Name: _____

Math 181-Quest 2 Dr. Rebin Muhammad ID: _____

Do not use any unapproved aids while taking this assessment. Read each question carefully and be sure to show all work in the space provided.

1. Sketch the graph of a function $f(x)$ that meets all of the following criteria. Be sure to scale your axes and label any important features of your graph.

   - $f(x)$ has a vertical asymptote at -1, but $\lim_{x\to -1^+} f(x)$ is finite.
   - $\lim_{x\to -5} f(x) = -4$, but $f(-5) = -3$.
   - $\lim_{x\to -6^-} f(x) = 3$ and $\lim_{x\to -6^+} f(x) = 2$.

   Answers vary. Here are some things to check for:

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   - The graph should have a vertical asymptote at $x = -1$. As the function approaches the vertical asymptote from the left, the function outputs should grow infinitely large in magnitude (either in the postive or negative y-direction). As the function approaches the vertical asymptote from the right, the function should bump straight into the vertical asymptote.
   - The graph should approach the point $(-5, -4)$, but there should be a removable discontinuity (hole) in the graph right at that point (indicated by an open circle). The actual value of the function at $x = -5$ should be -3, so the point $(-5, -3)$ should be included on the graph.
   - There is a jump discontinuity at $x = -6$ with the graph approaching the point $(-6, 3)$ as x approaches -6 from the left and approaching the point $(-6, 2)$ as x approaches -6 from the right.

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2. (a) Classify the type of discontinuity present at $x = 5$ for the function $f(x)$. Explain your reasoning using limits.

$$f(x) = \begin{cases} 8\,x - 37, & x < 5 \\ -6, & x = 5 \\ 4\,x - 17, & x > 5 \end{cases}$$

The function $f(x)$ has a removable discontinuity.

(b) Determine the value of b to make $g(x)$ continuous at $x = 5$. Explain your reasoning using limits.

$$g(x) = \begin{cases} bx - \frac{1}{3}, & x \leq 5 \\ x^2 - 11\,x + 32, & x > 5 \end{cases}$$

To make $g(x)$ continuous at $x = 5$, let $b = \frac{7}{15}$.

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3. Explain how to find the value of each limit.

(a)

$$\lim_{x\to-\infty}\frac{8\,x^2-2\,x^5+7}{9\,x-5\,x^3-7}\ \text{ and }\ \lim_{x\to+\infty}\frac{8\,x^2-2\,x^5+7}{9\,x-5\,x^3-7}$$

$$\lim_{x\to-\infty}\frac{8\,x^2-2\,x^5+7}{9\,x-5\,x^3-7}=+\infty\ \text{ and }\ \lim_{x\to+\infty}\frac{8\,x^2-2\,x^5+7}{9\,x-5\,x^3-7}=+\infty$$

(b)

$$\lim_{x\to-\infty}-\frac{4\,x^3-8\,x^2+5}{x-9\,x^3+8}\ \text{ and }\ \lim_{x\to+\infty}-\frac{4\,x^3-8\,x^2+5}{x-9\,x^3+8}$$

$$\lim_{x\to-\infty}-\frac{4\,x^3-8\,x^2+5}{x-9\,x^3+8}=\frac{4}{9}\ \text{ and }\ \lim_{x\to+\infty}-\frac{4\,x^3-8\,x^2+5}{x-9\,x^3+8}=\frac{4}{9}$$

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4. Explain how to find the value of each limit.

(a)

$$\lim_{x\to -2^-} \frac{(x+3)^2(x-4)^5}{(x+2)^4(x-5)^5}$$

$$\lim_{x\to -2^-} \frac{(x+3)^2(x-4)^5}{(x+2)^4(x-5)^5} = +\infty$$

(b)

$$\lim_{x\to -2^+} \frac{(x+3)^2(x-4)^5}{(x+2)^4(x-5)^5}$$

$$\lim_{x\to -2^+} \frac{(x+3)^2(x-4)^5}{(x+2)^4(x-5)^5} = +\infty$$

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5. Demonstrate and explain how to find the derivative of the following functions. Be sure to explicitly denote which derivative rules (product, quotient, sum and difference, etc.) you are using in your work.

(a)

$$f(y) = -\frac{3\,y^2 - 3\,y + 5}{y^4}$$

$$f'(y) = \frac{6\,y^2 - 9\,y + 20}{y^5}$$

(b)

$$h(y) = \frac{\cos(y)}{2\,(2\,y^2 - 3\,y + 2)}$$

$$h'(y) = -\frac{(4\,y - 3)\cos(y)}{2\,(2\,y^2 - 3\,y + 2)^2} - \frac{\sin(y)}{2\,(2\,y^2 - 3\,y + 2)}$$

(c)

$$g(t) = -\left(3\,t^2 + 4\,t - 1\right)\sin(t)$$

$$g'(t) = -\left(3\,t^2 + 4\,t - 1\right)\cos(t) - 2\,(3\,t + 2)\sin(t)$$

(d)

$$g(x) = (5\,x - 5\,e^x - 2)^4$$

$$g'(x) = -20\,(5\,x - 5\,e^x - 2)^3(e^x - 1)$$

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(e)

$$f(y) = 6\cos\left(y^{\frac{2}{3}}\right)$$

$$f'(y) = -\frac{4\sin\left(y^{\frac{2}{3}}\right)}{y^{\frac{1}{3}}}$$

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6. Suppose the function $p(x)$ satisfies $p(7) = 49$, $p'(7) = 8$, and $p''(x) > 0$ for x values nearby 7.

   (a) Explain and demonstrate how to find the linearization $L(x)$ of $p(x)$ at $x = 7$.
   $L(x) = 8\,x - 7$

   (b) Explain and demonstrate how to estimate the value of $p(6.96)$ using this linearization.
   $p(6.96) \approx 48.68$

   (c) Explain why your estimate of $p(6.96)$ is greater than or less than the actual value. The

   estimate is less than than the actual value.