

Do not use any unapproved aids while taking this assessment. Read each question carefully and be sure to show all work in the space provided.

1. Consider the following maps of polynomials $S : \mathcal{P} \rightarrow \mathcal{P}$ and $T : \mathcal{P} \rightarrow \mathcal{P}$ defined by

$$S(f(x)) = -f(x) + 2f'(x) \quad \text{and} \quad T(f(x)) = -3x - 2f(x^3)$$

Explain why one these maps is a linear transformation and why the other map is not.

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2. (a) Find the standard matrix for the linear transformation $S : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by

$$S \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} x_1 + 3x_2 + 7x_3 \\ x_1 + 4x_2 + 8x_3 \\ x_1 + x_2 + 6x_3 \end{bmatrix}.$$

- (b) Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^1$ be the linear transformation given by the standard matrix

$$[1 \ 0 \ 2 \ 1].$$

Compute $T \left(\begin{bmatrix} 2 \\ 8 \\ -2 \\ -8 \end{bmatrix} \right)$.

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3. Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be the linear transformation given by

$$T \left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \right) = \begin{bmatrix} x - 2y + 3w \\ y + z - w \\ x - 5y - 3z + 6w \end{bmatrix}.$$

- (a) Explain and demonstrate how to find the image of T and the kernel of T .
- (b) Explain and demonstrate how to find a basis of the image of T and a basis of the kernel of T .
- (c) Explain and demonstrate how to the rank and nullity of T , and why the rank-nullity theorem holds for T .

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4. Let $T : \mathbb{R}^5 \rightarrow \mathbb{R}^5$ be the linear transformation given by the standard matrix $\begin{bmatrix} -5 & 3 & -15 & -2 & -11 \\ 3 & -2 & 9 & 2 & 8 \\ 2 & -2 & 6 & 5 & 12 \\ -5 & 5 & -15 & -5 & -15 \\ 2 & -1 & 6 & 4 & 11 \end{bmatrix}$.

- (a) Explain and demonstrate why T is or is not injective.
- (b) Explain and demonstrate why T is or is not surjective.