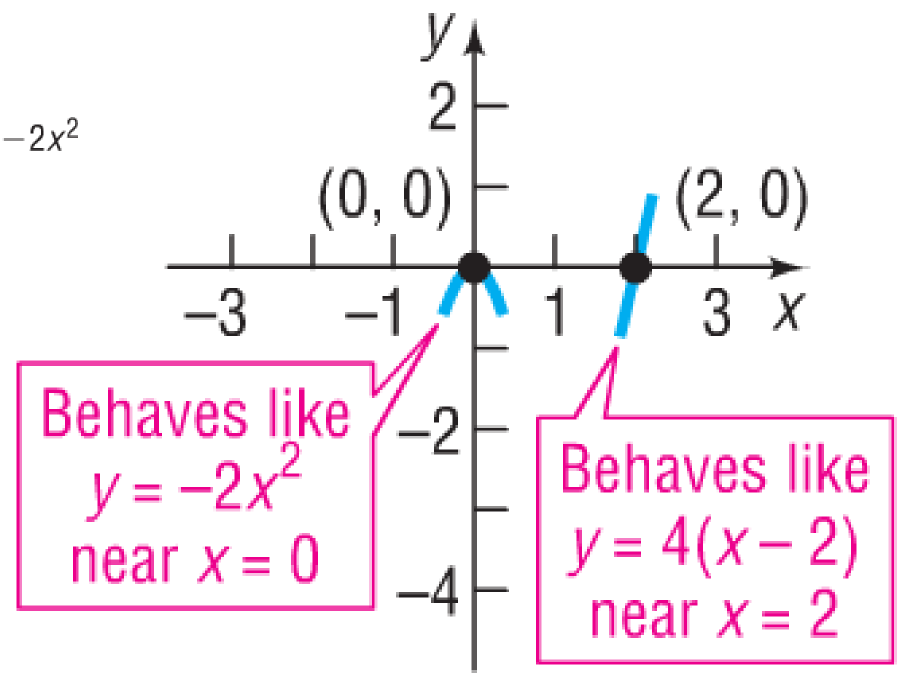




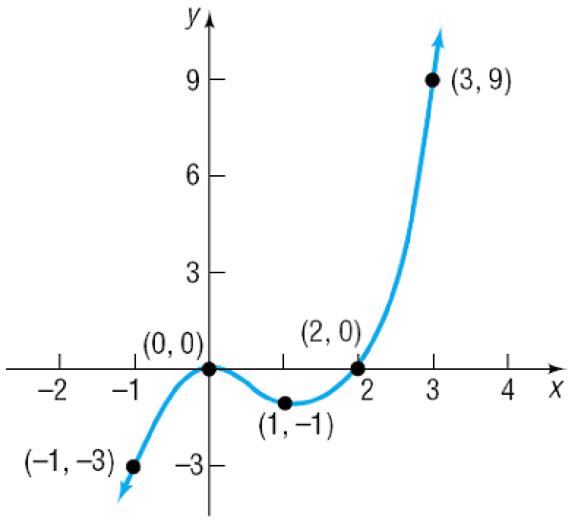
- The multiplicity of a real zero determines whether the graph crosses or touches the x-axis at the zero.
- The behavior of the graph near a real zero determines how the graph touches or crosses the x-axis.







Turning Points



Theorem

Turning Points

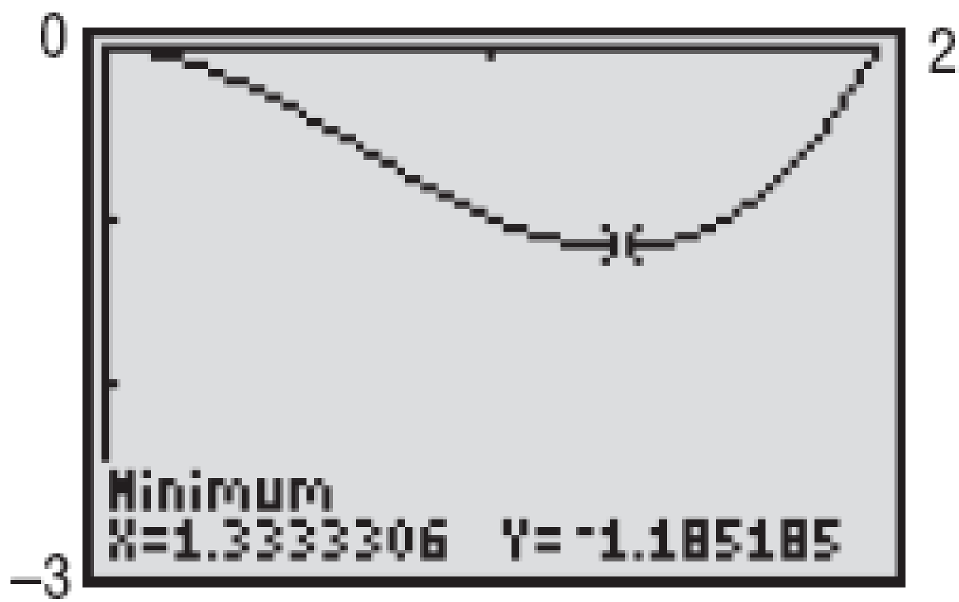
If f is a polynomial function of degree n, then f has at most $n - 1$ turning points.

If the graph of a polynomial function f has $n - 1$ turning points, the degree of f is at least n.

Exploration

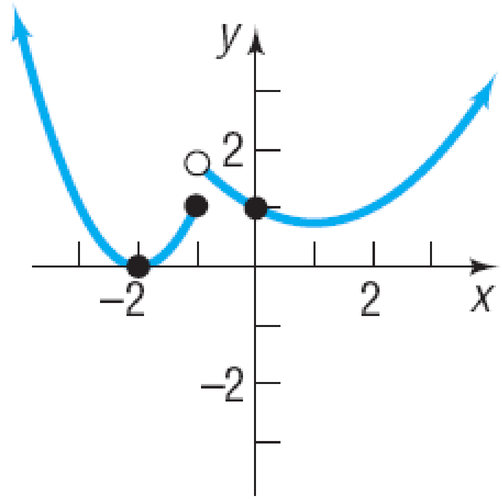
A graphing utility can be used to locate the turning points of a graph. Graph $Y_1 = x^2(x - 2)$. Use MINIMUM to find the location of the turning point for $0 < x < 2$. See Figure 15.

Figure 15



EXAMPLE Identifying the Graph of a Polynomial Function

Which of the graphs in Figure 16 could be the graph of a polynomial function? For those that could, list the zeros and state the least degree the polynomial can have. For those that could not, say why not.

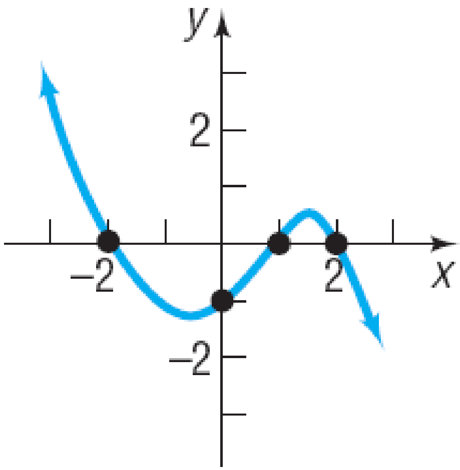


(a)

(a) The graph in Figure 16(a) cannot be the graph of a polynomial function because of the gap that occurs at $x = -1$. Remember, the graph of a polynomial function is continuous—no gaps or holes.

EXAMPLE Identifying the Graph of a Polynomial Function

Which of the graphs in Figure 16 could be the graph of a polynomial function? For those that could, list the zeros and state the least degree the polynomial can have. For those that could not, say why not.

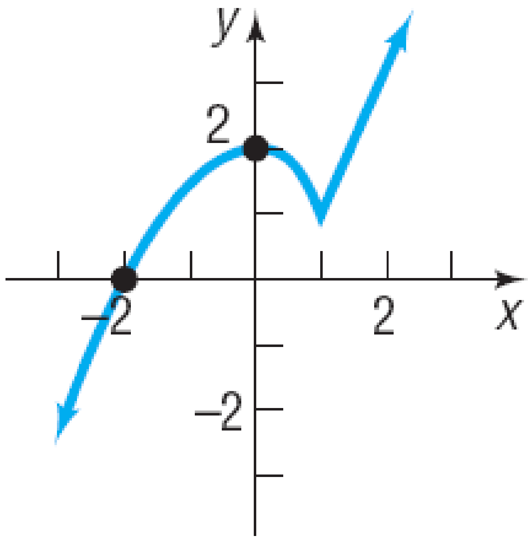


(b)

(b) The graph in Figure 16(b) could be the graph of a polynomial function because the graph is smooth and continuous. It has three real zeros, at -2, at 1, and at 2. Since the graph has two turning points, the degree of the polynomial function must be at least 3.

EXAMPLE Identifying the Graph of a Polynomial Function

Which of the graphs in Figure 16 could be the graph of a polynomial function? For those that could, list the zeros and state the least degree the polynomial can have. For those that could not, say why not.

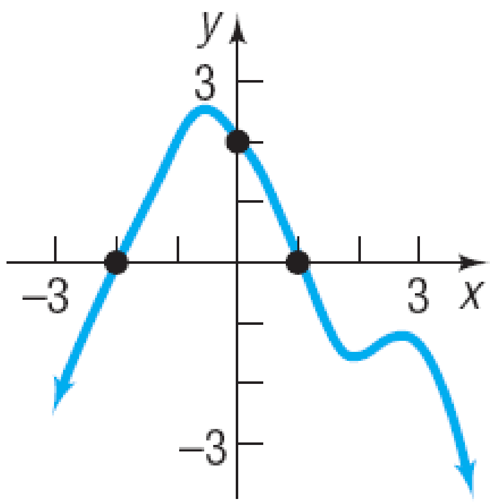


(c)

(c) The graph in Figure 16(c) cannot be the graph of a polynomial function because of the cusp at $x = 1$. Remember, the graph of a polynomial function is smooth.

EXAMPLE **Identifying the Graph of a Polynomial Function**

Which of the graphs in Figure 16 could be the graph of a polynomial function? For those that could, list the zeros and state the least degree the polynomial can have. For those that could not, say why not.



(d)

(d) The graph in Figure 16(d) could be the graph of a polynomial function. It has two real zeros, at -2 and at 1. Since the graph has three turning points, the degree of the polynomial function is at least 4.

End Behavior

Theorem

End Behavior

For large values of x, either positive or negative, that is, for large $|x|$, the graph of the polynomial

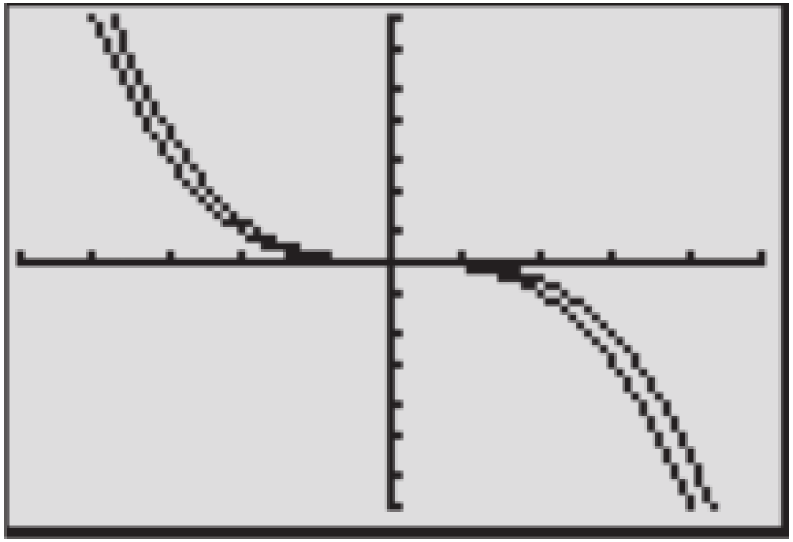
$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

resembles the graph of the power function

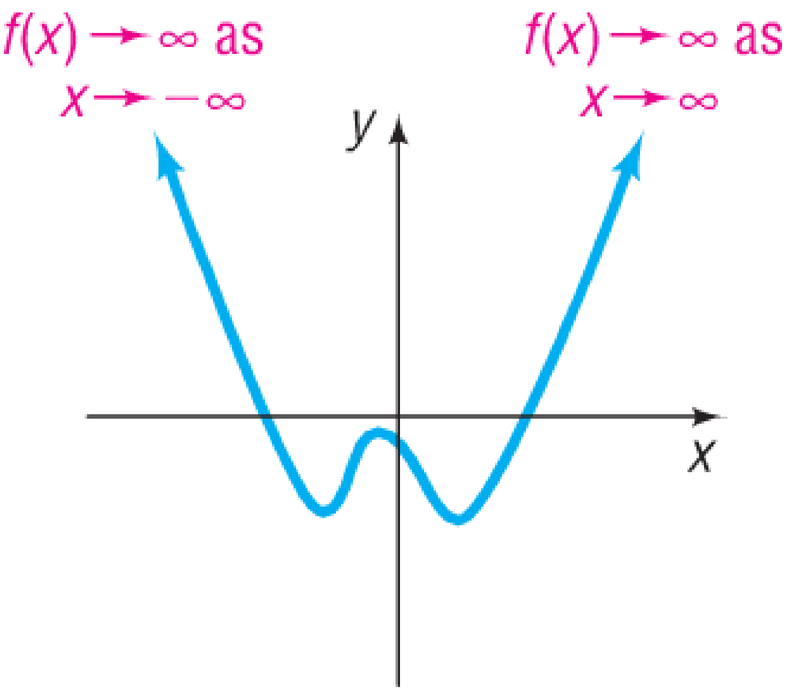
$$y = a_n x^n$$

$$f(x) = -2x^3 + 5x^2 + x - 4$$

x	$f(x)$	$y = -2x^3$
10	−1,494	−2,000
100	−1,949,904	−2,000,000
500	−248,749,504	−250,000,000
1,000	−1,994,999,004	−2,000,000,000

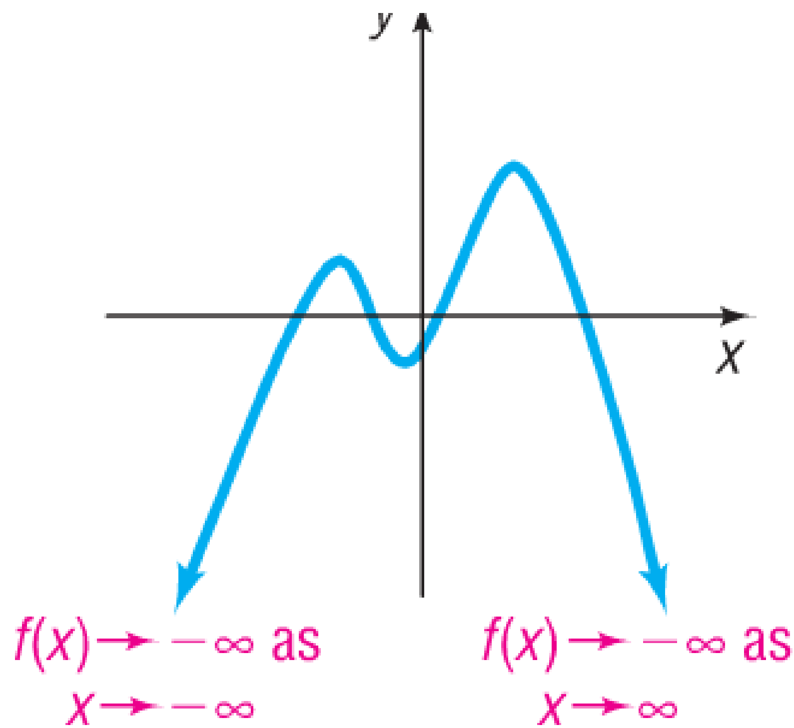


End behavior of $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$



(a)

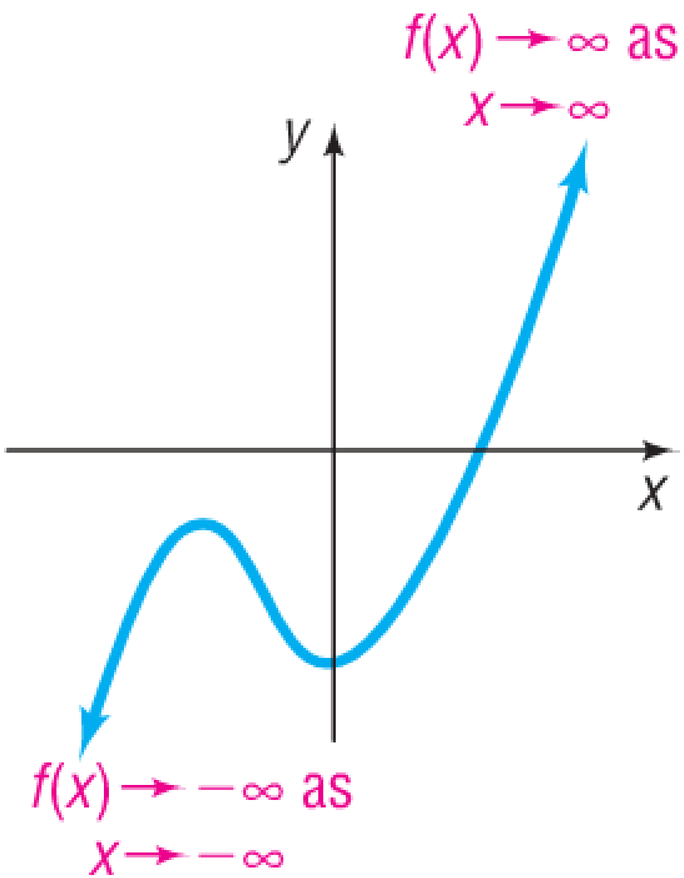
$n \geq 2$ even; $a_n > 0$



(b)

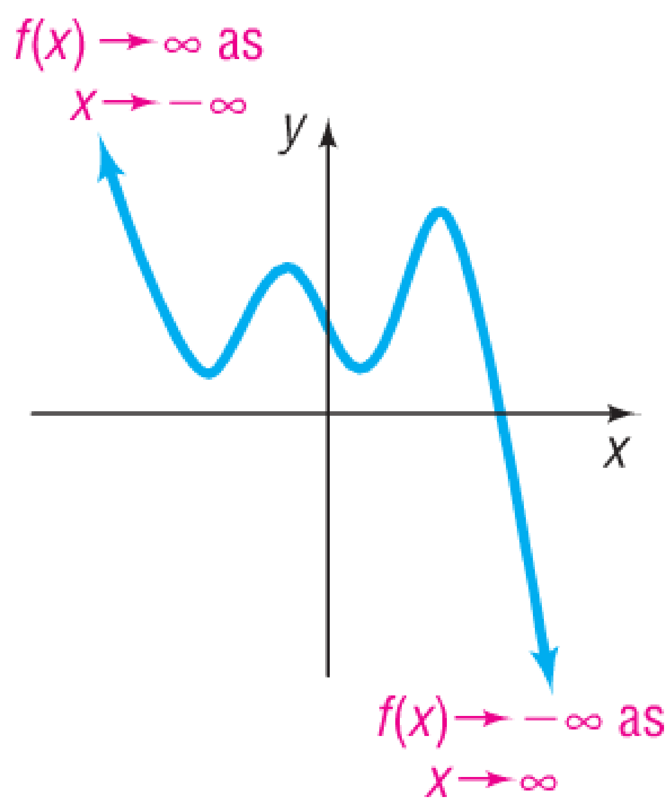
$n \geq 2$ even; $a_n < 0$

End behavior of $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$



(c)

$n \geq 3$ odd; $a_n > 0$



(d)

$n \geq 3$ odd; $a_n < 0$

EXAMPLE Identifying the Graph of a Polynomial Function

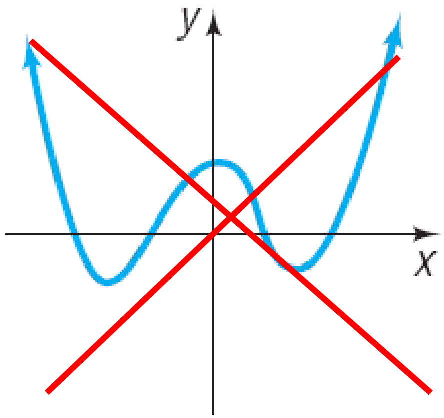
Which of the graphs in Figure 18 could be the graph of

$$f(x) = x^4 + 5x^3 + 5x^2 - 5x - 6?$$

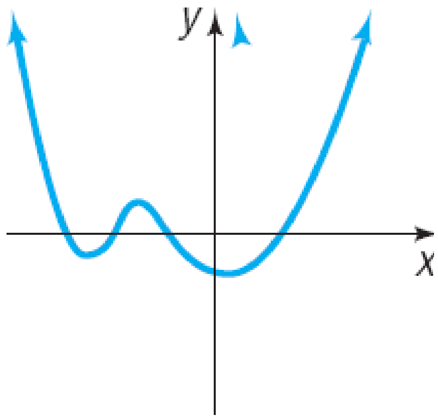
$f(0) = -6$ so the y intercept is -6.

The degree is 4 so the graph can turn at most 3 times.

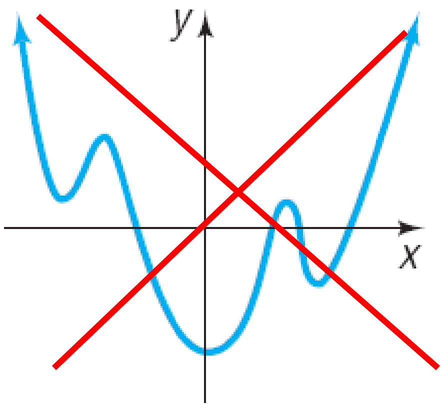
For large values of x, end behavior is like x^4 (both ends approach ∞)



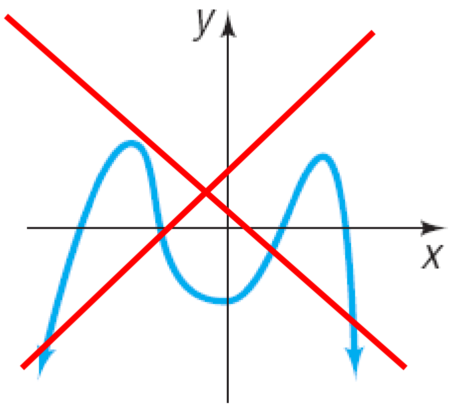
(a)



(b)



(c)



(d)

SUMMARY

Graph of a Polynomial Function $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \quad a_n \neq 0$

Degree of the polynomial function f: n

Graph is smooth and continuous.

Maximum number of turning points: $n - 1$

At a zero of even multiplicity: The graph of f touches the x-axis.

At a zero of odd multiplicity: The graph of f crosses the x-axis.

Between zeros, the graph of f is either above or below the x-axis.

End behavior: For large $|x|$, the graph of f behaves like the graph of $y = a_n x^n$.

4 Analyze the Graph of a Polynomial Function

EXAMPLE **How to Analyze the Graph of a Polynomial Function**

Analyze the graph of the polynomial function $f(x) = (2x + 1)(x - 3)^2$.

Step-by-Step Solution

Step 1: Determine the end behavior of the graph of the function.

Expand the polynomial to write it in the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

$$\begin{aligned} f(x) &= (2x + 1)(x - 3)^2 \\ &= (2x + 1)(x^2 - 6x + 9) \\ &= 2x^3 - 12x^2 + 18x + x^2 - 6x + 9 \\ &= 2x^3 - 11x^2 + 12x + 9 \end{aligned}$$

The polynomial function f is of degree 3. The graph of f behaves like $y = 2x^3$ for large values of $|x|$.

EXAMPLE **How to Analyze the Graph of a Polynomial Function**

Analyze the graph of the polynomial function $f(x) = (2x + 1)(x - 3)^2$.

Step 2: Find the x- and y-intercepts of the graph of the function.

The y-intercept is $f(0) = 9$.

To find the x-intercepts, we solve $f(x) = 0$.

$$(2x + 1)(x - 3)^2 = 0$$

$$2x + 1 = 0 \quad \text{or} \quad (x - 3)^2 = 0$$

$$x = -\frac{1}{2} \quad \text{or} \quad x - 3 = 0$$

$$x = 3$$

The x-intercepts are $-\frac{1}{2}$ and 3.

Step 3: Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the x-axis at each x-intercept.

The zero $-\frac{1}{2}$ has multiplicity 1 so the graph crosses there.

The zero 3 has multiplicity 2 so the graph touches there.

EXAMPLE How to Analyze the Graph of a Polynomial Function

Analyze the graph of the polynomial function $f(x) = (2x+1)(x-3)^2$.

Step 4: Determine the maximum number of turning points on the graph of the function.

The polynomial is degree 3 so the graph can turn at most 2 times.

Step 5: Determine the behavior of the graph of f near each x-intercept.

Near $-\frac{1}{2}$:

$$
\begin{aligned}
f(x) &= (2x+1)(x-3)^2 \\
&\approx (2x+1)\left(-\frac{1}{2}+3\right)^2 \\
&= (2x+1)\left(\frac{25}{4}\right) \\
&= \frac{25}{2}x + \frac{25}{4}
\end{aligned}
$$

A line with slope $\frac{25}{2}$

Near 3:

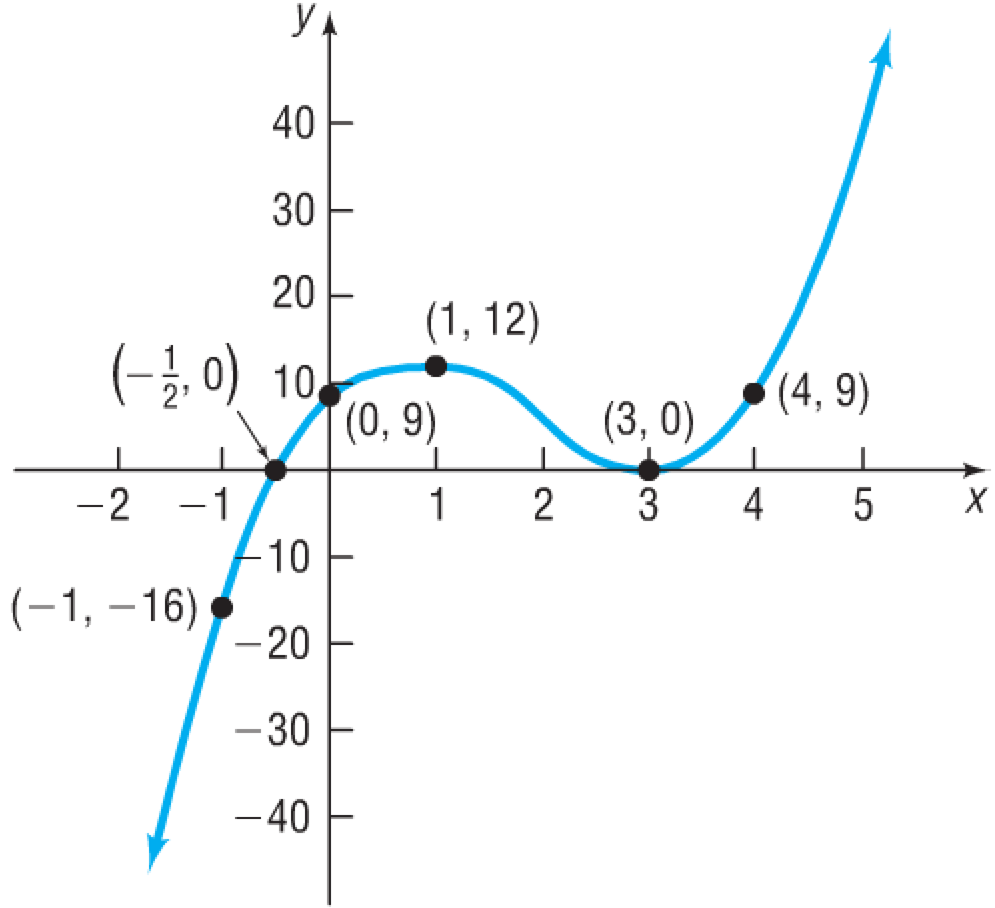
$$
\begin{aligned}
f(x) &= (2x+1)(x-3)^2 \\
&\approx (2\cdot 3+1)(x-3)^2 \\
&= 7(x-3)^2
\end{aligned}
$$

A parabola that opens up

EXAMPLE **How to Analyze the Graph of a Polynomial Function**

Analyze the graph of the polynomial function $f(x) = (2x + 1)(x - 3)^2$.

Step 6: Put all the information from Steps 1 through 5 together to obtain the graph of f.



SUMMARY Analyzing the Graph of a Polynomial Function

STEP 1: Determine the end behavior of the graph of the function.

STEP 2: Find the x- and y-intercepts of the graph of the function.

STEP 3: Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the x-axis at each x-intercept.

STEP 4: Determine the maximum number of turning points on the graph of the function.

STEP 5: Determine the behavior of the graph near each x-intercept.

STEP 6: Use the information in Steps 1 through 5 to draw a complete graph of the function.

EXAMPLE Analyzing the Graph of a Polynomial Function

Analyze the graph of the polynomial function $f(x) = x^2(x-4)(x+1)$

Step 1: End behavior: the graph of f resembles that of the power function $y = x^4$ for large values of $|x|$.

Step 2: The y-intercept is $f(0) = 0$. The x-intercepts satisfy the equation

$$f(x) = x^2(x-4)(x+1) = 0$$

$x^2 = 0$ or $x - 4 = 0$ or $x + 1 = 0$ $\quad x = 0$ or $x = 4$ or $x = -1$

Step 3: The intercept 0 is a zero of multiplicity 2, so the graph of f will touch the x-axis at 0; 4 and -1 are zeros of multiplicity 1, so the graph of f will cross the x-axis at 4 and -1.

Step 4: The graph of f will contain at most three turning points.

Step 5: The three x-intercepts are -1, 0, and 4.

Near -1: $f(x) = x^2(x-4)(x+1) \approx (-1)^2(-1-4)(x+1) = -5(x+1)$ A line with slope -5

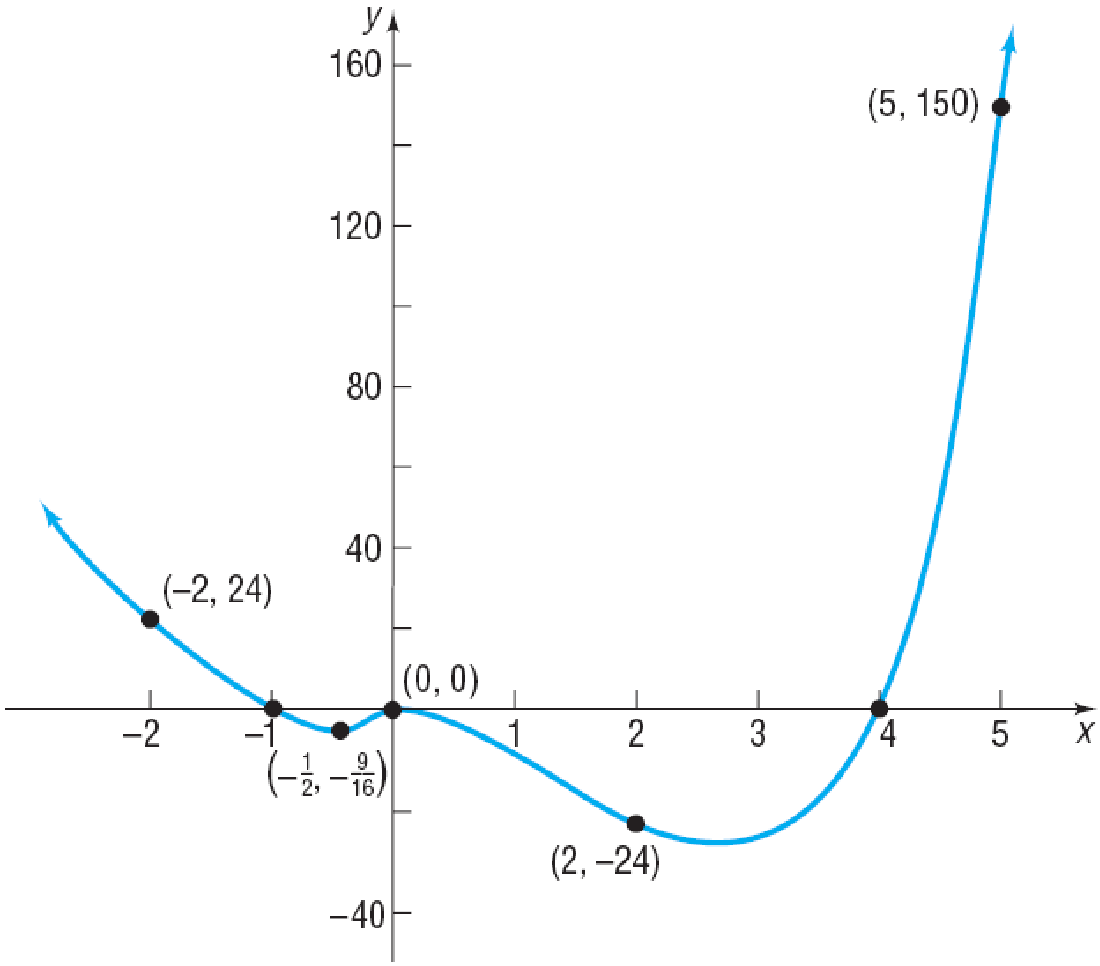
Near 0: $f(x) = x^2(x-4)(x+1) \approx x^2(0-4)(0+1) = -4x^2$ A parabola opening down

Near 4: $f(x) = x^2(x-4)(x+1) \approx 4^2(x-4)(4+1) = 80(x-4)$ A line with slope 80

EXAMPLE Analyzing the Graph of a Polynomial Function

Analyze the graph of the polynomial function

$$f(x) = x^2(x - 4)(x + 1)$$



EXAMPLE **How to Use a Graphing Utility to Analyze the Graph of a Polynomial Function**

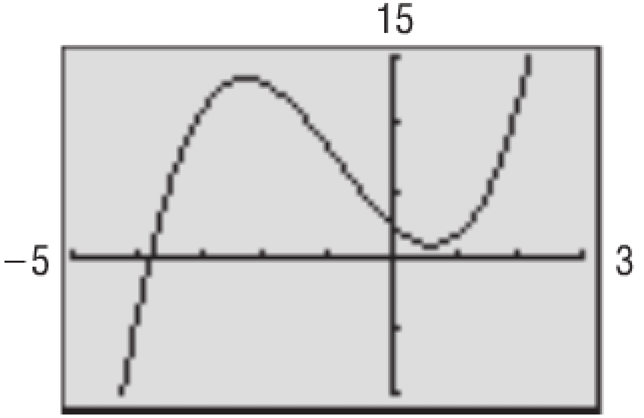
Analyze the graph of the polynomial function

$$f(x) = x^3 + 2.48x^2 - 4.3155x + 2.484406$$

Step 1: Determine the end behavior of the graph of the function.

The polynomial function f is of degree 3. The graph of f behaves like $y = x^3$ for large values of $|x|$.

Step 2: Graph the function using a graphing utility.



Step 3: Use a graphing utility to approximate the x- and y-intercepts of the graph.

The y-intercept is $f(0) = 2.484406$. In Examples 9 and 10, the polynomial function was factored, so it was easy to find the x-intercepts algebraically. However, it is not readily apparent how to factor f in this example. Therefore, we use a graphing utility's ZERO (or ROOT or SOLVE) feature and find the lone x-intercept to be -3.79, rounded to two decimal places.

EXAMPLE How to Use a Graphing Utility to Analyze the Graph of a Polynomial Function

Analyze the graph of the polynomial function

$$f(x) = x^3 + 2.48x^2 - 4.3155x + 2.484406$$

Step 4: Use a graphing utility to create a TABLE to find points on the graph around each x-intercept.

Table 7 shows values of x on each side of the x-intercept. The points $(-4, -4.57)$ and $(-2, 13.04)$ are on the graph.

Table 7

X	Y1	
-4	-4.574	
-2	13.035	

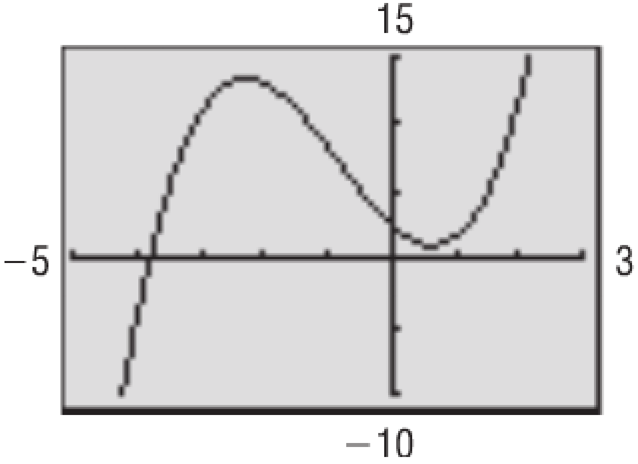
Y1=X^3+2.48X²-4...

EXAMPLE **How to Use a Graphing Utility to Analyze the Graph of a Polynomial Function**

Analyze the graph of the polynomial function

$$f(x) = x^3 + 2.48x^2 - 4.3155x + 2.484406$$

Step 5: Approximate the turning points of the graph.



From the graph of f shown in Figure 22, we can see that f has two turning points. Using MAXIMUM, one turning point is at $(-2.28, 13.36)$, rounded to two decimal places. Using MINIMUM, the other turning point is at $(0.63, 1)$, rounded to two decimal places.

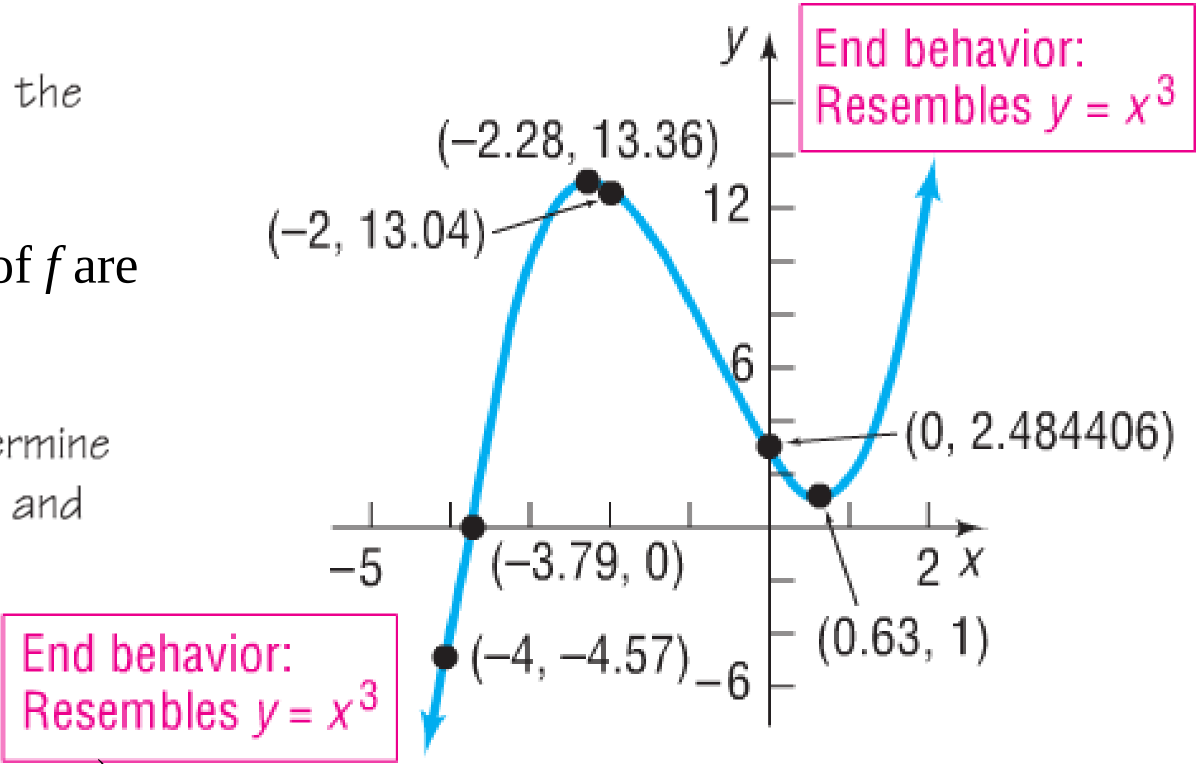
EXAMPLE

How to Use a Graphing Utility to Analyze the Graph of a Polynomial Function

Analyze the graph of the polynomial function

$$f(x) = x^3 + 2.48x^2 - 4.3155x + 2.484406$$

Step 6: Use the information in Steps 1 through 5 to draw a complete graph of the function by hand.



Step 7: Find the domain and the range of the function.

The domain and the range of *f* are the set of all real numbers.

Step 8: Use the graph to determine where the function is increasing and where it is decreasing.

Decreasing: $(-2.28, 0.63)$

Increasing: $(-\infty, -2.28)$ and $(0.63, \infty)$

Using a Graphing Utility to Analyze the Graph of a Polynomial Function

STEP 1: Determine the end behavior of the graph of the function.

STEP 2: Graph the function using a graphing utility.

STEP 3: Use a graphing utility to approximate the x- and y-intercepts of the graph.

STEP 4: Use a graphing utility to create a TABLE to find points on the graph around each x-intercept.

STEP 5: Approximate the turning points of the graph.

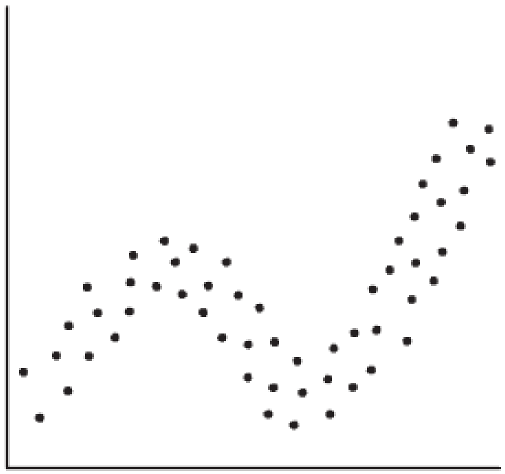
STEP 6: Use the information in Steps 1 through 5 to draw a complete graph of the function by hand.

STEP 7: Find the domain and the range of the function.

STEP 8: Use the graph to determine where the function is increasing and where it is decreasing.

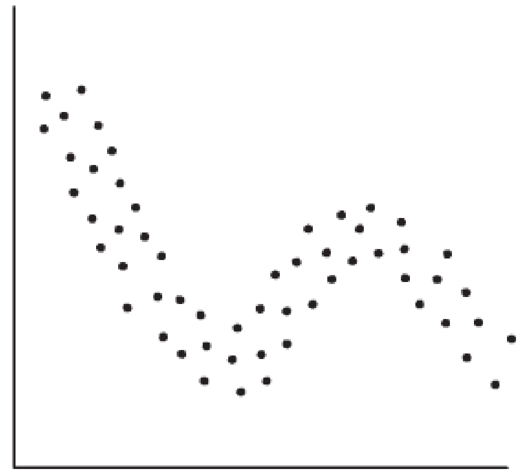
5 Build Cubic Models from Data

Data that follow a cubic relation should look like Figure 24(a) or (b).



$y = ax^3 + bx^2 + cx + d, a > 0$

(a)



$y = ax^3 + bx^2 + cx + d, a < 0$

(b)

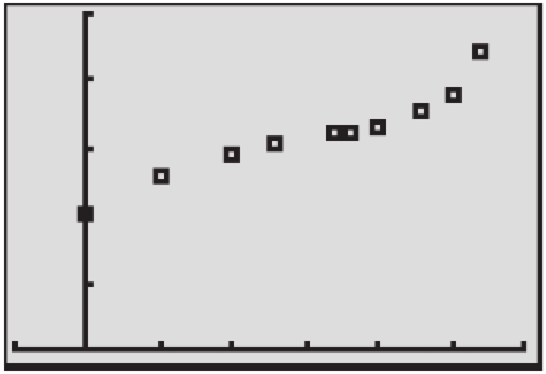
EXAMPLE A Cubic Function of Best Fit

The data in Table 8 represent the weekly cost C (in thousands of dollars) of printing x thousand textbooks.

(a) Draw a scatter diagram of the data using x as the independent variable and C as the dependent variable. Comment on the type of relation that may exist between the two variables x and C.

Number of Textbooks, x	Cost, C
0	100
5	128.1
10	144
13	153.5
17	161.2
18	162.6
20	166.3
23	178.9
25	190.2
27	221.8

Figure 25



A cubic relation may exist between the two variables.

EXAMPLE A Cubic Function of Best Fit

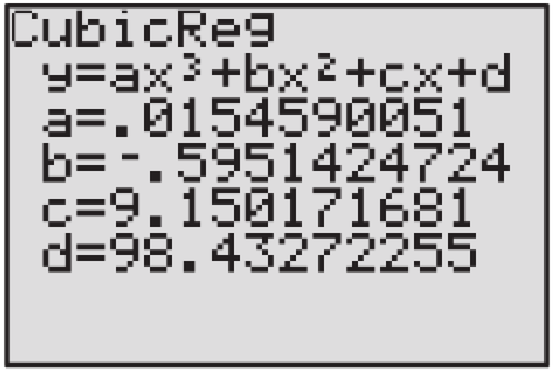
(b) Using a graphing utility, find the cubic function of best fit $C = C(x)$ that models the relation between number of texts and cost.

(c) Graph the cubic function of best fit on your scatter diagram.

(d) Use the function found in part (b) to predict the cost of printing 22 thousand texts per week.

Number of Textbooks, *x*	Cost, *C*
0	100
5	128.1
10	144
13	153.5
17	161.2
18	162.6
20	166.3
23	178.9
25	190.2
27	221.8

Figure 26



Cubic function of best fit:

$$C(x) = 0.0155x^3 - 0.5951x^2 + 9.1502x + 98.4327.$$

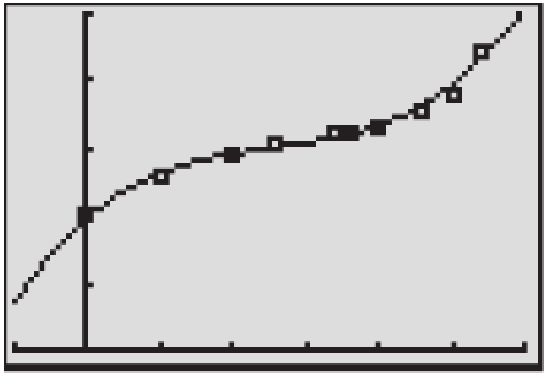
EXAMPLE A Cubic Function of Best Fit

(b) Using a graphing utility, find the cubic function of best fit $C = C(x)$ that models the relation between number of texts and cost.

(c) Graph the cubic function of best fit on your scatter diagram.

(d) Use the function found in part (b) to predict the cost of printing 22 thousand texts per week.

Figure 27



Cubic function of best fit:

$$C(x) = 0.0155x^3 - 0.5951x^2 + 9.1502x + 98.4327.$$

(d) Evaluate the function $C(x)$ at $x = 22$.

$$C(22) = 0.0155(22)^3 - 0.5951(22)^2 + 9.1502(22) + 98.4327 \approx 176.8$$

The model predicts that the cost of printing 22 thousand textbooks in a week will be 176.8 thousand dollars, that is $176,800.