Math 182-31546 Final May 11, 2021 Name: _____

No notes, calculators, or other aids are allowed. Read all directions carefully and write your answers in the space provided. To receive full credit, you must show all of your work.

Question 1: (5pts). Find $g(x)$ such that make the equation below correct:

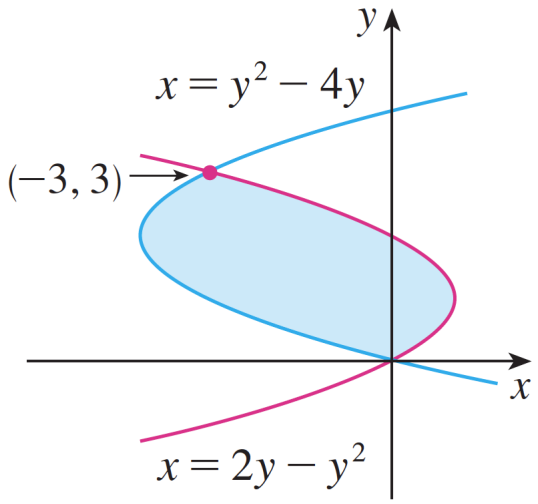
$$\int \frac{e^{\sqrt{x}}}{g(x)} dx = e^{\sqrt{x}} + C$$

Question 2: (5pts). Use integration by part to solve the integral below:

$$\int \theta \cos\theta d\theta$$

Question 3: (5pts). Use integral to find area between the function $f(x) = \frac{1}{x}$ and $x-$axis from $x = 0$ to $x = 100$. What do we call this kind of integral?

Question 4: (5pts). Find the shaded area between the curves:



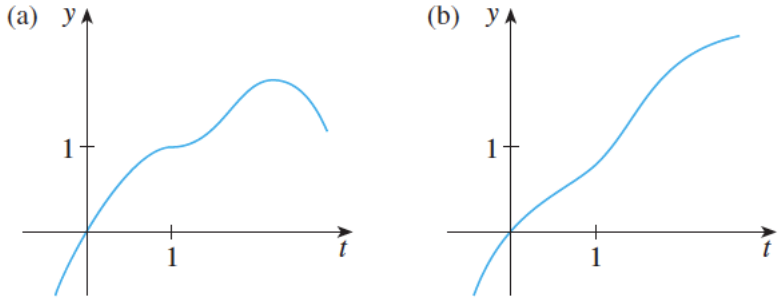
Question 5: (5pts). The integral below represents the volume of a solid. Describe the solid

$$\pi \int_0^1 \left(y^4 - y^8\right) dy$$

Question 6: (5pts). Use two different formula to find length of the line $y = ax + b$ from 0 to 1.

Question 7: (5pts). Explain why the functions with the given graphs can't be solutions of the differential equation

$$\frac{dy}{dt} = e^t(y-1)^2$$

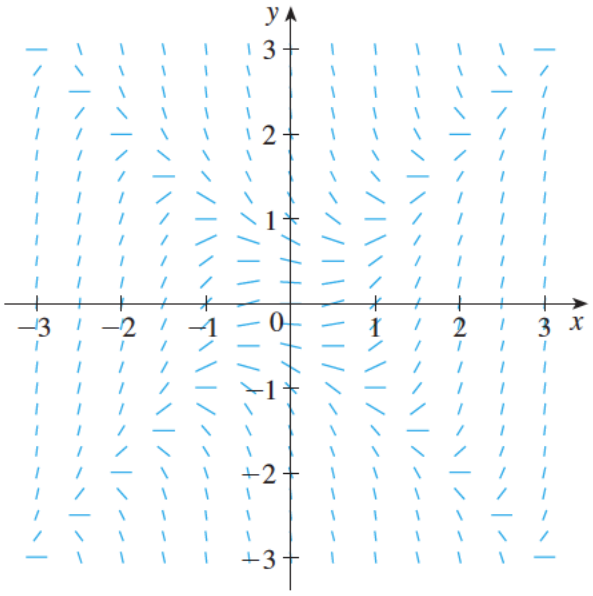


Question 8: (5pts). 1. A direction field for the differential equation $y' = x^2 - y^2$ is shown. Sketch the solution of the initial-value problem

$$y' = x^2 - y^2 \quad y(0) = 1$$

Use your graph to estimate the value of $y(0.3)$.

2. Use Euler's method with step size 0.1 to estimate $y(0.3)$, where $y(x)$ is the solution of the initial-value problem in part (1). Compare with your estimate from part (1).



Question 9: (5pts). Solve the initial-value problem

$$\frac{dr}{dt} + 2tr = r, \quad r(0) = 5$$

Question 10: (5pts). 1. Use series to evaluate the following limit:

$$\lim_{x \to 0} \frac{\sin x - x}{x^3}$$

2. The following integral is not possible to evaluated using the techniques of integration we studied in chapter 5, but its possible to evaluate it using series, evaluate the integral below:

$$\int \frac{e^x}{x} dx$$

Question 11: (5pts). Consider the series $\sum_{k=1}^{\infty}(-1)^k\frac{\ln(k)}{k}$.

1. Does this series converge? Explain.

2. Does this series converge absolutely? Explain what test you use to determine your answer.

Question 12: (5pts). Show that the function

$$f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$$

is a solution of the differential equation

$$f''(x)+f(x)=0$$