Math 182 Quiz 4 **25 Feb 2022**

1. Consider the region bound by $y = \sqrt{x+1}, x = 3, x = 6$.

   (a) Find an integral which computes the volume of the solid formed by rotating this region about the x-axis.

   (b) Find an integral which computes the volume of the solid formed by rotating this region about the y-axis.

2. Find the volume of the described solid .The base of S is the region enclosed by the parabola $y = 1 - x^2$ and the x-axis. Cross-sections perpendicular to the y-axis are squares.

1 Answers

1. Answer:

   (a) Using the washer/disk method, we obtain

   $$\int_3^6 \pi(\sqrt{x+1})^2 dx$$

   and using the shell method, we obtain

   $$\int_0^2 2\pi y(6-3)dy + \int_2^{\sqrt{7}} 2\pi y\left(6-\left(y^2-1\right)\right)dy$$

   (b) Using the washer/disk method, we obtain

   $$\int_0^2 \pi\left(6^2-3^2\right)dy + \int_2^{\sqrt{7}} \pi\left(6^2-\left(y^2-1\right)^2\right)dy$$

   and using the shell method, we obtain

   $$\int_3^6 2\pi x(\sqrt{x+1})dx$$

2. The cross-section of the base corresponding to the coordinate y has length $2x = 2\sqrt{1-y}$. $\left[y = 1-x^2 \quad \Leftrightarrow \quad x = \pm\sqrt{1-y}\right]$ The corresponding square with side s has area $A(x) = s^2 = (2\sqrt{1-y})^2 = 4(1-y)$. Therefore, $V = \int_0^1 A(y)dy = \int_0^1 4(1-y)dy = 4\left[y - \frac{1}{2}y^2\right]_0^1 = 4\left[\left(1-\frac{1}{2}\right)-0\right] = 2$