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Road to Calculus: The Work of Pierre de Fermat

On December 1, 1955 Rosa Parks boarded a Montgomery, Alabama city bus and refused to give up her seat for a white passenger. Her arrest and the boycott that followed marked a pivotal turning point in the Civil Rights Movement, but her action did not come out of nowhere. For the 12 years leading up to the boycott, Parks worked for social justice at her local NAACP chapter. In fact, she was carefully chosen by the organization as a catalyst for the movement. The illusion that her demonstration stemmed from a spontaneous idea in her head that day was largely created by the media who were able to generate more attention through novelty, rather than describing the complexity of the Civil Rights Movement. So what does this have to do with the history of calculus? Few names in calculus come to mind more than those of Gottfried Leibnitz and Isaac Newton. The greatly popularized debate in the history of calculus is whether it was Leibnitz or Newton who should be credited with its development. In fact, the progression of mathematics up until that point was so ripe for the development of calculus that it is not difficult to see how they could have been simultaneously developed. Evidence of this can be observed through the work of a far lesser known man by the name Pierre de Fermat.

As many scholars of his day, Fermat was born into a family of wealth and privilege, allowing him the opportunity to pursue a lifetime of scholarly research. Born August 20, 1601 Fermat was the son of a leather merchant in a small town in southern France, Beaumont-de-Lomagne. Little is known of his early education, although according to archives he received his primary and secondary education at the monastery of Grandselve. He later matriculated into the University of Toulouse. According to historical record, the wealth and status of Fermat's family had been gaining steadily. The natural professional course for a young man of his standing was to pursue a legal career. Fermat's study of law took him to the University of Law at Orleans, though his true calling seemed to be mathematics.

As a man of wealth Fermat was able to dedicate extensive time to his study of mathematics without being burdened by the shackles of the proletariat. During that time there was little to be gained in the way of social status for these efforts since mathematics was still not considered a professional discipline. However, Fermat made mathematics and its study into a lifetime challenge and excelled at it far more than he did in the field of law. A major contribution in his extensive work with mathematics was his development of analytic geometry.

As the invention of calculus is credited to both Newton and Leibnitz, Fermat and fellow scholar Rene Descartes are both recognized with the concurrent development of analytic geometry (although Descartes is credited with its invention and the Cartesian plane is named for him). Fermat lacked necessary elements for his system of Analytic Geometry until as late as 1635, while the publication by Descartes of *Rules of the Direction of the Mind* suggest his work laid the foundations for analytic geometry in the late 1620's. Fermat's work extended to his *Introduction to Plane and Solid Loci* which asserts that in every indeterminate algebraic expression of two unknowns a uniquely determined geometric curve exists called a locus. Loci or functions as we know them today are the backbone of calculus. While Newton and Leibnitz used the idea of the function extensively in their work, Fermat set the foundation. To further understand the behavior of functions, Fermat set out to develop a method of determining maxima and minima.

One of the most controversial times in Fermat's career was following the publication of his paper, *Method for Determining Maxima and Minima and Tangents to Curved Lines.* Spearheaded by Descartes, the controversy had all of Paris' mathematical society in an uproar accusing Fermat of stumbling upon his results through trial and error and lacking the scientific rigor of Descartes' works. Fermat proposed the determination of maxima and minima by observing the point at which a tangent line to a curve was parallel to the *x*-axis. His method is very close to the differential calculus method used today which determines maxima and minima by calculating the point at which the derivative of a function (slope of a tangent line) is zero. Although it lacked the clarity of proofs and substantiated documentation upon its release, the key concepts of its content were still used as a basis for subsequent research into maxima, minima, and tangents to curves. During the debate of his method's validity to determine maxima and minima, Fermat was inspired to write the *Analytic Investigation of the Maxima and Minima* to both detail and expand his method of tangents.

According to Michael S. Mahoney in *The Mathematical Career of Pierre de Fermat, 1601-1665*, "there is for the method of tangents no text like the *Analytic Investigation of Maxima and Minima*, which dispels the shadows surrounding its genesis and original foundations." Fermat's application of tangent lines to a curve and his insight on maxima and minima are obvious precursors to central concepts in calculus. Through expansive notes and memoirs, Fermat detailed these basic building blocks of calculus in precise detail. Even Isaac Newton credited Fermat's work in these areas stating that his ideas about calculus came directly from, "Fermat's way of drawing tangents" (Mahoney). It was these ideas that led to the infinite number of curves that could be defined by indeterminate equations in two unknowns. New realms of thought were opening for Fermat. He moved his focus from that of tangent lines to finding the area under a curve and began to expand what the Greeks had started with, the process of quadrature.

The ancient Greeks had experimented with quadrature of bounded geometric shapes, but now Fermat moved forward trying to calculate the area under such unbounded curves. Mahoney observes that, "in his *Treatise of Quadrature,* Fermat grouped together all of his mathematical forces- his analytic geometry, his method of maxima and minima, his method of tangents, and his direct quadrature of higher parabolas and hyperbolas- to construct a brilliant 'reduction analysis' for the quadrature of curves." With his treatise, Fermat intended to transform and alter the indeterminate equations of curves to determine their quadrability. Though his work on finding the area under curves is substantial, it was tedious and his writings were often incomprehensible to his colleagues. Mahoney states that, "characteristically, Fermat abbreviates where contemporaries and posterity cry for detail." Although Fermat's methods in calculating and conveying his findings in respect to quadrature are not fully understood, they foreshadow the later work of Leibnitz and Newton.

Fermat was within the grasp of developing what we now know as calculus, but his reasoning would never bring him past the brink. After showing how to determine the tangent to any given curve, Fermat notes in passing, "finally one could seek the converse of the proposition and, being given the property of the tangent, seek the curve that this property fits." Here Fermat is discussing the inverse to the idea of the tangent lines, his method of quadrature which we know today as analogous to the integral. However, Mahoney goes on to say that:

> The thought never seems to occur to Fermat. The method of tangents serves as a tool for the method of quadrature, but nowhere in his writings does Fermat even hint that the two methods may be inversely related. No other aspect of his work so clearly denies him the right to usurp the honor accorded to Leibnitz and Newton for the invention of the calculus.

The closeness to developing calculus may be better understood by the fact that Fermat seemed to be asking the wrong questions. Perhaps he failed to recognize the relationship between the integral and the derivative because he did not search for a greater application like Newton did for physics. Fermat's objectives were simply to measure the distance of the tangent line between the x-axis and the point of tangency and separately to calculate the area under a curve, two aims critically linked by numerous applications in physics and central to the work of Newton and Leibnitz.

Pierre de Fermat made tremendous contributions to mathematics, including those that would influence both Newton and Leibnitz in their development of infinitesimal calculus. His work, as many mathematicians before him, both grew upon an existing body of knowledge and advanced the future development of calculus.  Breakthroughs are often remembered through the heroic symbols they evoke, as the example of Rosa Parks. However, in crediting these accomplishments to a single inventor, one forfeits a greater wisdom that can be gained only in understanding the whole context of the discovery.

## Bibliography

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