Math 182-Quest 3 April 8, 2022 Name: ______________________________

Read all directions carefully and write your answers in the space provided. To receive full credit, you must show all of your work.

Question 1: (40pts). use appropriate tests to determine the convergence or divergence of the following series. Throughout, if a series is a convergent geometric series, find its sum.

1. $\displaystyle\sum_{k=3}^{\infty} \frac{3}{\sqrt{k+1}}$

2. $\displaystyle\sum_{k=0}^{\infty} \frac{e^k}{k!!}$

3. $\displaystyle\sum_{k=0}^{\infty} \frac{2k^{2021}+1}{k^{2022}+k+1}$

4. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln(k))^2}$

Question 2: (30pts). Answer only one (second one counted as a bonus)

1. Find the partial sum S_5 of the series $\sum_{n=1}^{\infty} \frac{1}{n^6}$ and estimate the error in using it as approximation to the sum of the series.

2. Determine a value of n so that the nth partial sum S_n of the alternating series $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$ approximates the sum to within 10^{-4}.

Question 3: (20pts). Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

1. $\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots\right\}$

2. $\left\{-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, \dots\right\}$

Question 4: (20pts). Determine whether the sequence converges or diverges. If it converges, find the limit. If it diverges write state it.

1. $a_n = \frac{1+n^k}{n+3n^{k+1}}$ (Where k is positive integer)

2. $a_n = \cos\left(\frac{1}{n}\right)$

Question 5: (30pts). 1. Consider the following expression. Find the value of c.

$$\sum_{n=2}^{\infty}(1+c)^{-n} = 6$$

2. Find the sum of the series

$$\sum_{n=1}^{\infty}\frac{1}{(n+1)(n+2)}$$

Question 6: (30pts). Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

1. $\displaystyle\sum_{n=0}^{\infty} \frac{(-12)^n}{n!}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

Question 7: (15pts). Determine whether the Ratio Test is inconclusive (that is, it fails to give a definite answer), conclusive (convergent), or conclusive (divergent) for the given series.

$$\sum_{n=2}^{\infty} \frac{3}{n^3}$$

Question Bonus: (30pts). Show that if $\displaystyle\sum_{n=1}^{\infty} a_n$ absolutely converges, then $\displaystyle\sum_{n=1}^{\infty} a_n^2$ converges.

Question 2: (30pts). Question 1: (30pts). 1. Suppose $\sum a_n = 3$ and s_n is the nth partial sum of the series. What is $\lim_{n\to\infty} a_n$? What is $\lim_{n\to\infty} S_n$?

2. If $\sum c_n 6^n$ is convergent, then $\sum c_n(-2)^n$ is convergent.
3. If $\sum c_n 6^n$ is convergent, then $\sum c_n(-6)^n$ is convergent.
4. If $\{a_n\}$ and $\{b_n\}$ are divergent, then $\{a_n + b_n\}$ is divergent.
5. If $\{a_n\}$ and $\{b_n\}$ are divergent, then $\{a_n b_n\}$ is divergent.

Determine whether the sequence is convergent or divergent. If it is convergent, find its limit.

1. $a_n = \frac{2+n^3}{1+2n^3}$
2. $a_n = \frac{9^{n+1}}{10^n}$
3.

Question 3: (30pts). 9-18 Determine whether the series is convergent or divergent.

1. $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$
2. $\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$
3. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$
4. $\sum_{n=1}^{\infty} \ln\left(\frac{n}{3n+1}\right)$

Question F. ind the sum of the series: $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{2^{3n}}$