Math 165 – Section 5.1 – Composition of functions

1) **Write the definition** – section 5.1, page 258, new edition.

   (f o g) (x) =

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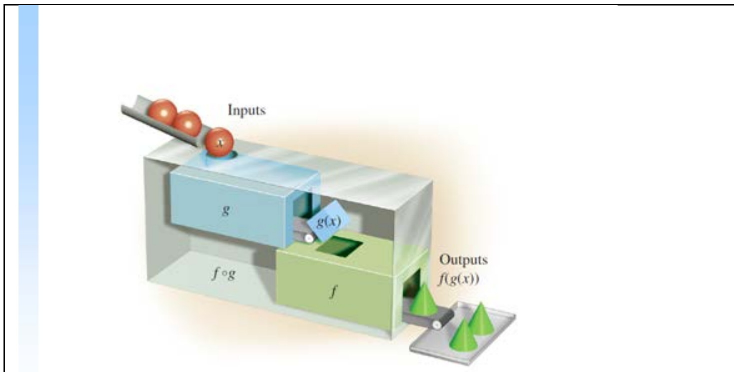
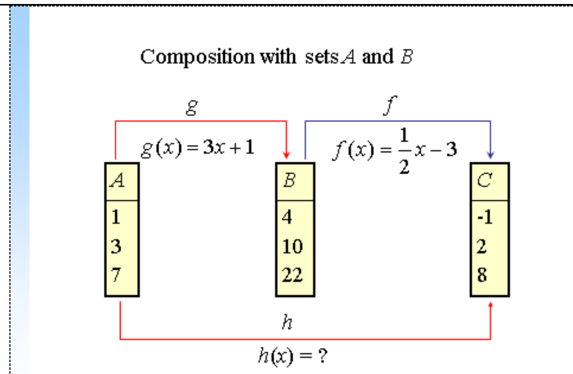
2) **Composition: "x goes into g", "the output from g is the input into f".**





Look at the tables A, B, and C above.

a) Show how you go from the number 1 listed on table A, to the number 4 in table B.

b) Show how you go from the number 4 in table B to the number -1 in table C.

c) If we put together steps (a) and (b) above, we can say that *(f o g) (1) = f(g(1)) = f(4) = -1*.
   This means, first we evaluate g(1) = 3(1) + 1 = 4; then evaluate f(4) = (1/2)(4) – 3 = -1.
   Now, using the same diagram above, complete the following:

   a. *What is (f o g)(3)?* b. *What is (f o g)(7)?*

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3) Given *f(x) = 2x – 7 and g(x) = -3x + 1,* ***show all work to find each of the following:***

   a. *Find (f o g) (-2)* *b. Find (g o f)(-1)*

Math 165 – Section 5.1 – Composition of functions

What is composition?

Putting one function into another – where? In the "variable place" of the other

EXAMPLE **Finding a Composite Function and Its Domain**

Suppose that $f(x) = 2x^2 - x + 4$ and $g(x) = 4x + 1$.

Find: (a) $f \circ g$ (b) $g \circ f$

Then find the domain of each composite function.

(a) $(f \circ g)(x) = f(g(x)) = f(4x+1) = 2(4x+1)^2 - (4x+1) + 4$

$g(x) = 4x + 1$ →

$= 2(16x^2 + 8x + 1) - 4x - 1 + 4$

$= 32x^2 + 16x + 2 - 4x + 3$

$= 32x^2 + 12x + 5$

The domain of g is all real numbers as is the domain of the composite function, so the domain of $f \circ g$ is the set of all real numbers.

EXAMPLE **Finding a Composite Function and Its Domain**

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Find: (a) $f \circ g$ (b) $g \circ f$

Then find the domain of each composite function.

(b) $(g \circ f)(x) = g(f(x)) = g(2x^2 - x + 4) = 4(2x^2 - x + 4) + 1$

$f(x) = 2x^2 - x + 4$ →

$= 8x^2 - 4x + 16 + 1$

$= 8x^2 - 4x + 17$

The domain of f is all real numbers as is the domain of the composite function, so the domain of $g \circ f$ is the set of all real numbers.

Math 165 – Section 5.1 – Composite functions - Applications

4) In the real world there are many situations in which some quantity depends on a variable which at the same time, depends on another variable.

Suppose that an oil tanker is leaking oil and we want to be able to determine the area o the circular oil patch around the ship. It is determined that the oil is leaking from the tanker in such a way that the radius of the circular oil patch around the ship is increasing at a rate of 3 feet per minute.

$$r(t) = 3t$$

$$A(r) = \pi r^2$$

a) What will be the area of the spill after 2 minutes? After 5 minutes? After t-minutes?

b) NOTE: (we'll do in class)

5) The volume of a balloon is given by $V(r) = \frac{4}{3}\pi r^3$ and the radius is increasing with time, in seconds, according to the formula: $r(t) = \frac{1}{2}t^3$.

a) What is the volume of the balloon after 3 seconds? After 5 seconds? After t seconds?

c) NOTE: (we'll do in class)

Math 165 – Section 5.1 – Composite functions – From Tables and Graphs

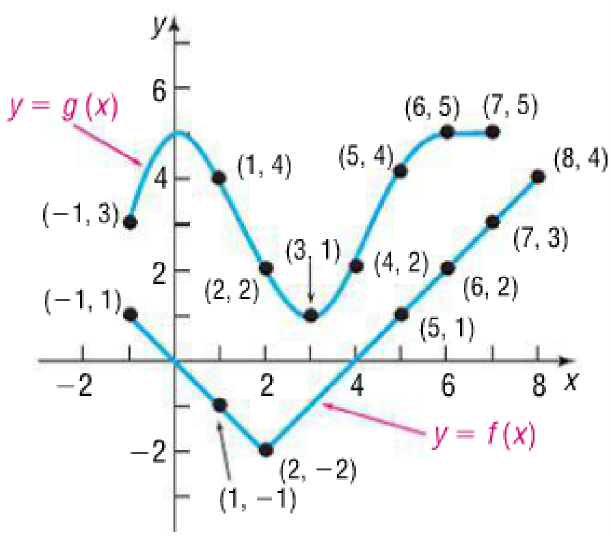
6) Use the values in the table to evaluate the indicated composition of functions.

8.

x	−3	−2	−1	0	1	2	3
$f(x)$	11	9	7	5	3	1	−1
$g(x)$	−8	−3	0	1	0	−3	−8

(a) $(f \circ g)(1)$ (b) $(f \circ g)(2)$
(c) $(g \circ f)(2)$ (d) $(g \circ f)(3)$
(e) $(g \circ g)(1)$ (f) $(f \circ f)(3)$

7) Use the graphs to evaluate the composition of functions.



10. (a) $(g \circ f)(1)$ (b) $(g \circ f)(5)$
(c) $(f \circ g)(0)$ (d) $(f \circ g)(2)$

Math 165 – Section 5.1 – Composite functions and their Domain

8) Find the composite function and its domain. Also, make up some evaluation problems using the given functions.

In Problems 23–38, for the given functions f and g, find:
(a) $f \circ g$ *(b)* $g \circ f$ *(c)* $f \circ f$ *(d)* $g \circ g$

State the domain of each composite function.

23. $f(x) = 2x + 3;\quad g(x) = 3x$

24. $f(x) = -x;\quad g(x) = 2x - 4$

25. $f(x) = 3x + 1;\quad g(x) = x^2$

26. $f(x) = x + 1;\quad g(x) = x^2 + 4$

27. $f(x) = x^2;\quad g(x) = x^2 + 4$

28. $f(x) = x^2 + 1;\quad g(x) = 2x^2 + 3$

29. $f(x) = \frac{3}{x - 1};\quad g(x) = \frac{2}{x}$

30. $f(x) = \frac{1}{x + 3};\quad g(x) = -\frac{2}{x}$

31. $f(x) = \frac{x}{x - 1};\quad g(x) = -\frac{4}{x}$

32. $f(x) = \frac{x}{x + 3};\quad g(x) = \frac{2}{x}$

33. $f(x) = \sqrt{x};\quad g(x) = 2x + 3$

34. $f(x) = \sqrt{x - 2};\quad g(x) = 1 - 2x$

35. $f(x) = x^2 + 1;\quad g(x) = \sqrt{x - 1}$

36. $f(x) = x^2 + 4;\quad g(x) = \sqrt{x - 2}$

37. $f(x) = \frac{x - 5}{x + 1};\quad g(x) = \frac{x + 2}{x - 3}$

38. $f(x) = \frac{2x - 1}{x - 2};\quad g(x) = \frac{x + 4}{2x - 5}$

Math 165 – Section 5.1 – Components of a Composite functions

9) Find the components *f and g*, so that *H = fog*

In Problems 47–52, find functions f and g so that $f \circ g = H$.

47. $H(x) = (2x + 3)^4$

48. $H(x) = (1 + x^2)^3$

49. $H(x) = \sqrt{x^2 + 1}$

50. $H(x) = \sqrt{1 - x^2}$

51. $H(x) = |2x + 1|$

52. $H(x) = |2x^2 + 3|$

10) Applications

61. Automobile Production The number N of cars produced at a certain factory in one day after t hours of operation is given by $N(t) = 100t - 5t^2, 0 \le t \le 10$. If the cost C (in dollars) of producing N cars is $C(N) = 15{,}000 + 8000N$, find the cost C as a function of the time t of operation of the factory.

62. Environmental Concerns The spread of oil leaking from a tanker is in the shape of a circle. If the radius r (in feet) of the spread after t hours is $r(t) = 200\sqrt{t}$, find the area A of the oil slick as a function of the time t.

Math 165 – Section 5.1 – Composite functions – Applications

64. Cost of a Commodity The price p, in dollars, of a certain commodity and the quantity x sold obey the demand equation

$$p = -\frac{1}{5}x + 200 \quad 0 \le x \le 1000$$

Suppose that the cost C, in dollars, of producing x units is

$$C = \frac{\sqrt{x}}{10} + 400$$

Assuming that all items produced are sold, find the cost C as a function of the price p.

65. Volume of a Cylinder The volume V of a right circular cylinder of height h and radius r is $V = \pi r^2 h$. If the height is twice the radius, express the volume V as a function of r.

66. Volume of a Cone The volume V of a right circular cone is $V = \frac{1}{3}\pi r^2 h$. If the height is twice the radius, express the volume V as a function of r.

Functions from Functions (Professor McCullough)

Let $f(x)=x^2$; $g(x)=\frac{1}{x}$; $h(x)=\sqrt{x}$; $k(x)=e^x$; $A(x)=\ln(x)$

Express each of the following as a combination of the functions above. Use the operations addition, subtraction, multiplication, division and composition.

1. $e^{\sqrt{x}}$

2. x^2e^x

3. $\frac{e^x}{x^2}$

4. $x^2+\sqrt{x}$

5. $e^{\ln(x)}$

6. $\ln(x)-e^x$

7. $\ln\left(\frac{1}{x}\right)$

8. $\ln\left(x^2+\sqrt{x}\right)$

9. $\sqrt{x^2+e^x}$

10. $e^{x^2-\sqrt{x}}$