Section 7.6

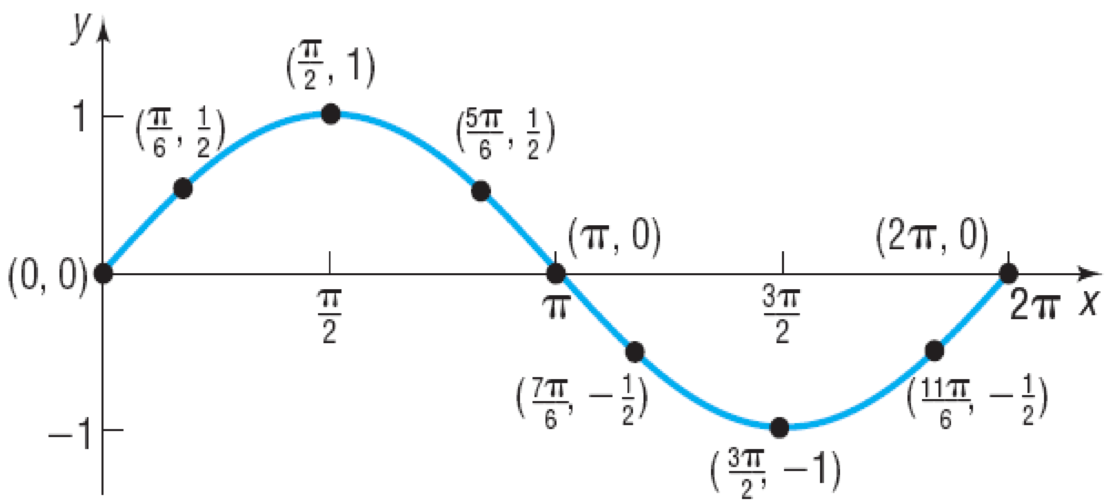
Graphs of the Sine and Cosine Functions

$$y = f(x) = \sin x \qquad y = f(x) = \cos x \qquad y = f(x) = \tan x$$

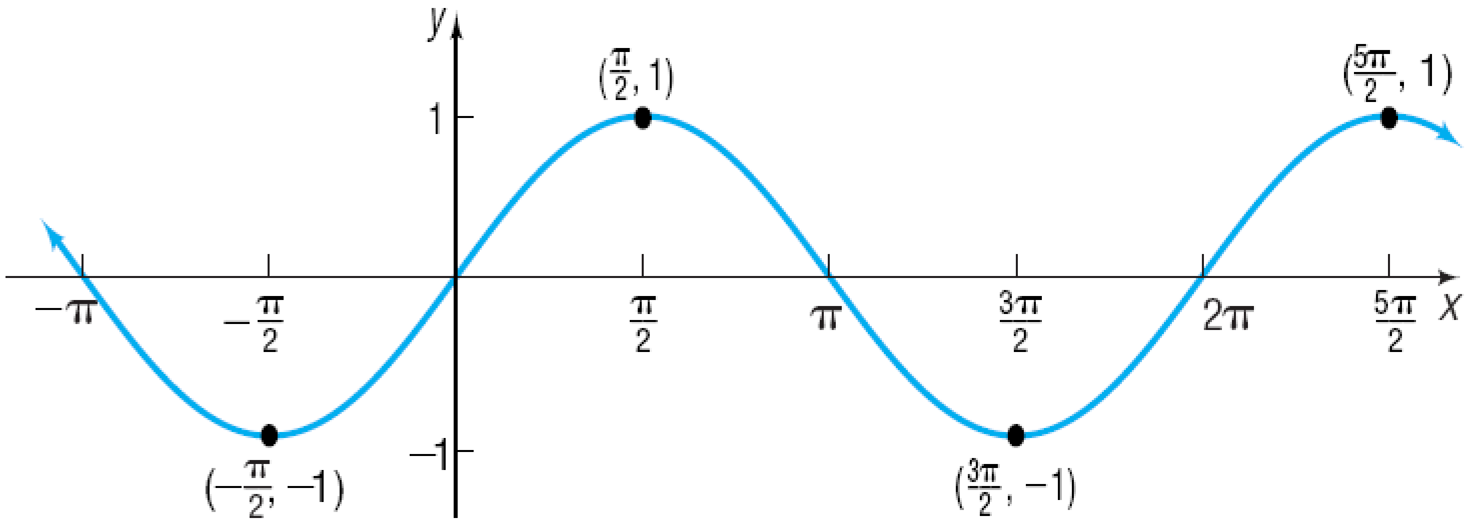
$$y = f(x) = \csc x \qquad y = f(x) = \sec x \qquad y = f(x) = \cot x$$

The Graph of the Sine Function $y = \sin x$

x	$y = \sin x$	(x, y)
0	0	$(0, 0)$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\left(\frac{\pi}{6}, \frac{1}{2}\right)$
$\frac{\pi}{2}$	1	$\left(\frac{\pi}{2}, 1\right)$
$\frac{5\pi}{6}$	$\frac{1}{2}$	$\left(\frac{5\pi}{6}, \frac{1}{2}\right)$
π	0	$(\pi, 0)$
$\frac{7\pi}{6}$	$-\frac{1}{2}$	$\left(\frac{7\pi}{6}, -\frac{1}{2}\right)$
$\frac{3\pi}{2}$	-1	$\left(\frac{3\pi}{2}, -1\right)$
$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$
2π	0	$(2\pi, 0)$



$$y = \sin x, 0 \leq x \leq 2\pi$$

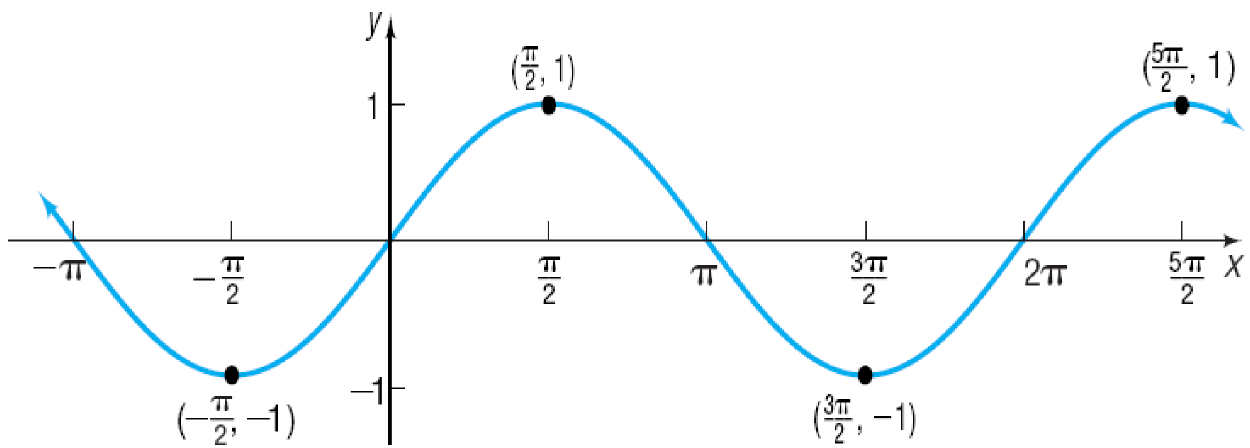


$$y = \sin x, -\infty < x < \infty$$

Properties of the Sine Function

1. The domain is the set of all real numbers.
2. The range consists of all real numbers from -1 to 1, inclusive.
3. The sine function is an odd function, as the symmetry of the graph with respect to the origin indicates.
4. The sine function is periodic, with period 2π.
5. The x-intercepts are $\ldots, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots$; the y-intercept is 0.
6. The maximum value is 1 and occurs at $x = \ldots, -\dfrac{3\pi}{2}, \dfrac{\pi}{2}, \dfrac{5\pi}{2}, \dfrac{9\pi}{2}, \ldots$;

   the minimum value is -1 and occurs at $x = \ldots, -\dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{7\pi}{2}, \dfrac{11\pi}{2}, \ldots$.

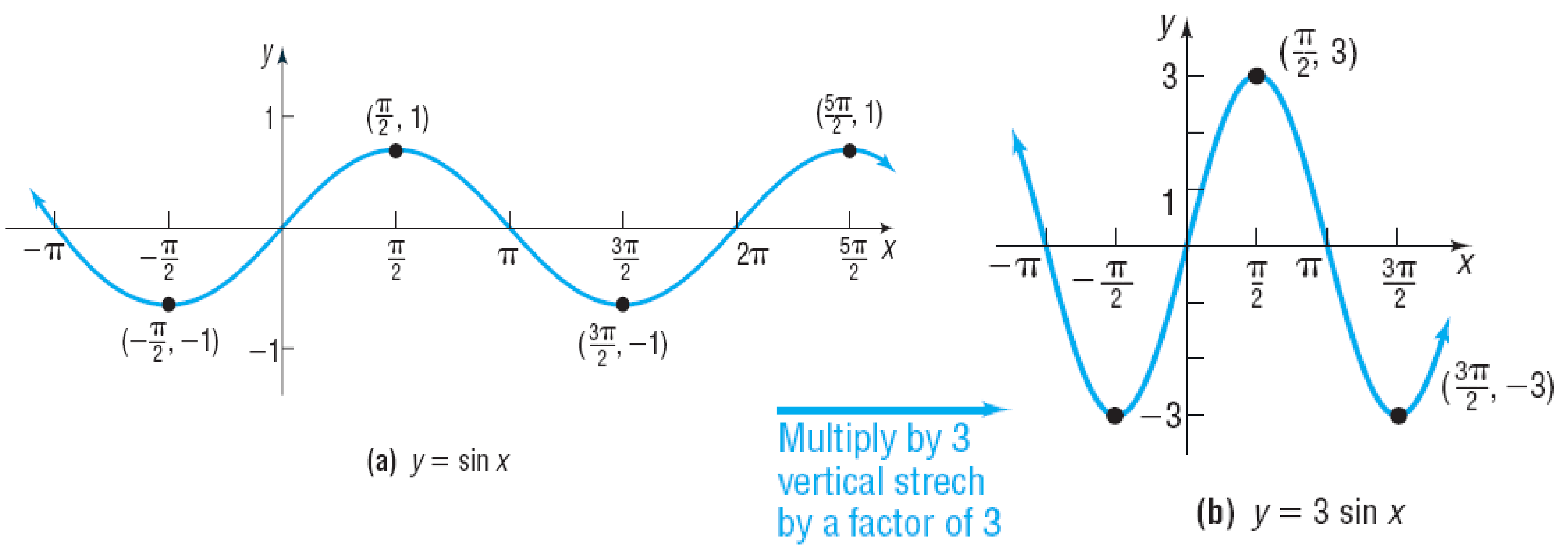


1 Graph Functions of the Form $y = A\sin(\omega x)$ Using Transformations

EXAMPLE

Graphing Functions of the Form $y = A\sin(\omega x)$ Using Transformations

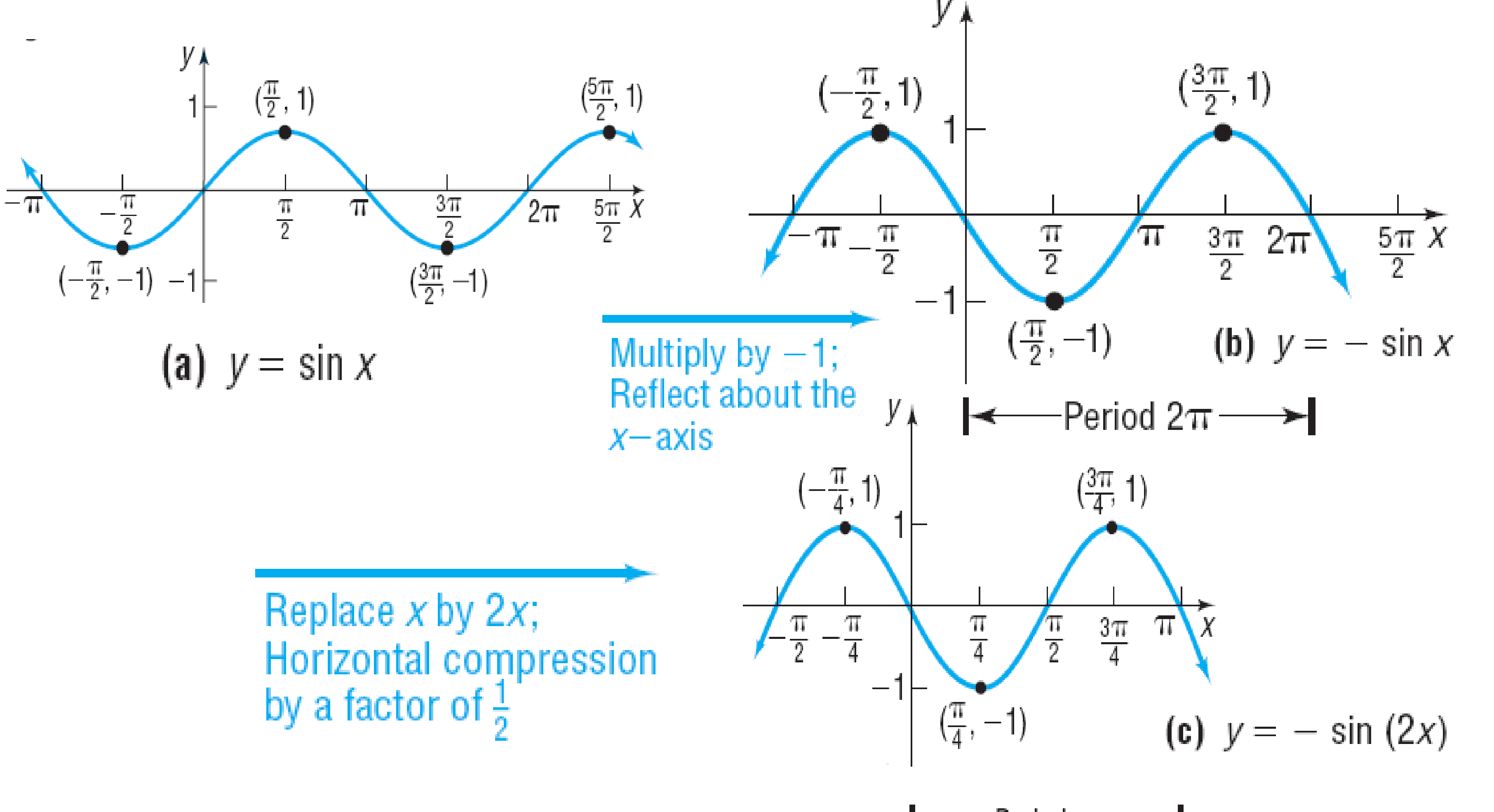
Graph $y = 3\sin x$ using transformations.



EXAMPLE

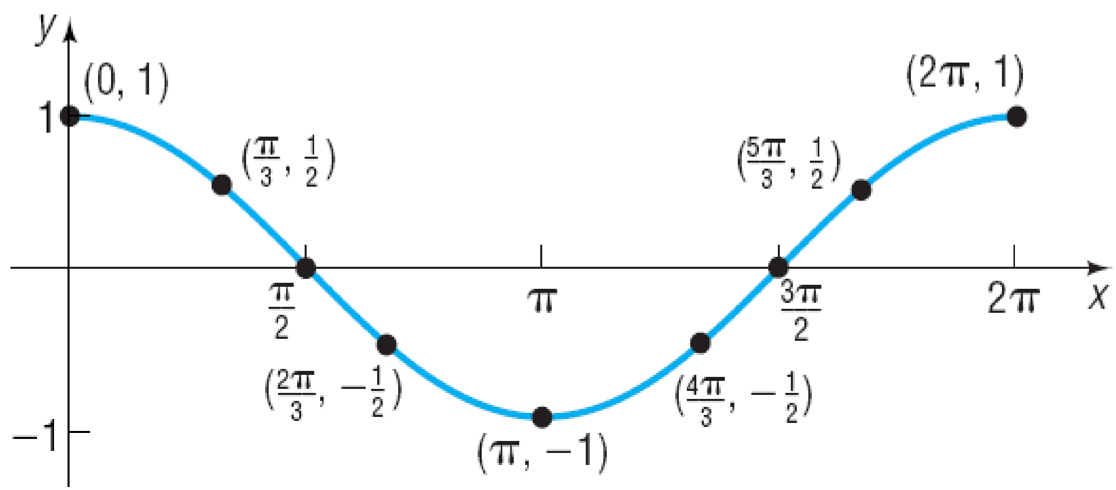
Graphing Functions of the Form $y = A\sin(\omega x)$ Using Transformations

Graph $y = -\sin(2x)$ using transformations.

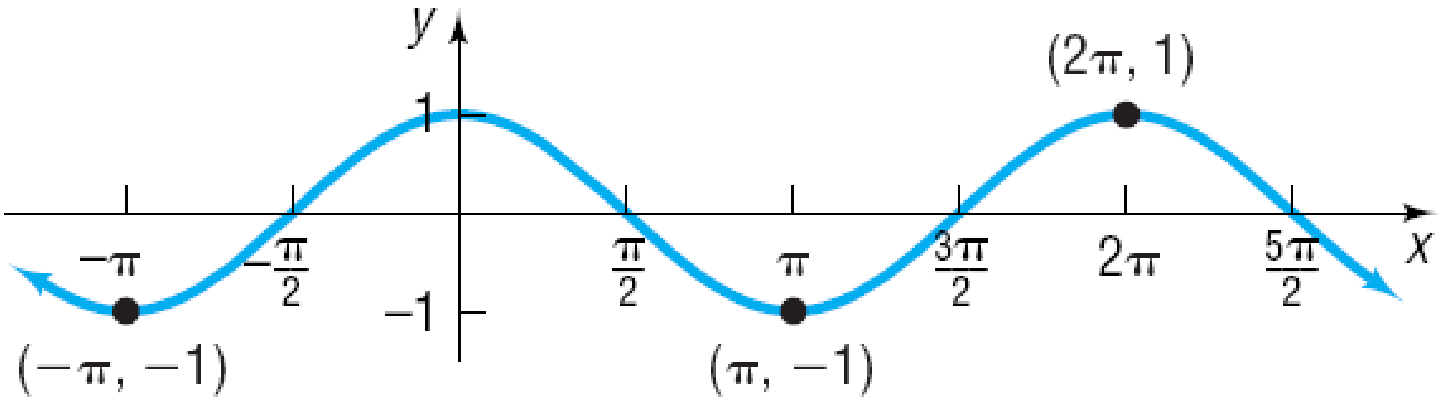


The Graph of the Cosine Function

x	$y = \cos x$	(x, y)
0	1	(0, 1)
$\frac{\pi}{3}$	$\frac{1}{2}$	$\left(\frac{\pi}{3}, \frac{1}{2}\right)$
$\frac{\pi}{2}$	0	$\left(\frac{\pi}{2}, 0\right)$
$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\left(\frac{2\pi}{3}, -\frac{1}{2}\right)$
π	-1	$(\pi, -1)$
$\frac{4\pi}{3}$	$-\frac{1}{2}$	$\left(\frac{4\pi}{3}, -\frac{1}{2}\right)$
$\frac{3\pi}{2}$	0	$\left(\frac{3\pi}{2}, 0\right)$
$\frac{5\pi}{3}$	$\frac{1}{2}$	$\left(\frac{5\pi}{3}, \frac{1}{2}\right)$
2π	1	$(2\pi, 1)$



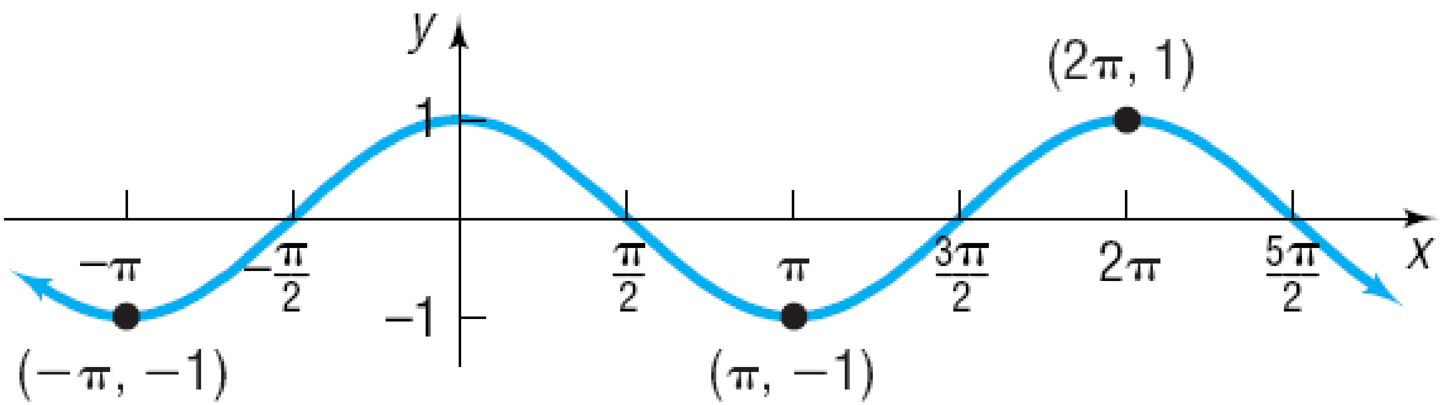
$y = \cos x, 0 \leq x \leq 2\pi$



$$y = \cos x, \ -\infty < x < \infty$$

Properties of the Cosine Function

1. The domain is the set of all real numbers.
2. The range consists of all real numbers from -1 to 1, inclusive.
3. The cosine function is an even function, as the symmetry of the graph with respect to the y-axis indicates.
4. The cosine function is periodic, with period 2π.
5. The x-intercepts are $\ldots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$; the y-intercept is 1.
6. The maximum value is 1 and occurs at $x = \ldots, -2\pi, 0, 2\pi, 4\pi, 6\pi, \ldots$; the minimum value is -1 and occurs at $x = \ldots, -\pi, \pi, 3\pi, 5\pi, \ldots$.

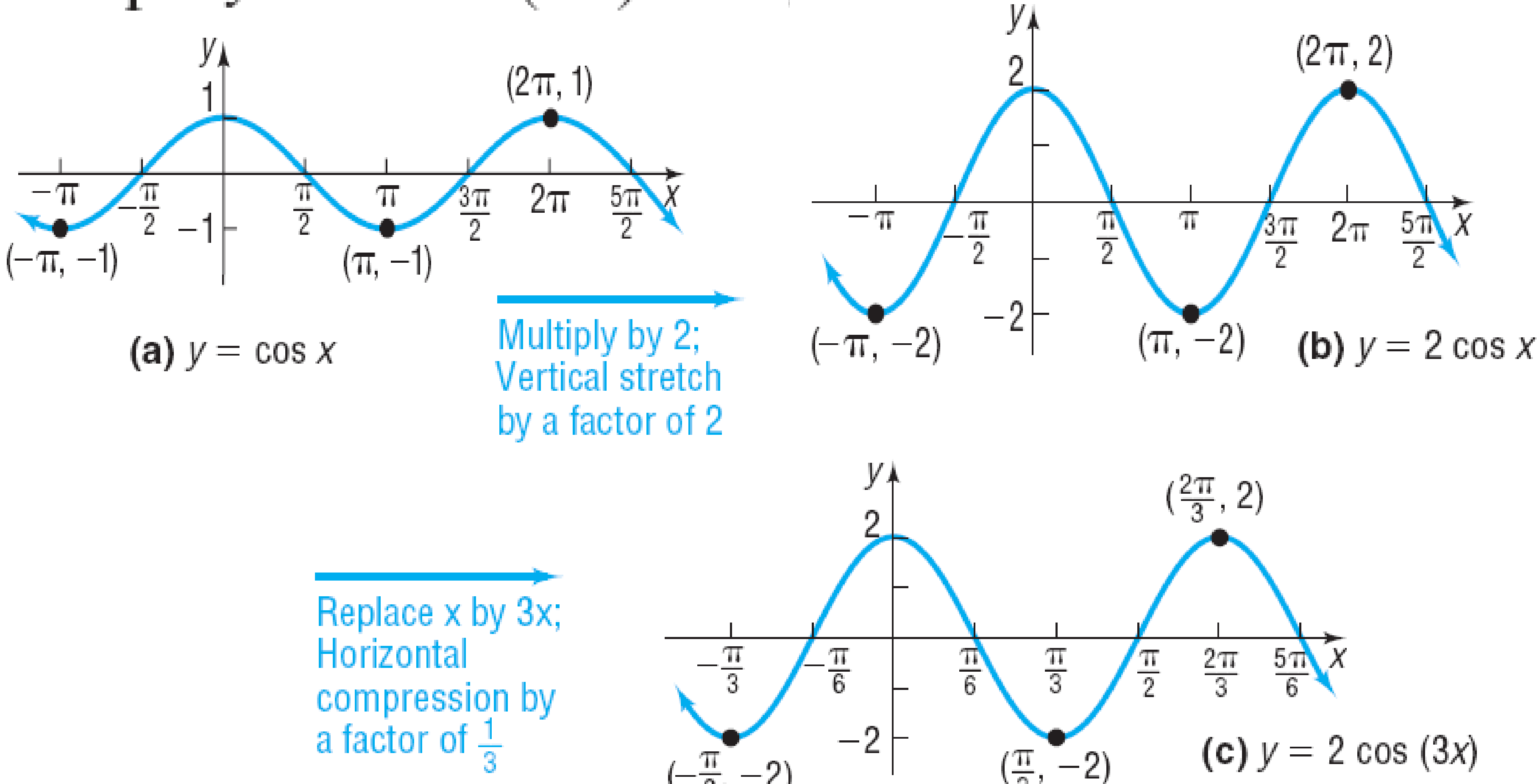


2 Graph Functions of the Form $y = A\cos(\omega x)$ Using Transformations

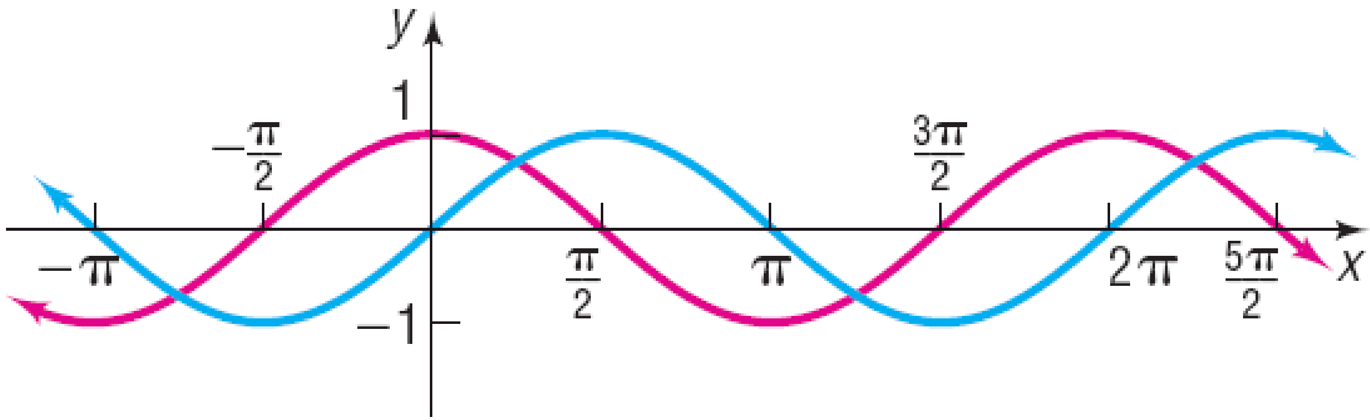
EXAMPLE

Graphing Functions of the Form $y = A\cos(\omega x)$ Using Transformations

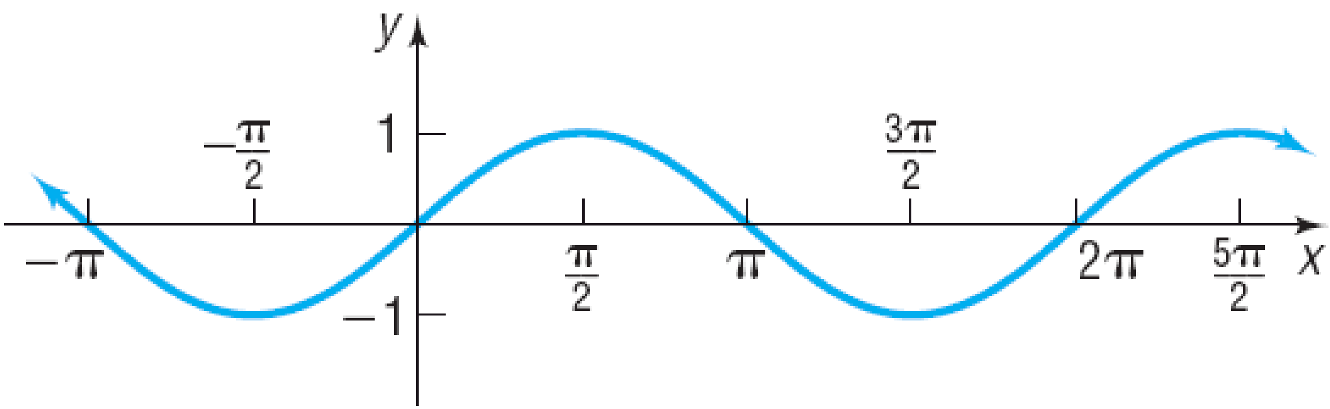
Graph $y = 2\cos(3x)$ using transformations.



Sinusoidal Graphs



(a) $y = \cos x$ $y = \cos\left(x - \frac{\pi}{2}\right)$



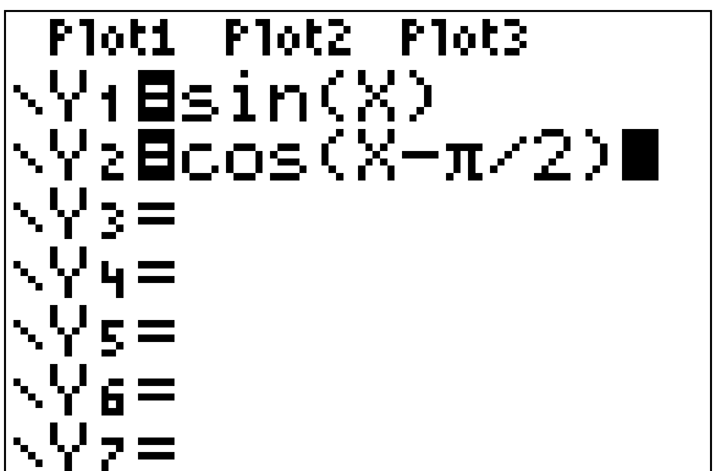
(b) $y = \sin x$

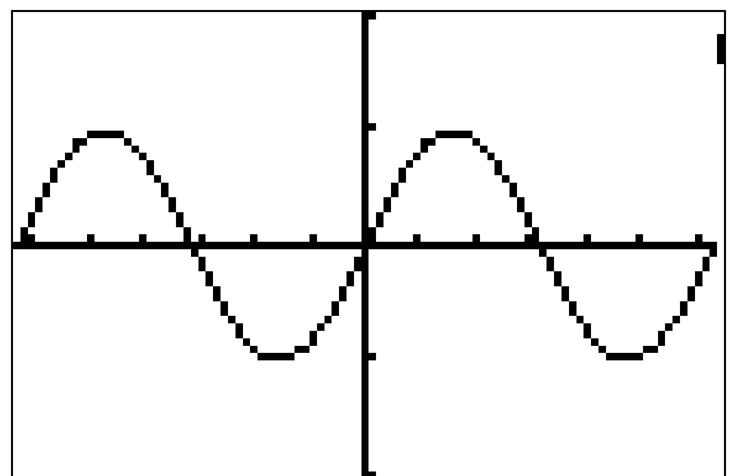
$$\boxed{\sin x = \cos\left(x - \frac{\pi}{2}\right)}$$

Seeing the Concept

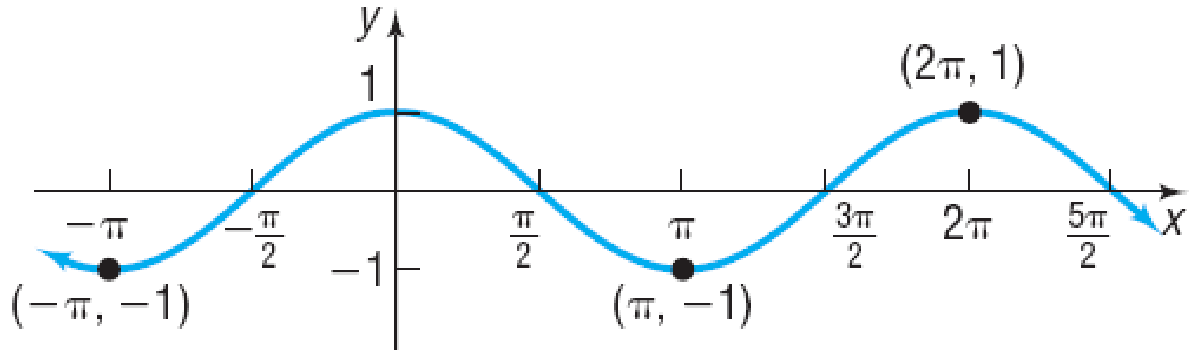
Graph $Y_1 = \sin x$ and $Y_2 = \cos\left(x - \frac{\pi}{2}\right)$.

How many graphs do you see?



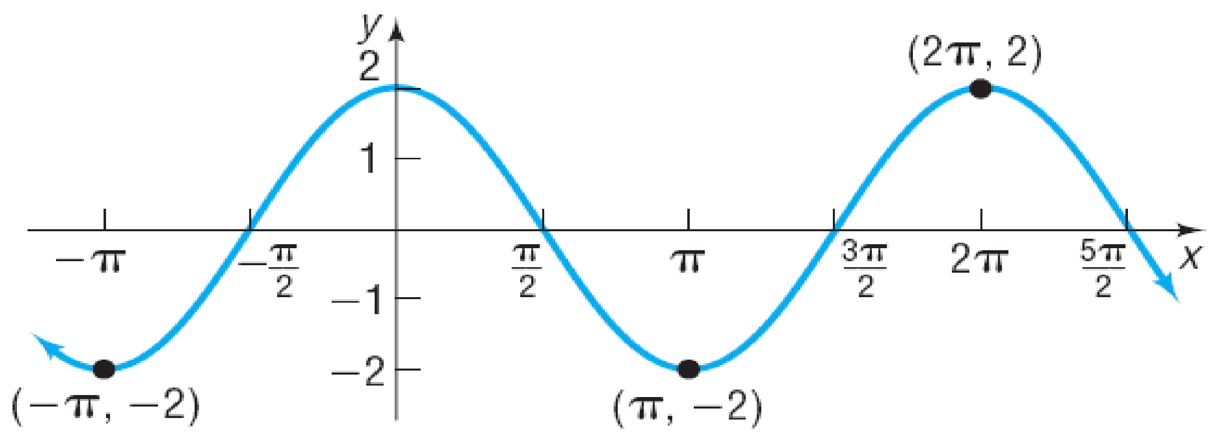


3 Determine the Amplitude and Period of Sinusoidal Functions



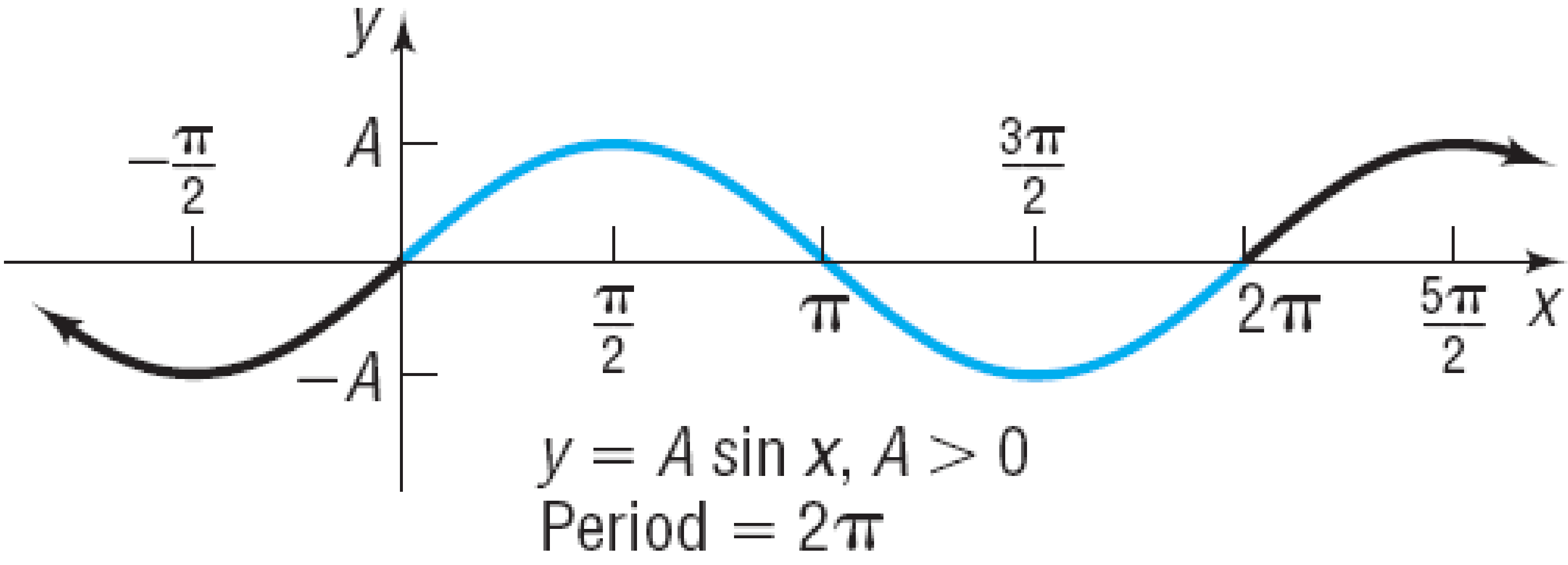
$y = \cos x$

Multiply by 2;
Vertical stretch
by a factor of 2

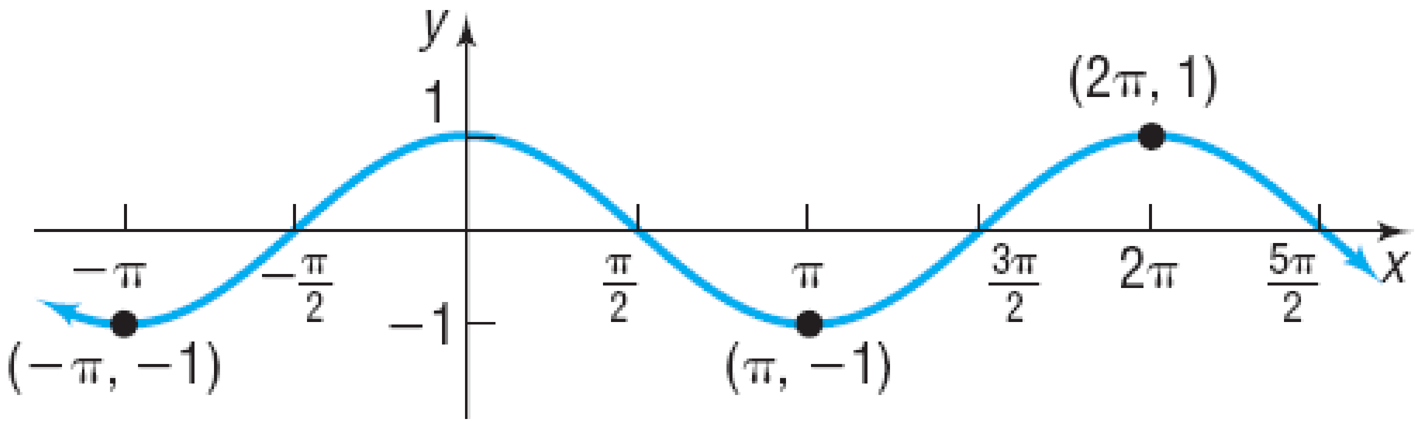


$y = 2 \cos x$

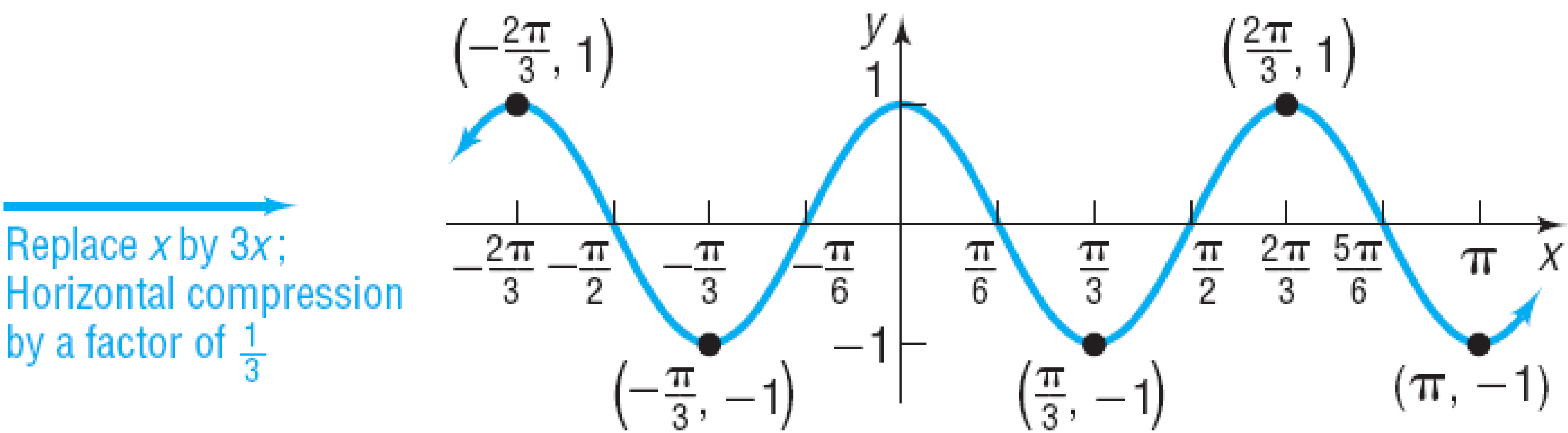
Amplitude



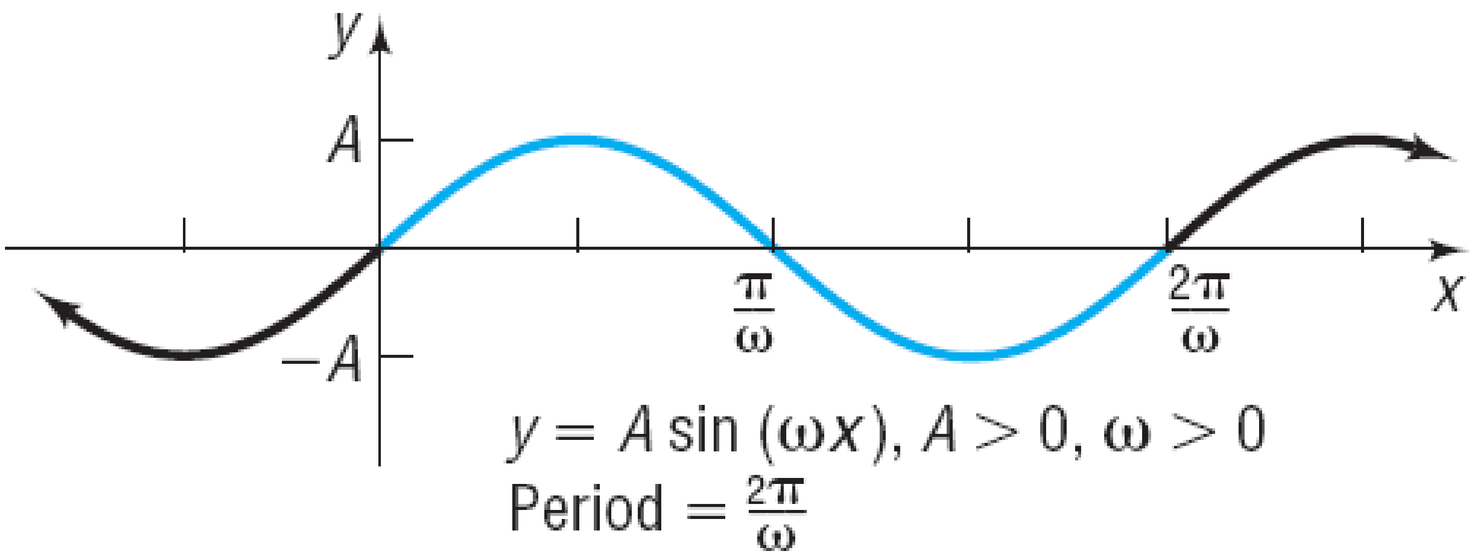
y
A
$-\frac{\pi}{2}$
$\frac{3\pi}{2}$
$-A$
$\frac{\pi}{2}$
π
2π
$\frac{5\pi}{2}$
x
$y = A \sin x$, $A > 0$
Period $= 2\pi$



$y = \cos x$



$y = \cos(3x)$



THEOREM

If $\omega > 0$, the amplitude and period of $y = A\sin(\omega x)$ and $y = A\cos(\omega x)$ are

$$\boxed{\text{Amplitude} = |A| \qquad \text{Period} = T = \frac{2\pi}{\omega}}$$

EXAMPLE

Finding the Amplitude and Period of a Sinusoidal Function

Determine the amplitude and period of $y = -4\cos(3x)$

$$\text{Amplitude} = |-4| = 4$$

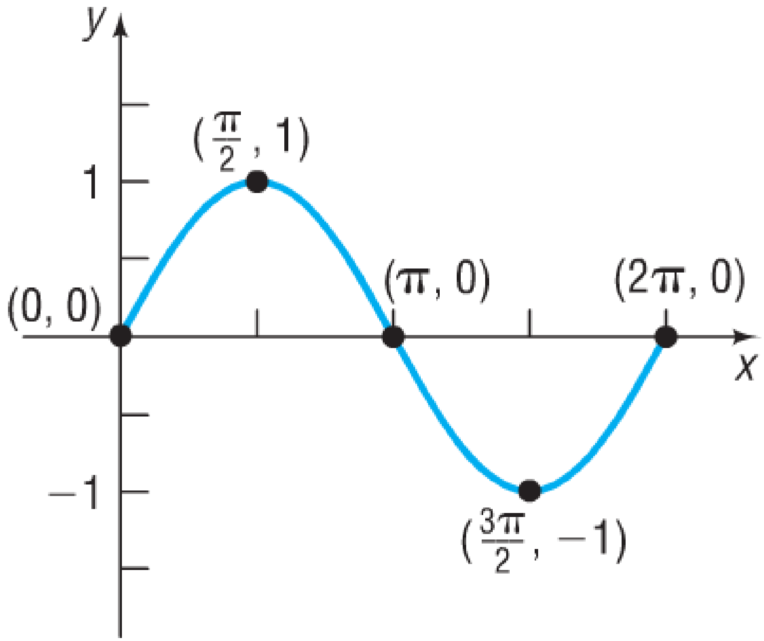
$$\text{Period} = T = \frac{2\pi}{\omega} = \frac{2\pi}{3}$$

If $\omega > 0$, the amplitude and period of $y = A\sin(\omega x)$ and $y = A\cos(\omega x)$ are

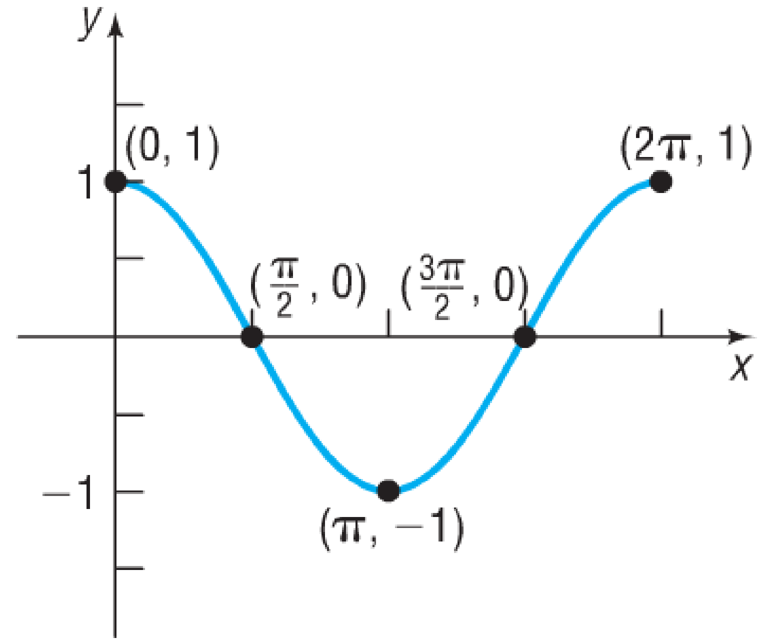
$$\boxed{\text{Amplitude} = |A| \qquad \text{Period} = T = \frac{2\pi}{\omega}}$$

4 Graph Sinusoidal Functions Using Key Points

$$\left[0, \frac{\pi}{2}\right], \quad \left[\frac{\pi}{2}, \pi\right], \quad \left[\pi, \frac{3\pi}{2}\right], \quad \left[\frac{3\pi}{2}, 2\pi\right]$$



(a) $y = \sin x$



(b) $y = \cos x$

For $y = \sin x$: $\quad (0, 0), \left(\frac{\pi}{2}, 1\right), (\pi, 0), \left(\frac{3\pi}{2}, -1\right), (2\pi, 0)$

For $y = \cos x$: $\quad (0, 1), \left(\frac{\pi}{2}, 0\right), (\pi, -1), \left(\frac{3\pi}{2}, 0\right), (2\pi, 1)$

EXAMPLE **How to Graph a Sinusoidal Function Using Key Points**

Graph $y = 4\cos(2x)$ using key points.

Step 1: Determine the amplitude and period of the sinusoidal function.

$$\text{Amplitude} = |4| = 4 \qquad \text{Period} = T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$$

Step 2: Divide the interval $\left[0, \frac{2\pi}{\omega}\right]$ into four subintervals of the same length.

$$\pi \div 4 = \frac{\pi}{4}$$

$$\left[0, \frac{\pi}{4}\right], \left[\frac{\pi}{4}, \frac{\pi}{2}\right], \left[\frac{\pi}{2}, \frac{3\pi}{4}\right], \left[\frac{3\pi}{4}, \pi\right]$$

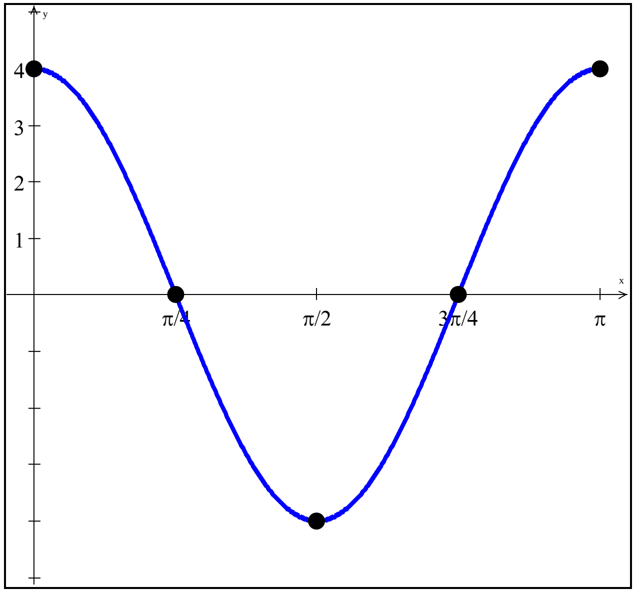
Step 3: Use the endpoints of these subintervals to obtain five key points on the graph.

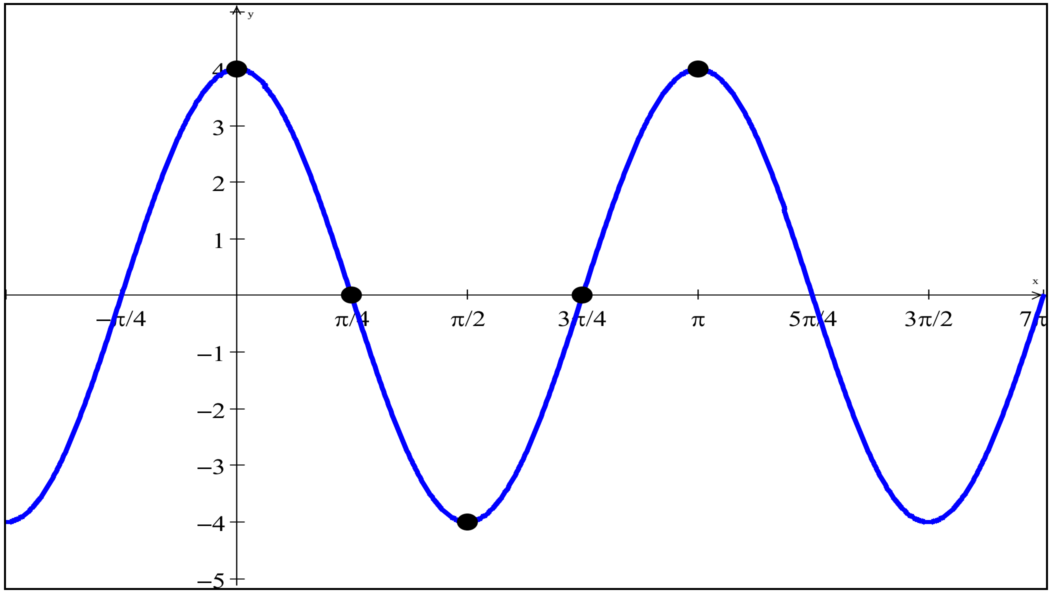
$$(0,4), \left(\frac{\pi}{4}, 0\right), \left(\frac{\pi}{2}, -4\right), \left(\frac{3\pi}{4}, 0\right), (\pi, 4)$$

EXAMPLE How to Graph a Sinusoidal Function Using Key Points

Graph $y = 4\cos(2x)$ using key points.

Step 4: Plot the five key points and draw a sinusoidal graph to obtain the graph of one cycle. Extend the graph in each direction to make it complete.





$$(0,4), \left(\frac{\pi}{4}, 0\right), \left(\frac{\pi}{2}, -4\right), \left(\frac{3\pi}{4}, 0\right), (\pi, 4)$$

SUMMARY Steps for Graphing a Sinusoidal Function of the Form $y = A\sin(\omega x)$ or $y = A\cos(\omega x)$ Using Key Points

STEP 1: Determine the amplitude and period of the sinusoidal function.

STEP 2: Divide the interval $\left[0, \dfrac{2\pi}{\omega}\right]$ into four subintervals of the same length.

STEP 3: Use the endpoints of these subintervals to obtain five key points on the graph.

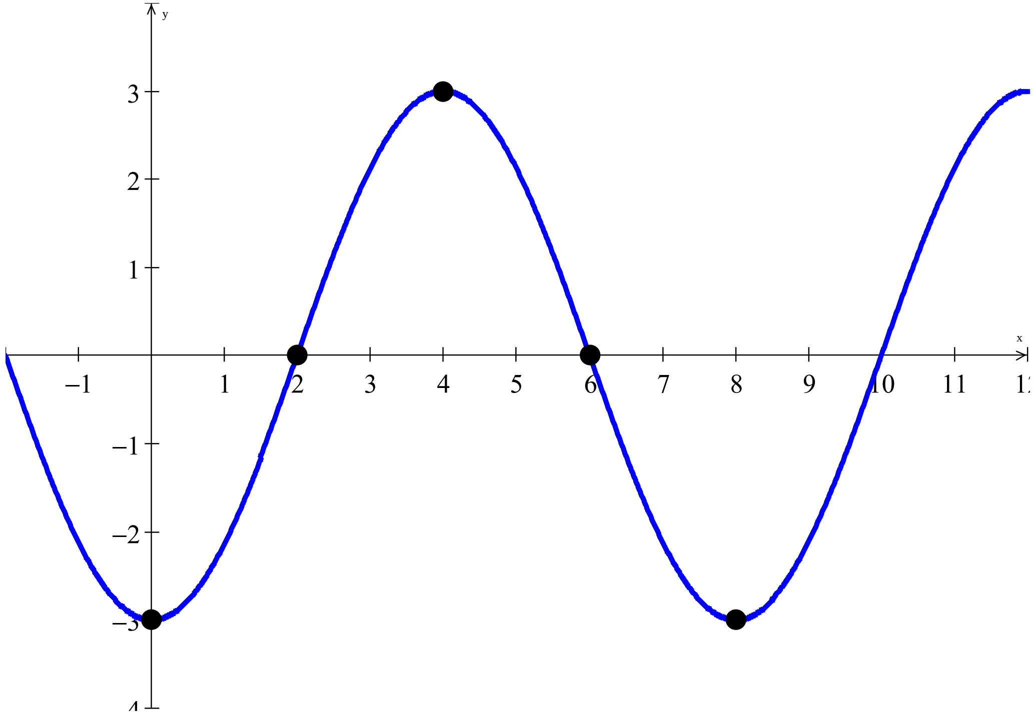
STEP 4: Plot the five key points and draw a sinusoidal graph to obtain the graph of one cycle. Extend the graph in each direction to make it complete.

EXAMPLE **Graphing a Sinusoidal Function Using Key Points**

Graph $y = -3\cos\left(\frac{\pi}{4}x\right)$ using key points.

$$\text{Amplitude} = \left|-3\right| = 3 \qquad \text{Period} = T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\pi}{4}} = 8 \qquad 8 \div 4 = 2$$

$$[0,2],[2,4],[4,6],[6,8] \qquad (0,-3),(2,0),(4,3),(6,0),(8,-3)$$



EXAMPLE Graphing a Sinusoidal Function Using Key Points

Graph $y = 5\sin(3x)$ using key points.

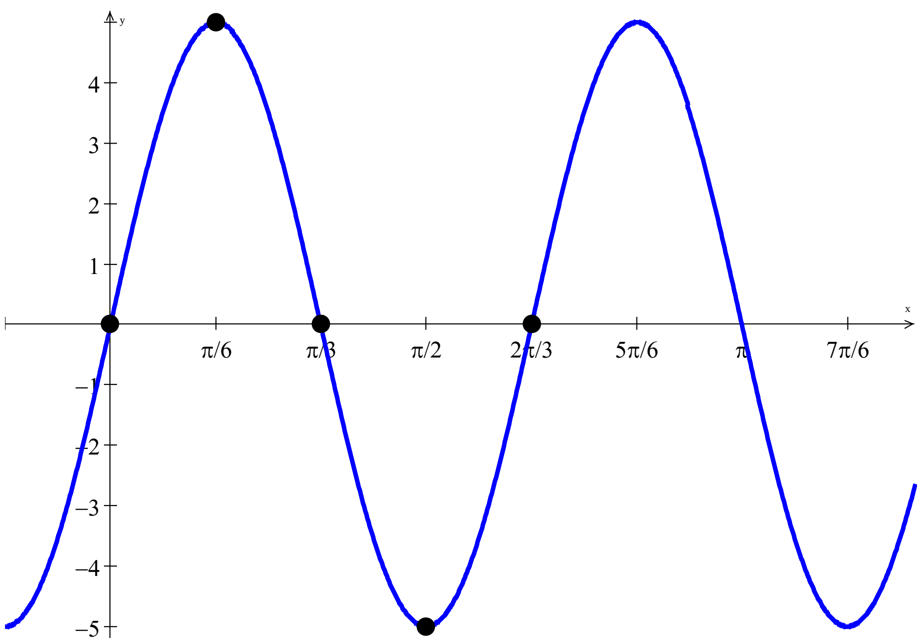
Amplitude = $|5| = 5$

Period = $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{3}$

$\dfrac{2\pi}{3} \div 4 = \dfrac{\pi}{6}$

$\left[0, \dfrac{\pi}{6}\right], \left[\dfrac{\pi}{6}, \dfrac{\pi}{3}\right], \left[\dfrac{\pi}{3}, \dfrac{\pi}{2}\right], \left[\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right]$

$(0,0), \left(\dfrac{\pi}{6}, 5\right), \left(\dfrac{\pi}{3}, 0\right), \left(\dfrac{\pi}{2}, -5\right), \left(\dfrac{2\pi}{3}, 0\right)$

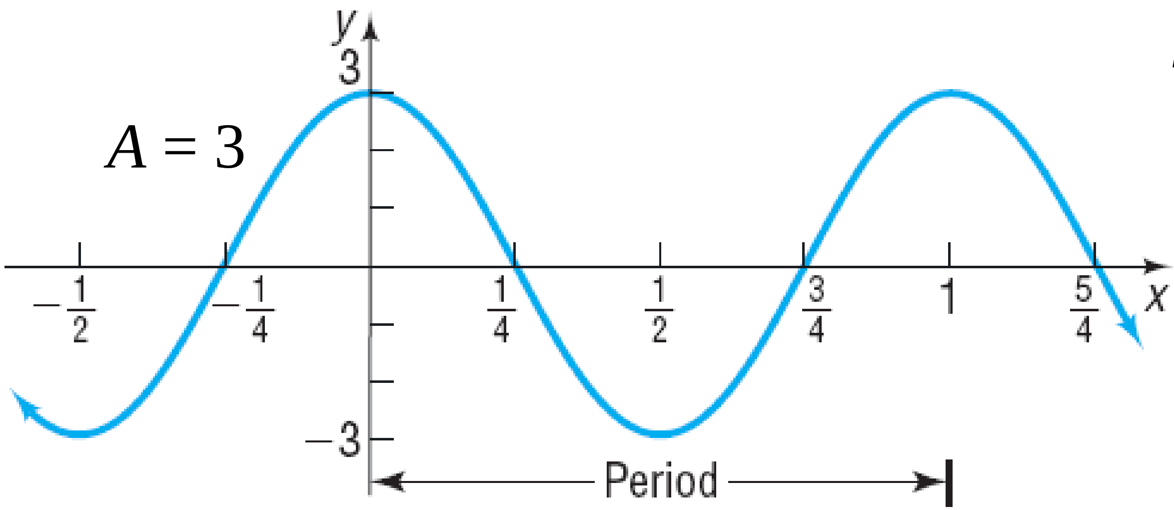


5 Find an Equation for a Sinusoidal Graph

EXAMPLE **Finding an Equation for a Sinusoidal Graph**

Find an equation for the graph shown

$$y = A\cos(\omega x)$$



$$T = \frac{2\pi}{\omega} = 1$$

$$\omega = 2\pi$$

$$y = A\cos(\omega x) = 3\cos(2\pi x)$$

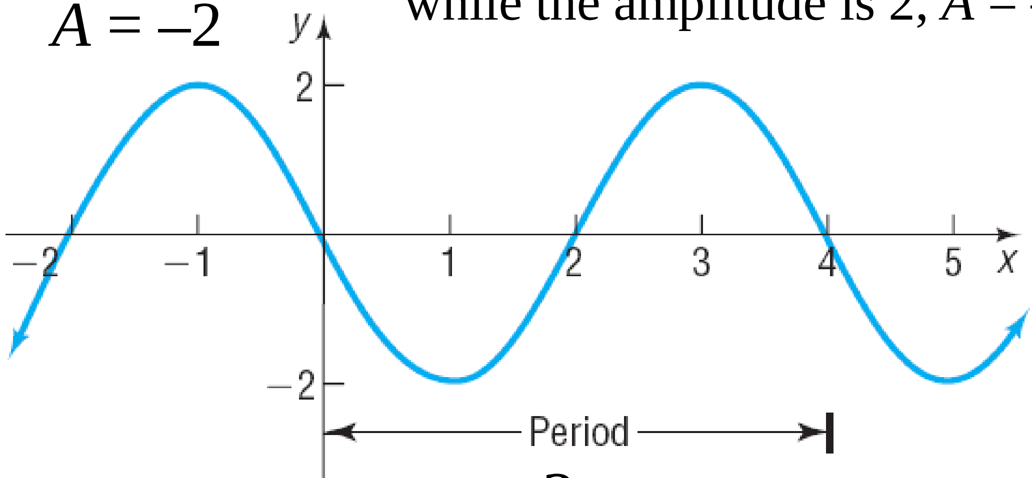
EXAMPLE **Finding an Equation for a Sinusoidal Graph**

Find an equation for the graph shown

Note that this is a reflection over the x-axis of the sine function so while the amplitude is 2, $A = -2$.

$$y = A\sin(\omega x)$$

$A = -2$



$$T = \frac{2\pi}{\omega} = 4 \qquad \omega = \frac{\pi}{2}$$

$$y = A\sin(\omega x) = -2\sin\left(\frac{\pi}{2}x\right)$$