

Math 165 – Section 5.5 - Properties of Logarithms

To do at home - Go to section 5.5 and copy the contents of all the yellow boxes of the section.

$$\log_a 1 = 0 \quad \log_a a = 1$$

Properties of Logarithms

In the properties given next, M and a are positive real numbers, $a \neq 1$, and r is any real number.

The number $\log_a M$ is the exponent to which a must be raised to obtain M . That is,

$$a^{\log_a M} = M \quad (1)$$

The logarithm with base a of a raised to a power equals that power. That is,

$$\log_a a^r = r \quad (2)$$

Properties of Logarithms

In the following properties, M , N , and a are positive real numbers, $a \neq 1$, and r is any real number.

The Log of a Product Equals the Sum of the Logs

$$\log_a(MN) = \log_a M + \log_a N \quad (3)$$

The Log of a Quotient Equals the Difference of the Logs

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N \quad (4)$$

The Log of a Power Equals the Product of the Power and the Log

$$\log_a M^r = r \log_a M \quad (5)$$

$$a^r = e^{r \ln a} \quad (6)$$

Section 5.5 - Properties of Logarithms – Expanding

1) Expand using properties of logarithms

45. $\log_a(u^2v^3) \quad u > 0, v > 0$

48. $\ln(x\sqrt{1+x^2}) \quad x > 0$

51. $\log\left[\frac{x(x+2)}{(x+3)^2}\right] \quad x > 0$

46. $\log_2\left(\frac{a}{b^2}\right) \quad a > 0, b > 0$

49. $\log_2\left(\frac{x^3}{x-3}\right) \quad x > 3$

52. $\log\left[\frac{x^3\sqrt{x+1}}{(x-2)^2}\right] \quad x > 2$

47. $\ln(x^2\sqrt{1-x}) \quad 0 < x < 1$

50. $\log_5\left(\frac{\sqrt[3]{x^2+1}}{x^2-1}\right) \quad x > 1$

53. $\ln\left[\frac{x^2-x-2}{(x+4)^2}\right]^{1/3} \quad x > 2$

Section 5.5 - Properties of Logarithms – Condensing

2) Rewrite as a single logarithmic expression

57. $3 \log_5 u + 4 \log_5 v$

60. $\log_2\left(\frac{1}{x}\right) + \log_2\left(\frac{1}{x^2}\right)$

58. $2 \log_3 u - \log_3 v$

61. $\log_4(x^2 - 1) - 5 \log_4(x + 1)$

59. $\log_3 \sqrt{x} - \log_3 x^3$

62. $\log(x^2 + 3x + 2) - 2 \log(x + 1)$

Section 5.5 - Properties of Logarithms – Write y as a function of x

3) Express y as a function of x . The constant C is a positive number

87. $\ln y = \ln x + \ln C$

89. $\ln y = \ln x + \ln(x + 1) + \ln C$

91. $\ln y = 3x + \ln C$

93. $\ln(y - 3) = -4x + \ln C$

88. $\ln y = \ln(x + C)$

90. $\ln y = 2 \ln x - \ln(x + 1) + \ln C$

92. $\ln y = -2x + \ln C$

94. $\ln(y + 4) = 5x + \ln C$

Section 5.5 - Properties of Logarithms – Mixed Practice

4) Solve the following problems

Mixed Practice

85. If $f(x) = \ln x$, $g(x) = e^x$, and $h(x) = x^2$, find:

- (a) $(f \circ g)(x)$. What is the domain of $f \circ g$?
- (b) $(g \circ f)(x)$. What is the domain of $g \circ f$?
- (c) $(f \circ g)(5)$
- (d) $(f \circ h)(x)$. What is the domain of $f \circ h$?
- (e) $(f \circ h)(e)$

86. If $f(x) = \log_2 x$, $g(x) = 2^x$, and $h(x) = 4x$, find:

- (a) $(f \circ g)(x)$. What is the domain of $f \circ g$?
 - (b) $(g \circ f)(x)$. What is the domain of $g \circ f$?
 - (c) $(f \circ g)(3)$
 - (d) $(f \circ h)(x)$. What is the domain of $f \circ h$?
 - (e) $(f \circ h)(8)$
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