

# HONR 105AG

## Fundamental Concepts of Inquiry in Science and Math

### The Art of Mathematical Proof

#### Instructor Information

Dr. Zhou Dong  
HT 134 Germantown Campus  
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Office hours	MWF 11:00 am – 11:50 am
Appointment hours	TR 12:30 pm – 1:30 pm
<a href="#">Click here to book an appointment</a>	

#### Course Information

HONR 105AG  
Fall 2024 CRN 23333  
Thursdays 2-3 pm HT401

#### Catalog Description:

Selected themes and topics in the natural sciences and mathematics will be used to help students develop a better understanding of the concepts, terminology, and methodology of the study of natural sciences and mathematics. Students may take this course twice to fulfill the requirements of the Honors Scholar Program, provided each time it is taken, a different topic is covered. Specific information about each section of this course will be published prior to the start of each registration and may be obtained from the campus honors coordinator.

*1 semester hour*

#### Course Outcomes:

Upon course completion, a student will be able to:

- Demonstrate an understanding of the concepts, terminology, and methodologies associated with proof-writing in mathematics.
- Explain the importance and limitations of proofs in the field of mathematics.
- Explain the differences and similarities in approach to art and mathematics.
- Explain the differences and similarities in approach to science and mathematics.
- Communicate proof concepts effectively in both oral and written presentations.

### Course Topic:

We will explore the idea of mathematical proof. What is a mathematical proof and why are they necessary? Can a mathematical proof be beautiful? Beginning with Aristotle's ideas of logical deduction, we will study proofs by Euclid, Euler, Cantor, and others. We will also discuss what cannot be proven mathematically as proved by Gödel.

### Prerequisites:

Students must have completed 12 MC credits, including an A or B in ENGL 101, with a minimum GPA of 3.4. Students must also be eligible for college-level math; this means eligibility for any MATH course at the 100 level or above without support (e.g. MATH 117 without MATH 017, MATH 120 without MATH 020, or MATH 165).

### Texts:

- Dunham, W. (1990). *Journey through genius: The great theorems of mathematics*. New York: Wiley. (You will receive a copy of this book to use for the semester, courtesy of the math department)
- Hamkins, J. (2020). *Proof and the Art of Mathematics*. MIT Press. (The MC Library has this ebook with unlimited access)
- Hamkins, J. (2021). *Proof and the Art of Mathematics: Examples and Extensions*. MIT Press. (The MC Library has this ebook with unlimited access)

### Attendance:

Class meets every Thursday, 2-3 pm in HT 401, Germantown campus.

You are expected to be at every class meeting. Attendance is critical to doing well in this course. Missing class will have a devastating effect on your ability to comprehend the course material and enjoyment of the class, so please let me know as soon as possible if you are going to miss class.

### Communication:

I will use your MC email and Microsoft Teams to communicate with you. All class materials and assignments will be on our class Team: [The Art of Mathematical Proof Fall 2024](#).

### Assignments:

- Weekly reading assignments – necessary for class participation
- Weekly proof writing assignments
- Weekly guided journal entries
- Weekly online discussions
- Final Exam: Proof presentation
- Comparison paper: (5-8 pages) – compare a piece of art to a mathematical proof.

### Grading:

Class Participation	10%
Writing Proofs	15%
Journal Entries	15%
Discussion Board	10%

Proof Presentation	25%
Comparison Paper	25%
<b>Total</b>	<b>100%</b>

90% – 100%	A
80% – 89%	B
70% – 79%	C

60% – 69%	D
Below 60%	F

**Grading rubric for class participation: 0 – 2 points per week.**

2 points	Came to class prepared and actively participated in class discussion
1 point	Came to class but not fully prepared or did not participate in class discussion
0 points	Missed class

There are 14 class meetings. You will be graded out of 25 total participation points. This means you can earn 3 “bonus” points if you came to all 14 class meetings prepared and actively participated.

**Grading rubric for proof problems: 0 – 2 points per week.**

2 points	Good faith effort on the proof and submitted on time
1 point	Incomplete proof or submitted one week late
0 points	Nothing submitted

There are 14 proof assignments. You will be graded out of 25 total participation points. This means you can earn 3 “bonus” points if you submitted all 14 problems with good faith efforts on time.

**Grading rubric for journal entries: 0 – 2 points per week.**

2	Thoughtful and thorough response, submitted on-time.
1	Thoughtful and thorough response but submitted one week late. Or Incomplete response submitted on-time.
0	Response not submitted within two weeks or incomplete response submitted late.

There are 14 journal entries. You will be graded out of 25 total journal points. This means you can earn 3 “bonus” points if you submit 14 well-thought entries on time.

**Grading rubric for discussion board: 0-2 points per week.**

2	Made at least 2 quality posts and response to the discussion board.
1	Made only 1 quality posts or response to the discussion board.
0	Made no quality posts or responses.

There are 14 weeks. You will be graded out of 25 total discussion board points. This means you can earn 3 “bonus” points if you earn 2 points for all 14 weeks.

**Grading rubric for comparison paper: 16 points total**

Category \ Points	4	3	2	1
<b>Purpose &amp; Supporting details</b>	The paper compares and contrasts items clearly. The paper points to specific examples to illustrate the comparison. The	The paper compares and contrasts items clearly, but the supporting information is general. The paper includes	The paper compares and contrasts items clearly, but the supporting information is incomplete. The paper may include	The paper compares or contrasts, but does not include both. There is no supporting information or

	paper includes only the information relevant to the comparison.	only the information relevant to the comparison.	information that is not relevant to the comparison.	support is incomplete.
<b>Organization &amp; Structure</b>	The paper breaks the information into whole-to-whole, similarities-to-differences, or point-by-point structure. It follows a consistent order when discussing the comparison.	The paper breaks the information into whole-to-whole, similarities-to-differences, or point-by-point structure but does not follow a consistent order when discussing the comparison.	The paper breaks the information into whole-to-whole, similarities - to-differences, or point-by-point structure, but some information is in the wrong section. Some details are not in a logical or expected order, and this distracts the reader.	Many details are not in a logical or expected order. There is little sense that the writing is organized.
<b>Transitions</b>	The paper moves smoothly from one idea to the next. The paper uses comparison and contrast transition words to show relationships between ideas. The paper uses a variety of sentence structures and transitions.	The paper moves from one idea to the next, but there is little variety. The paper uses comparison and contrast transition words to show relationships between ideas.	Some transitions work well; but connections between other ideas are fuzzy.	The transitions between ideas are unclear or nonexistent.
<b>Grammar &amp; Spelling (Conventions)</b>	Writer makes no errors in grammar or spelling that distract the reader from the content.	Writer makes 1-2 errors in grammar or spelling that distract the reader from the content.	Writer makes 3-4 errors in grammar or spelling that distract the reader from the content.	Writer makes more than 4 errors in grammar or spelling that distract the reader from the content.

[Rubric taken from ReadWriteThink.org]

**Grading rubric for proof presentation: 12 points total**

<b>Category \ Points</b>	<b>4</b>	<b>3</b>	<b>2</b>	<b>1</b>
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<b>Delivery</b>	<ul style="list-style-type: none"> <li>• Holds attention of entire audience with the use of direct eye contact, seldom looking at notes</li> <li>• Speaks with fluctuation in volume and inflection to maintain audience interest and emphasize key points</li> </ul>	<ul style="list-style-type: none"> <li>• Consistent use of direct eye contact with audience, but still returns to notes</li> <li>• Speaks with satisfactory variation of volume and inflection</li> </ul>	<ul style="list-style-type: none"> <li>• Displays minimal eye contact with audience, while reading mostly from the notes</li> <li>• Speaks in uneven volume with little or no inflection</li> </ul>	<ul style="list-style-type: none"> <li>• Holds no eye contact with audience, as entire report is read from notes</li> <li>• Speaks in low volume and/or monotonous tone, which causes audience to disengage</li> </ul>
<b>Content / Organization</b>	<ul style="list-style-type: none"> <li>• Demonstrates full knowledge by answering all class questions with explanations and elaboration</li> <li>• Clearly presents completed proof with logical flow and full discussion of the creative process</li> </ul>	<ul style="list-style-type: none"> <li>• Is at ease with expected answers to all questions, without elaboration</li> <li>• Presents completed proof and some discussion of the creative process</li> </ul>	<ul style="list-style-type: none"> <li>• Is uncomfortable with information and is able to answer only rudimentary questions</li> <li>• Presents partial proof and some discussion of the creative process</li> </ul>	<ul style="list-style-type: none"> <li>• Does not have grasp of information and cannot answer questions about subject</li> <li>• Does not have logical flow or missing discussion of creative process</li> </ul>
<b>Enthusiasm / Audience Awareness</b>	<ul style="list-style-type: none"> <li>• Demonstrates strong enthusiasm about topic during entire presentation</li> <li>• Significantly increases audience understanding and knowledge of topic; convinces an audience to recognize the validity and importance of the subject</li> </ul>	<ul style="list-style-type: none"> <li>• Shows some enthusiastic feelings about topic</li> <li>• Raises audience understanding and awareness of most points</li> </ul>	<ul style="list-style-type: none"> <li>• Shows little or mixed feelings about the topic being presented</li> <li>• Raises audience understanding and knowledge of some points</li> </ul>	<ul style="list-style-type: none"> <li>• Shows no interest in topic presented</li> <li>• Fails to increase audience understanding of knowledge of topic</li> </ul>

[Rubric taken and adapted from ReadWriteThink.org]

### Academic Dishonesty:

All students are expected to achieve their goals with academic honor. Cheating, plagiarism, and/or other forms of academic dishonesty or misconduct, examples of which can be found in The Student

Code of Conduct, are not tolerated. Students who engage in any act of academic dishonesty or misconduct are subject to sanctions ranging from an F on the assignment to failing the course. Each student should read and comply with the policies and procedures in The Student Code of Conduct.

Note on collaboration: students are encouraged to discuss ideas with each other and even share their work with one another for feedback, especially on written proofs. However, you should never take someone else's idea or copy another person's work and turn it in as your own.

### Late/Make-up Work:

Since I expect to see you every class period, there should be little need for late or make-up work. Journal entries may be submitted the week after its original due date. Weekly discussion board posts cannot be made-up.

### Classroom Conduct:

To do well in this course you should do each reading assignment and be on time for every class. Be prepared to ask questions, take notes and participate in classroom activities. In other words, be actively involved in the learning process. Of course, I will expect all students to comply with the behaviors outlined in The Student Code of Conduct.

### Disability:

Any student who may need an accommodation due to a disability, please make an appointment to see me during my office hour. A letter from Disability Support Services authorizing your accommodations will be needed.

### Tentative Course Outline/Schedule:

Week	Topic, reading assignments, and proof assignments
1 9/5	Math, Art, and Science <ul style="list-style-type: none"> <li>• A Mathematician's Lament <a href="https://www.maa.org/sites/default/files/pdf/devlin/LockhartsLament.pdf">https://www.maa.org/sites/default/files/pdf/devlin/LockhartsLament.pdf</a></li> <li>• The Unreasonable Effectiveness of Mathematics in the Natural Sciences <a href="https://www.maths.ed.ac.uk/~v1ranick/papers/wigner.pdf">https://www.maths.ed.ac.uk/~v1ranick/papers/wigner.pdf</a></li> <li>• Why we want proof <a href="https://plus.maths.org/content/brief-introduction-proofs">https://plus.maths.org/content/brief-introduction-proofs</a></li> </ul> Proof Assignment – geometry puzzle
2 9/12	The Aesthetics of Mathematical Proof <ul style="list-style-type: none"> <li>• The Universal Aesthetics of Mathematics <a href="https://montgomerycollege.primo.exlibrisgroup.com/permalink/01MONTGOMERY_IN/ST/1tg2j/cdi_proquest_journals_2158402193">https://montgomerycollege.primo.exlibrisgroup.com/permalink/01MONTGOMERY_IN/ST/1tg2j/cdi_proquest_journals_2158402193</a></li> <li>• Dunham: Preface</li> <li>• Hamkins: Preface</li> <li>• Hamkins: A Note to the Student</li> <li>• Hamkins: Index of Mathematical Habits</li> </ul> Proof Assignment – Mathematical Habits applied to geometry puzzle
3 9/19	Euclid (1 of 2) <ul style="list-style-type: none"> <li>• The origins of proof <a href="https://plus.maths.org/content/origins-proof">https://plus.maths.org/content/origins-proof</a></li> </ul>

	<ul style="list-style-type: none"> <li>Dunham: Chapter 2. Euclid's Proof of the Pythagorean Theorem (ca. 300 B.C.)</li> <li>Seeing Pythagoras <a href="https://plus.maths.org/content/seeing-pythagoras">https://plus.maths.org/content/seeing-pythagoras</a></li> <li>Painting - Proof of the Pythagorean Theorem (Euclid) <a href="https://americanhistory.si.edu/collections/search/object/nmah_694620">https://americanhistory.si.edu/collections/search/object/nmah_694620</a></li> </ul> <p>Proof Assignment – Hamkins Exercise 6.2</p>
4 9/26	<p>Euclid (2 of 2)</p> <ul style="list-style-type: none"> <li>Dunham: Chapter 3. Euclid and the Infinitude of Primes (ca. 300 B.C.)</li> <li>Hamkins: 3.5 Infinitely many primes</li> <li>Maths in a minute: How many primes? <a href="https://plus.maths.org/content/math-minute-how-many-primes">https://plus.maths.org/content/math-minute-how-many-primes</a></li> <li>Poem: Stopping by Euclid's Proof of the Infinitude of Primes <a href="https://montgomerycollege.primo.exlibrisgroup.com/permalink/01MONTGOMERY_IN ST/1tg2j/cdi_crossref_primary_10_1080_0025570X_2005_11953322">https://montgomerycollege.primo.exlibrisgroup.com/permalink/01MONTGOMERY_IN ST/1tg2j/cdi_crossref_primary_10_1080_0025570X_2005_11953322</a></li> </ul> <p>Proof Assignment – Hamkins Exercise 3.11</p>
5 10/3	<p>Archimedes</p> <ul style="list-style-type: none"> <li>Dunham: Chapter 4. Archimedes' Determination of Circular Area (ca. 225 B.C.)</li> <li>Hamkins: 6.10 Area of a circle</li> <li>What is the area of a circle? <a href="https://plus.maths.org/content/os/issue43/features/korner/index">https://plus.maths.org/content/os/issue43/features/korner/index</a></li> </ul> <p>Proof Assignment – Hamkins Exercise 6.7</p>
6 10/10	<p>Calculus (1 of 2)</p> <ul style="list-style-type: none"> <li>Dunham: Chapter 7. A Gem from Isaac Newton (Late 1660s)</li> <li>Hamkins: 6.2 Binomial Square</li> <li>Painting - Fluxions (Newton) <a href="https://americanhistory.si.edu/collections/search/object/nmah_694638">https://americanhistory.si.edu/collections/search/object/nmah_694638</a></li> <li>The Pi Day Recipe Book <a href="https://mathwithbaddrawings.com/2016/03/14/the-pi-day-recipe-book/">https://mathwithbaddrawings.com/2016/03/14/the-pi-day-recipe-book/</a></li> </ul> <p>Proof Assignment – Hamkins Exercise 6.2</p>
7 10/17	<p>Calculus (2 of 2)</p> <ul style="list-style-type: none"> <li>Dunham: Chapter 8. The Bernoullis and the Harmonic Series (1689)</li> <li>Outer space: The rule of two <a href="https://plus.maths.org/content/outer-space-series">https://plus.maths.org/content/outer-space-series</a></li> <li>Painting - Harmonic Series from a Quadrilateral (Pappus) <a href="https://americanhistory.si.edu/collections/search/object/nmah_694642">https://americanhistory.si.edu/collections/search/object/nmah_694642</a></li> </ul> <p>Proof Assignment – Harmonic Series Proof</p>
8 10/24	<p>Euler (1 of 3)</p> <ul style="list-style-type: none"> <li>Euler's polyhedron formula <a href="https://plus.maths.org/content/eulers-polyhedron-formula">https://plus.maths.org/content/eulers-polyhedron-formula</a></li> <li>Painting - Polyhedron Formula (Euler) <a href="https://americanhistory.si.edu/collections/search/object/nmah_694651">https://americanhistory.si.edu/collections/search/object/nmah_694651</a></li> <li>Hamkins: Chapter 8 Pick's Theorem</li> </ul> <p>Proof Assignment – Hamkins Exercise 8.3</p>
9 10/31	<p>Euler (2 of 3)</p> <ul style="list-style-type: none"> <li>Dunham Chapter 9. The Extraordinary Sums of Leonhard Euler (1734)</li> </ul> <p>Proof Assignment – TBA</p>
10	<p>Euler (3 of 3)</p>

11/7	<ul style="list-style-type: none"> <li>• Dunham Chapter 10. A Sampler of Euler's Number Theory (1736)</li> <li>• Hutchings: Introduction to Mathematical Arguments</li> </ul> Proof Assignment – Hutchings Exercise 1
11 11/14	Cantor (1 of 2) <ul style="list-style-type: none"> <li>• Dunham Chapter 11. The Non-Denumerability of the Continuum (1874)</li> <li>• A glimpse of Cantor's paradise <a href="https://plus.maths.org/content/os/issue47/features/macgregor/index">https://plus.maths.org/content/os/issue47/features/macgregor/index</a></li> </ul> Proof Assignment – TBA
12 11/21	Cantor (2 of 2) <ul style="list-style-type: none"> <li>• Dunham Chapter 12. Cantor and the Transfinite Realm (1891)</li> <li>• Cantor and Cohen: Infinite investigators part I <a href="https://plus.maths.org/content/os/issue47/features/elwes1/index">https://plus.maths.org/content/os/issue47/features/elwes1/index</a></li> <li>• Cantor and Cohen: Infinite investigators part II <a href="https://plus.maths.org/content/os/issue47/features/elwes2/index">https://plus.maths.org/content/os/issue47/features/elwes2/index</a></li> </ul> Proof Assignment – Paradox
13 11/28	Thanksgiving Break NO CLASS
14 11/5	Limits of Mathematical Proof (1 of 2) <ul style="list-style-type: none"> <li>• This is not a carrot: Paraconsistent mathematics <a href="https://plus.maths.org/content/not-carrot">https://plus.maths.org/content/not-carrot</a></li> <li>• Gödel and the limits of logic <a href="https://plus.maths.org/issue39/features/dawson/index.html">https://plus.maths.org/issue39/features/dawson/index.html</a></li> <li>• Searching for the missing truth <a href="https://plus.maths.org/content/searching-missing-truth">https://plus.maths.org/content/searching-missing-truth</a></li> <li>• Picking holes in mathematics <a href="https://plus.maths.org/content/picking-holes-mathematics">https://plus.maths.org/content/picking-holes-mathematics</a></li> <li>• Hamkins: Chapter 4 Mathematical Induction</li> </ul> Proof Assignment – Hamkins Exercise 4.1
15 12/12	Limits of Mathematical Proof (2 of 2) <ul style="list-style-type: none"> <li>• We must know, we will know <a href="https://plus.maths.org/issue41/features/morris/index.html">https://plus.maths.org/issue41/features/morris/index.html</a></li> <li>• What computers can't do <a href="https://plus.maths.org/issue5/turing/index.html">https://plus.maths.org/issue5/turing/index.html</a></li> <li>• The origins of proof III: Proof and puzzles through the ages <a href="https://plus.maths.org/content/origins-proof-iii-proof-and-puzzles-through-ages">https://plus.maths.org/content/origins-proof-iii-proof-and-puzzles-through-ages</a></li> <li>• The future of proof <a href="https://plus.maths.org/content/future-proof">https://plus.maths.org/content/future-proof</a></li> </ul> Proof Assignment – Improve a proof
16 12/19	Proof Presentation