Name: _____

Math 264 Dr.Rebin Muhammad Exam 2 M#: _____

Do not use any unapproved aids while taking this assessment. Read each question carefully and be sure to show all work in the space provided.

1. Consider each of these claims about a vector equation.

   (a) "$\begin{bmatrix} -6 \\ -2 \\ -5 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} -3 \\ 0 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 0 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ -2 \\ 0 \end{bmatrix}$."

   i. Write a statement involving the solutions of a vector equation that's equivalent to this claim.

   ii. Determine if the statement you wrote is true or false.

   iii. If your statement was true, describe a linear combination of $\begin{bmatrix} -3 \\ 0 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 0 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ -2 \\ 0 \end{bmatrix}$ that equals $\begin{bmatrix} -6 \\ -2 \\ -5 \end{bmatrix}$.

   (b) "$\begin{bmatrix} -7 \\ -1 \\ -4 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} -3 \\ 0 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 0 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ -2 \\ 0 \end{bmatrix}$."

   i. Write a statement involving the solutions of a vector equation that's equivalent to this claim.

   ii. Determine if the statement you wrote is true or false.

   iii. If your statement was true, describe a linear combination of $\begin{bmatrix} -3 \\ 0 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 0 \\ -2 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ -2 \\ 0 \end{bmatrix}$ that equals $\begin{bmatrix} -7 \\ -1 \\ -4 \end{bmatrix}$.

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2. (a) Write a statement involving the solutions of a vector equation that's equivalent to each claim:

   i. "The set of vectors $\left\{ \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 15 \\ 12 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ -4 \\ 3 \end{bmatrix} \right\}$ **spans** $\mathbb{R}^4$."

   ii. "The set of vectors $\left\{ \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 15 \\ 12 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ -4 \\ 3 \end{bmatrix} \right\}$ does **not** span $\mathbb{R}^4$."

   (b) Explain how to determine which of these statements is true.

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3. Consider the following two sets of Euclidean vectors:

$$U = \left\{ \left[\begin{array}{c} x \\ y \end{array} \right] \middle| \, 6\,x + 2\,y = 0 \right\} \qquad W = \left\{ \left[\begin{array}{c} x \\ y \end{array} \right] \middle| \, 4\,x = y^3 \right\}$$

Explain why one of these sets is a subspace of $\mathbb{R}^2$ and one is not.

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4. (a) Write a statement involving the solutions of a vector equation that's equivalent to each claim:

   i. "The set of vectors $\left\{ \begin{bmatrix} -1 \\ 2 \\ -1 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} -5 \\ 3 \\ -3 \\ -2 \end{bmatrix} \right\}$ is linearly **independent**."

   ii. "The set of vectors $\left\{ \begin{bmatrix} -1 \\ 2 \\ -1 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} -5 \\ 3 \\ -3 \\ -2 \end{bmatrix} \right\}$ is linearly **dependent**."

   (b) Explain how to determine which of these statements is true.

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5. (a) Write a statement involving the solutions of a vector equation that's equivalent to each claim:

   i. "The set of vectors $\left\{ \begin{bmatrix} 1 \\ -2 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \\ 4 \end{bmatrix} \right\}$ is a **basis** for $\mathbb{R}^4$."

   ii. "The set of vectors $\left\{ \begin{bmatrix} 1 \\ -2 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \\ 4 \end{bmatrix} \right\}$ is **not** a basis for $\mathbb{R}^4$."

   (b) Explain how to determine which of these statements is true.

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6. Consider the following subspace W of $\mathbb{R}^4$:

$$W = \text{span}\left\{\begin{bmatrix} -3 \\ 1 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} -5 \\ -7 \\ -10 \\ -5 \end{bmatrix}\right\}.$$

(a) Explain and demonstrate how to find a basis of W.

(b) Explain and demonstrate how to find the dimension of W.

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7. (a) Write a statement involving the solutions to a polynomial equation that's equivalent to each claim about the following set of polynomials:

$$\left\{x^3 - x^2 + x - 1, -3\,x^3 + 3\,x^2 - 3\,x + 3, -x^2 + x - 2, -3\,x^3 + 3\,x^2 - 2\,x + 1\right\}$$

   i. "The set of polynomials **spans** $\mathcal{P}_3$. "

   ii. "The set of polynomials does **not** span $\mathcal{P}_3$. "

   (b) Explain how to determine which of these statements is true.

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8. (a) Write a statement involving the solutions to a polynomial equation that's equivalent to each claim about the following set of polynomials:

$$\left\{-x^2-2\,x+1, x^3+x^2+x, -x, -5\,x^2-4\,x+4\right\}$$

i. "The set of polynomials **spans** $\mathcal{P}_3$. "

ii. "The set of polynomials does **not** span $\mathcal{P}_3$. "

(b) Explain how to determine which of these statements is true.